THE EFFECT OF CERAMIC VOLUME FRACTION ON THE SHAKEDOWN THRESHOLD PRESSURE OF FGM SPHERES

Farid Vakili-Tahami[1], Mohammad Zehsaz[2], Mohammad-Ali Saeimi-Sadigh[3]

In this paper, Finite Element based numerical solution method has been used to study the mechanical behavior of a metal/ceramic FGM (functionally graded material) sphere which is subjected to variable internal pressure. Using the FE (Finite Element) method, for different case studies, the threshold internal pressure value has been obtained which causes reverse yielding of the critical point in the vessel or shakedown. The results show that by increasing the volume fraction of ceramic along the thickness of the spherical vessel, the internal pressure level or shakedown threshold that causes reverse yielding increases and therefore the load carrying capacity of the vessel increases and the risk of low cycle fatigue failure reduces.

Keywords: Functionally graded material, shakedown threshold, pressurized spheres.

1. Introduction

