ANALYTICAL COMPUTATION METHOD FOR ELECTRODYNAMIC FORCES ACTING OVER ELECTRICAL TRANSFORMER WINDINGS

Ioan-Dragoş DEACONU[1], Aurel-Ionuţ CHIRILĂ[2], Constantin GHIŢĂ[3], Valentin NĂVRĂPESCU[4], Gianfranco CHICCO[5]

În prezent există mai multe metode de calcul a forţelor electrodinamice de scurtcircuit din înfăşurările transformatoarelor electrice trifazate. Aceste metode fie sunt foarte precise (de exemplu metoda elementului finit) dar necesită un timp îndelungat de calcul fie permit un calcul rapid (de exemplu metoda clasică de calcul) însă furnizează rezultate aproximative. Lucrarea de faţă prezintă o metodă rapidă de calcul a acestor forţe, avantajul acesteia fiind că permite obţinerea unor rezultate precise. Pentru a demonstra eficienţa metodei a fost realizată o comparaţie între rezultatele obţinute cu ajutorul acesteia şi rezultatele obţinute cu ajutorul principalelor metode de calcul utilizate în prezent.

Different methods are used at present to compute the short-circuit electrodynamic forces exerted over the windings of three-phase electrical transformers. Methods giving quite accurate results (e.g., the finite element method) require significant computing resources and computation time. Classical analytical methods are fast but not so accurate. A fast computation method with accurate results is presented in this paper. Comparisons between the results given by this method and by other well-known methods applied to a three-phase transformer is performed in order to show and discuss the effectiveness of the method.

Keywords: electrical transformer, short-circuit electrodynamic forces, analytic method, finite element method

1. Introduction

The forces that stress the windings during a short-circuit are the most common cause for the destruction of power electrical transformers [1–2]. For this reason, the computation of these forces is becoming an important step in the

---

[1] Teach. Assis., Electrical Engineering Faculty., Universitatea POLITEHNICA Bucureşti, Romania, dragos.deaconu@gmail.com
[2] Teach. Assis., Electrical Engineering Faculty., Universitatea POLITEHNICA Bucureşti, Romania
[3] Prof., Electrical Engineering Faculty, Universitatea POLITEHNICA Bucureşti, Romania
[4] Prof., Electrical Engineering Faculty, Universitatea POLITEHNICA Bucureşti, Romania
[5] Prof., Department of Electrical Engineering, Politecnico di Torino, Italy

transformer's windings design including the fixing mechanical parts of the windings [3–4].

At present the transformers designers are using different methods in order to predetermine the short-circuit forces. The most common are the classical method and numerical methods.

There are various numerical methods that compute the transformer's leakage magnetic field (e.g. finite differences, finite elements, variational methods). Solving magnetic field problems with such methods can be found in [5], [6] for 2D and [7], [8] for 3D. Based on this field, different quantities are derived, such as leakage reactance [9–10], transformer's equivalent circuits [11], core's saturation and non-symmetrical loads influences over the leakage field [12], stray losses in different solid metallic parts of the transformer (tank, clamping plates, frame) [13] and of course short-circuit electrodynamic forces [3].

The classical method to estimate the short-circuit forces [14] is often used by the transformers designers/manufacturers due to its calculation simplicity. Unfortunately, this method provides only the global approximated value of the forces without any stress distribution along the windings.

This paper presents an analytical method to compute the distribution along the windings of the short-circuit electrodynamic forces (axial and radial) acting over the windings of a three-phase power electrical transformer. The method is based on the leakage magnetic field inside the transformer's window calculation. More exactly, the Poisson equation for the leakage magnetic field's vector-potential is integrated using the Green's function method [15–18].

The leakage magnetic field inside the window has been determined for the first time by E. Roth [19]. The two expressions found by Roth have poor convergence because they are expressed using double Fourier-series, Fourier-Bessel, respectively.

The calculations proved that for axial component of magnetic flux density both expressions give the same result. In case of radial component the results are similar only in cases with windings of equal magnetomotive forces. When the magnetomotive forces are not balanced, supposing a plane-parallel problem, the values obtained are greater than the measured ones [20].

The method presented in the paper is based on analytical expressions (in order to determine the transformers leakage magnetic field) containing simple Fourier-series that exhibit fast convergence.

2. Computation of the leakage magnetic field inside the window of a three-phase transformer

The leakage magnetic field inside the window is computed supposing the following simplifying hypotheses:

        - winding's curvature is neglected – magnetic field is independent over the z coordinate, thus a plane-parallel problem can be considered;
        - displacement currents between the coils turns and the eddy currents of the ferromagnetic core are neglected;
        - the core's magnetic permeability is supposed to have a high value ($\cong 10^6$);
        If the vector-potential of the leakage magnetic field is determined, then the magnetic flux density can be obtained using the following expression:

$$B(x, y) = rotA(x, y) \tag{1}$$

where $A(x, y)$ represents the vector-potential. This potential is computed by integrating the following linear constant coefficients partial derivatives equation over the window's surface Σ with Neumann defined conditions on the boundary Γ of the surface Σ :

$$\Delta A(x, y) = -\mu_0 \cdot J(x, y) \tag{2}$$

where $J(x, y)$ represents the current density. The integration of (2) is performed using the three-potentials formula, giving the vector-potential:

$$A(x, y) = \int_\Sigma J(x_0, y_0) \cdot G(x, y; x_0, y_0) \cdot dx_0 \cdot dy_0 +$$
$$\frac{1}{\mu_0} \cdot \int_\Gamma G(x, y; x_0, y_0) \cdot \frac{\partial A(x_0, y_0)}{\partial n} \cdot ds_0 + A_0 \tag{3}$$

where $G(x, y; x_0, y_0)$ represents the Green's function corresponding to the Laplace operator and Neumann boundary conditions, and A_0 is an arbitrary constant (for simplicity it is usually considered to be null).
        The Green's function is defined by the following relations:

$$\Delta G(x, y; x_0, y_0) = -\mu_0 \cdot \delta(x - x_0) \cdot \delta(y - y_0) \tag{4}$$

$$\partial G(x, y; x_0, y_0)/\partial n = -\mu_0/l_\Gamma \tag{5}$$

where $\delta(x - x_0) \cdot \delta(y - y_0)$ is called bi-dimensional Dirac's function for cartesian coordinates and l_Γ is the length of the curve Γ (see Fig. 1).
        The resulting leakage magnetic field inside the window $B(x, y)$ can be considered to be generated by the superimposing of two fields. One is defined in

the hypothesis of equal magnetomotive forces windings, denoted with $B'(x,y)$, thus neglecting the magnetizing magnetomotive force. The other one is defined in the hypothesis of the solely existing magnetizing magnetomotive force, and it is denoted with $B^*(x,y)$. In consequence:

$$B(x,y) = B'(x,y) + B^*(x,y) \tag{6}$$

The solution of (4) with boundary conditions (5), when balanced magnetomotive forces are considered, it is formed by two different expressions [21]. Each expression is valid for one of the two dividing sub-domains of surface Σ. The division is defined by the line $x = x_0$ (see Fig. 1) and for the Green's function expression this division is reflected only as a simple sum instead of a double one as it would have been in the case where no division was considered.
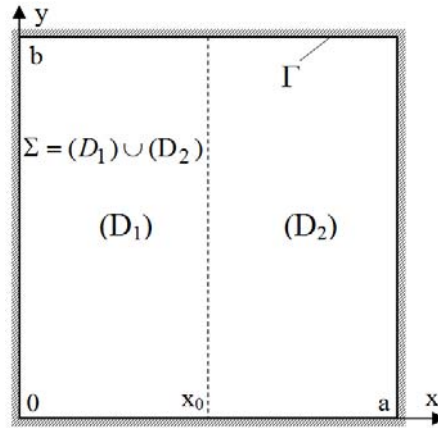

Fig. 1. Transformer's window (computation domain) split into sub-domains.

In order to compute the leakage magnetic field, a transformer with a different number of coils is considered for both high voltage (HV) and low voltage (LV) windings. Surface Σ is divided into five sub-domains as it is presented in Fig. 2. LV coils have the same radial dimension $a'_1 - a_1$ and different heights $b'_{1j} - b_{1j}$; HV coils have also the same radial dimension $a'_2 - a_2$ and different heights $b'_{2j} - b_{2j}$, where j is the order of the coil. For simplicity it is considered the same number of coils for both windings, denoted with l.

In consequence, the leakage magnetic field vector-potential will have five different expressions corresponding to each of the five sub-domains. The same is for magnetic flux density (whose expressions are resulting from (1)).
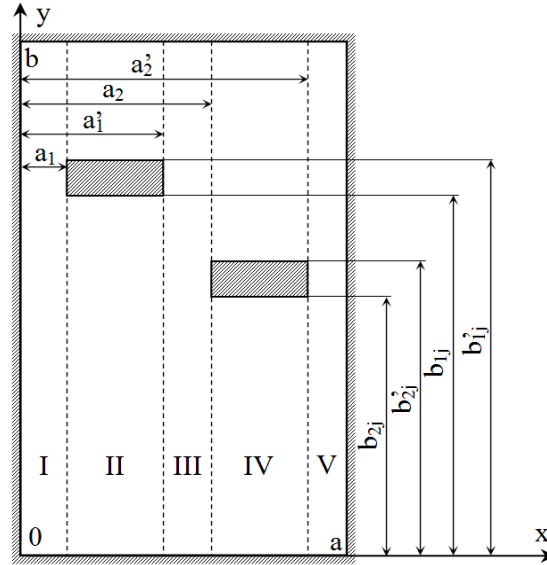

Fig. 2. Coils of order j, belonging to the two windings of the transformer.

The case of plane-parallel field, based on (1) and (3), yields that for each point belonging to the rectangular boundary Γ of surface Σ (see Fig. 1), the quotient between the normal derivative of the leakage magnetic fields vector-potential for that point and the void's magnetic permeability is equal to the tangential magnetic field, i.e. tangential component of the leakage magnetic field to boundary Γ.

Neglecting the magnetizing magnetomotive force causes the canceling of the tangential magnetic field to boundary Γ, and so (3) becomes:

$$A^{'}(x,y) = \int_{\Sigma} J(x_0, y_0) \cdot G(x, y; x_0, y_0) \cdot dx_0 \cdot dy_0 \tag{3'}$$

Equation (3') will have five different expressions, corresponding to the five sub-domains:

$$A_{I}^{'}(x,y) = \sum_{j=1}^{l} \left(\int_{a_1}^{a_1^{'}} \int_{b_{1j}}^{b_{1j}^{'}} J_{1j} \cdot G_1 dx_0 dy_0 + \int_{a_2}^{a_2^{'}} \int_{b_{2j}}^{b_{2j}^{'}} J_{2j} \cdot G_1 dx_0 dy_0 \right)$$

$$A'_{II}(x,y) = \sum_{j=1}^{l}\left(\int_{a_1}^{x}\int_{b_{1j}}^{b'_{1j}} J_{1j}\cdot G_2\,\mathrm{d}x_0\mathrm{d}y_0 + \int_{x}^{a'_1}\int_{b_{1j}}^{b'_{1j}} J_{1j}\cdot G_1\,\mathrm{d}x_0\mathrm{d}y_0 + \right.$$

$$\left. + \int_{a_2}^{a'_2}\int_{b_{2j}}^{b'_{2j}} J_{2j}\cdot G_1\,\mathrm{d}x_0\mathrm{d}y_0 \right)$$

$$A'_{III}(x,y) = \sum_{j=1}^{l}\left(\int_{a_1}^{a'_1}\int_{b_{1j}}^{b'_{1j}} J_{1j}\cdot G_2\,\mathrm{d}x_0\mathrm{d}y_0 + \int_{a_2}^{a'_2}\int_{b_{2j}}^{b'_{2j}} J_{2j}\cdot G_1\,\mathrm{d}x_0\mathrm{d}y_0 \right) \qquad (7)$$

$$A'_{IV}(x,y) = \sum_{j=1}^{l}\left(\int_{a_1}^{a'_1}\int_{b_{1j}}^{b'_{1j}} J_{1j}\cdot G_2\,\mathrm{d}x_0\mathrm{d}y_0 + \int_{a_2}^{x}\int_{b_{2j}}^{b'_{2j}} J_{2j}\cdot G_2\,\mathrm{d}x_0\mathrm{d}y_0 + \right.$$

$$\left. + \int_{x}^{a'_2}\int_{b_{2j}}^{b'_{2j}} J_{2j}\cdot G_1\,\mathrm{d}x_0\mathrm{d}y_0 \right)$$

$$A_{V}(x,y) = \sum_{j=1}^{l}\left(\int_{a_1}^{a'_1}\int_{b_{1j}}^{b'_{1j}} J_{1j}\cdot G_2\,\mathrm{d}x_0\mathrm{d}y_0 + \int_{a_2}^{a'_2}\int_{b_{2j}}^{b'_{2j}} J_{2j}\cdot G_2\,\mathrm{d}x_0\mathrm{d}y_0 \right)$$

In (7), J_{1j}, J_{2j} and G_1, G_2 have as arguments (x_0,y_0) and $(x,y;x_0,y_0)$ respectively. The current densities specific to the coil number j are obtained with following expressions:

$$J_{1j} = w_{1j}i_1 \big/ [(a'_1 - a_1)(b'_{1j} - b_{1j})], J_{2j} = -w_{2j}i_2 \big/ [(a'_2 - a_2)(b'_{2j} - b_{2j})] \quad (8)$$

where w_{1j} and w_{2j} represent the number of turns for j coils, while i_1 and i_2 represent the instantaneous values of the phase currents for LV and HV coils.

The integration of (7) allows computing the vector-potential for each sub-domain. Because the problem is plane-parallel, there will be only two components (for each sub-domain) for the magnetic flux density, over the x axis (radial component), and over the y axis (axial component) respectively:

$$B'_x(x,y) = \partial A'(x,y)\big/\partial y \qquad\qquad B'_y(x,y) = -\partial A'(x,y)\big/\partial x \qquad\qquad (9)$$

Along the separation lines between the regions, the axial and radial components of the magnetic flux density must meet the following equalities:

$$B'_{Ix}(x,y)\big|_{x=a_1} = B'_{IIx}(x,y)\big|_{x=a_1}, \; B'_{Iy}(x,y)\big|_{x=a_1} = B'_{IIy}(x,y)\big|_{x=a_1} \; \text{etc.} \quad (10)$$

The solution of (4) with boundary conditions (5), **when just magnetizing magnetomotive force is considered**, is formed by only one expression [21]. This time the computation domain (surface Σ – see Fig. 1) has not been divided into sub-domains and thus the Green's function contains one double-sum and two single-sums.

In order to compute the leakage magnetic field the same transformer shown in Fig. 2 is considered but inside the window is placed only one conductor. The current flowing through this conductor is equal to the magnetizing magnetomotive force ($w_1 \cdot i_{10} = w_1 \cdot i_1 - w_2 \cdot i_2$).

The tangential magnetic field (effective value of the leakage magnetic field tangential component over boundary Γ - inside the window) is defined by:

$$(1/\mu_0) \cdot \partial A(x_0, y_0)/\partial n = H_t^*(x_0, y_0) \tag{11}$$

Taking into account (3) and (11) the leakage magnetic field vector-potential given solely by the magnetizing magnetomotive force can be deducted with the following expression:

$$A^*(x, y) = \int_\Gamma G(x, y; x_0, y_0) \cdot H_t^*(x_0, y_0) \cdot ds_0 \tag{3''}$$

As a consequence, equation (3'') can be integrated after obtaining the tangential component of the leakage magnetic field over the boundary Γ. This component may be computed using a 2D finite element method.

The integration of (3'') allows computing the vector-potential for each of the five sub-domains (see Fig. 2) and then, because the problem is plane-parallel, the two components (for each sub-domain) of the magnetic flux density $B^*(x, y)$ (these components are obtained with expressions similar to (9)):

Based on the supposed hypotheses, the effect superposition principle can be applied. Thus, the resultant leakage magnetic field $B(x, y)$ inside the transformer's window can be obtained as a sum of two fields: one is generated by the windings b and B from Fig. 3 ($B_B(x, y)$) and the other one is given by the windings c and C from Fig. 3 ($B_C(x, y)$). Hence:

$$B(x, y) = B_B(x, y) + B_C(x, y) \tag{12}$$

The currents are flowing through the transformer's windings that are placed inside the transformer's leakage magnetic field. In consequence, axial and

radial forces are exerting over the coils. In case of short-circuit, per unit length radial forces (F_x) and axial forces (F_y), that are acting over the coil j, part of the LV winding (II region – Fig. 2) and HV winding (IV region – Fig. 2) respectively, are given by the following expressions:

$$F_{x_{1j}, y_{1j}} = \int_{a_1}^{a_1'} \int_{b_{1j}}^{b_{1j}'} J_{1j}(x,y) \cdot B_{\text{II}y, \text{II}x}(x,y) \cdot dx \cdot dy$$

$$F_{x_{2j}, y_{2j}} = \int_{a_2}^{a_2'} \int_{b_{2j}}^{b_{2j}'} J_{2j}(x,y) \cdot B_{\text{IV}y, \text{IV}x}(x,y) \cdot dx \cdot dy$$

(13)

where $B_{\text{II}y}$, $B_{\text{IV}y}$ and $B_{\text{II}x}$, $B_{\text{IV}x}$ are the axial and radial components of the resultant leakage magnetic fields B_{II} and B_{IV} (that are computed with (12)).
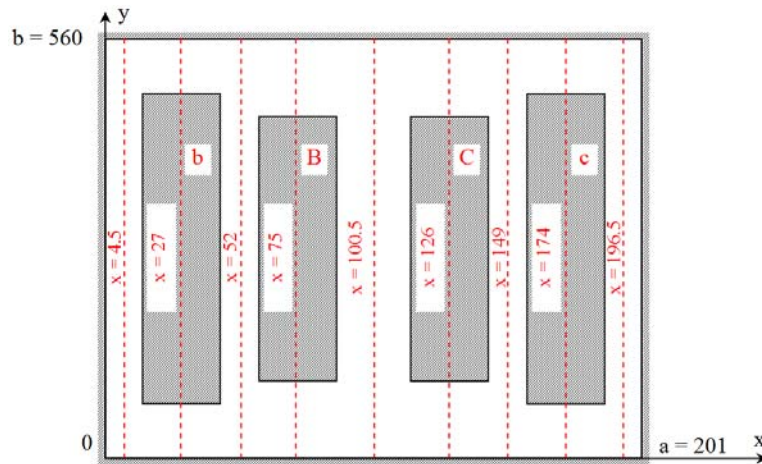

Fig. 3. The computational domain used to determine the resultant leakage magnetic field inside a three-phase power transformer window (dimensions are expressed in mm).

Double-integrals (13) can be evaluated with MATLAB using trapezoidal method [22]. The current densities specific to j parts of the windings where the short-circuit forces are computed are deducted using (8) where the currents i_1 and i_2 respectively represent the maximum possible values during the short-circuit period when the transformer is supplied at rated voltage [14].

3. Study case for a three-phase power transformer

In the following a comparison is presented between three different computational methods regarding the short-circuit electrodynamic forces (axial and radial components): numerical methods (2D and 3D finite element method), classical method and analytical method (presented in Section 2).

The considered three-phase electrical transformer has the following rated data: $S_n = 630$ kVA, $f_n = 50$ Hz, $U_{1n} = 10$ kV, $U_{2n} = 0.4$ kV, $u_k = 5.58$ %, $w_1 = 1082$, $w_2 = 25$, group $\Delta y5$. The main geometrical dimensions (expressed in mm) are shown in Fig. 4.
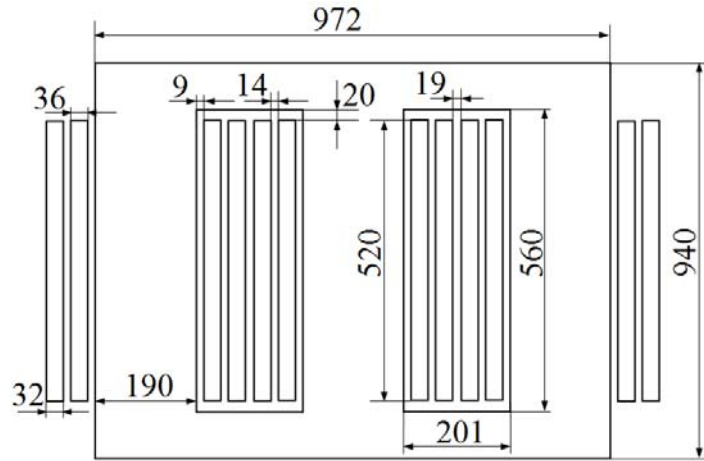

Fig. 4. Longitudinal cross-section of the transformer through the largest step of the yokes and columns (dimensions are expressed in mm).

The computer that has been used for computations has the following configuration: Intel Core 2 Quad processor clocked at 2.83 GHz and 4 GB RAM. The 2D and 3D finite element method have been applied using MagNet 6.26.1 designed by Infolytica company [23], the classical method and the presented analytical method have been applied using MATLAB designed by MathWorks company [24].

Fig. 5 shows the variation of resultant leakage magnetic field $B(x, y)$ components with respect to the transformer's window height for the right-side transformers window (for the 2D and 3D finite element method and the analytical method applied at the same coordinate $x = 52$ - see Fig. 3). In the case of 3D finite element method due to the symmetry of the problem just half of the variation has been represented.

It can be seen that the analytical method and the 2D/3D finite element methods yield similar results regarding the leakage magnetic field inside the transformer's window. For this reason the forces are comparable. Similar results have been achieved for all the coordinates shown in Fig. 3.

Table 1 presents the global values obtained for the electrodynamic short-circuit forces using the three methods.
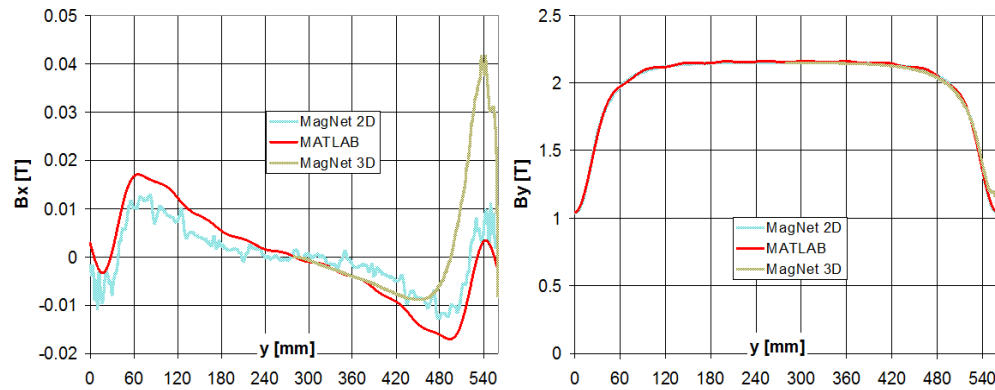

Fig. 5. Variation of resultant leakage magnetic field components along the transformer's widow height for $x = 52$.

Table 1

Electrodynamic short-circuit forces (global values)

2D/3D finite element method		
Radial force for winding B from Fig. 3.	-911.22 kN	
Radial force for winding b from Fig. 3.	913.4 kN	
Axial force for the bottom half of the winding B from Fig. 3.	20.67 kN	computation time
Axial force for the bottom half of the winding b from Fig. 3.	26.54 kN	~ 10800 s
Radial force for winding C from Fig. 3.	232.03 kN	
Radial force for winding c from Fig. 3.	-230.68 kN	
Axial force for the bottom half of the winding C from Fig. 3.	9.24 kN	
Axial force for the bottom half of the winding c from Fig. 3.	4.1 kN	
Classical method – only the windings placed on the middle column are considered		
Radial force	844.57 kN	computation
Axial force for half winding	-29.78 kN	time ~ 1 s
Analytical method		
Radial force for winding B from Fig. 3.	-915.17 kN	computation
Radial force for winding b from Fig. 3.	917.93 kN	time
Axial force for the bottom half of the winding B from Fig. 3.	19.41 kN	~ 600 s

Axial force for the bottom half of the winding b from Fig. 3.	28.47 kN	
Radial force for winding C from Fig. 3.	230.37 kN	
Radial force for winding c from Fig. 3.	– 229.25 kN	
Axial force for the bottom half of the winding C from Fig. 3.	8.72 kN	
Axial force for the bottom half of the winding c from Fig. 3.	4.6 kN	

6. Conclusions

Using the presented method it is possible, for each point inside a three-phase transformer's window, to compute the leakage magnetic field and the electrodynamic forces distribution along its windings in a fast and accurate way.

The simulations show that the RMS value of the leakage magnetic field generated by the magnetizing magnetomotive force $B^*(x, y)$ has low values for common three-phase power transformers when the short-circuit regime is considered, thus this component can be neglected.

The presented analytical method has been initially applied to a single-phase transformer. This paper presents how the method is developed and may be applied on a three-phase transformer. At present the results are validated through the finite element method (see Fig. 5) but in the near future it is also intended to validate the results provided by this method with experimental measurements of electrodynamic forces.

The computations involved by the developed method can be implemented on a computer without major difficulties by using a more affordable software and computational resources than the ones needed by finite element methods. Another advantage over the finite element method is that this analytical method does not require material specifications for the transformers components but only the instantaneous values of the windings flowing currents and the geometrical dimensions of the transformers window. Also it does not require a complex preprocessing stage (e.g. drawing a scaled model, setting the appropriate mesh, and so forth).

R E F E R E N C E S

[1]. *S. Januszewski, H. Swiatek, K. Zymmer*, Consequences of internal short-circuits in very high powerconverters, Industrial Electronics, ISIE '96, vol.1, pp. 519-524, 1996.
[2]. *M. Fortin, P. Riffon, J. Gerth, R. P. McLaughlin*, Short-Circuit Strength and Short Circuit Testing of Power and Distribution Transformers, IEEE/PES Transformers Committee, Charlotte, North Carolina, March 17, 2008.
[3]. *M. Ardebili, K. Abbaszadeh, S. Jamali, H.A. Toliyat*, Winding arrangement effects on electromagnetic forces and short-circuit reactance calculation in power transformers via numerical and analytical methods, 12th Biennial IEEE Conference on Electromagnetic Field Computation, CEFC 2006, art. no. 1633296, 2006.

[4]. *Z. Meng, Z. Wang*, The analysis of mechanical strength of HV winding using finite element method, part I calculation of electromagnetic forces, 39th International Universities Power Engineering Conference, UPEC 2004 - Conference Proceedings 1, pp. 170-174, 2004.

[5]. *C. Hernándeza, M. A. Arjona*, Design of distribution transformers based on a knowledge-based system and 2D finite elements, ELSEVIER Journal - Finite Elements in Analysis and Design, Volume 43, Iss. 8, pp. 659-665, May 2007.

[6]. *A. M. Kashtiban, A. Vahedi, A. Halvaei*, Investigation of winding type effect on leakage flux of single phase shell type transformer using FEM, Proc. of the 8th International Conference on Electrical Machines and Systems 3, art. no. 1575057, pp. 1755-1758, ICEMS 2005.

[7]. *K. Komeza, S. Wiak*, Electromagnetic field calculation of a leakage transformer by means of finite element method with hermitian elements, The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 13, iss. 1, pp. 97-100, 1994.

[8]. *X. Li, Q. Chen, R. Li, G. Long*, Calculation of leakage reactance for a transformer windings with uneven distribution of magnetic force, Gaodianya Jishu 31 (3), pp. 15-17, 2005.

[9]. *C. Ghiţă*, Calculul inductivităţii de dispersie la transformatoare, Buletinul Institutului Politehnic Bucureşti, 32, 3, pp. 47-51, 1980.

[10]. *A. Naderian, J. Faiz, J. Mohseni*, Calculation of distribution transformer leakage reactance using energy technique, IEEE AUPEC'02, Australasian, Melbourne, Australia, 2002.

[11]. *T. Henriksen*, Transformer leakage flux modeling, International Conference on Power Systems Transients – IPST 2001, Rio de Janeiro, Brazil, June 24 - 28, 2001.

[12]. *M. Dolinar, D. Dolinar, G.Štumberger, B.Polajžer*, Comparison of different iron core models of three-phase core-type transformers, The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 26, iss. 4, pp. 1049-1062, 2007.

[13]. *L. Susnjic, Z. Haznadar, Z. Valkovic*, Stray losses computation in power transformer, 12th Biennial IEEE Conference on Electromagnetic Field Computation, art. no. 1633280, 2006.

[14]. *C. Ghiţă*, Convertoare electromecanice, Editura ICPE, Bucureşti, 1998.

[15]. *V. S. Vladimirov*, Uravenenia matematiceskoi fiziki, Iyd-vo „Nauka", Moscva, 1967.

[16]. *R. Răduleţ, I. Ciric*, Solutions des problèmes aux limites de l'écuation de Poisson, utilisant deux expressions de la fonction de Green en deux sous-domaines, Rev. Roum. Sci. Techn. – Electrotechn. et Energ., 13, 4, 1968.

[17]. *C. Ghiţă*, Calculul câmpului magnetic de dispersie la transformatoare, Studii şi Cercetări de Energetică şi Electrotehnică, Edit. Acad. R. S. R., Tom 24, *2*, pp. 479-494, 1974.

[18]. *C. Ghiţă*, Dispersion magnetic field induced by the magnetization current in electrical transformers, Buletinul Institutului Politehnic, **28**, *3*, pp. 127-130, 1976.

[19]. *E. Roth*, Etude analytique du champ de fuites des transformateurs et des efforts mécaniques exercés sur les enroulements, R.G.E., pp. 158-166, Mai, 1928.

[20]. *V. N. Nedelcu*, Regimurile de funcţionare ale maşinilor de curent alternativ, Editura tehnică, Bucureşti, 1968.

[21]. *I. Ciric*, Green functions of the Laplace operator for internal Neumann problems, Rev. Roum. Sci. Techn. – Electrotechn. et Energ., 20, 3, 1970.

[22]. http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=16420.

[23]. http://www.infolytica.com.

[24]. http://www.mathworks.com.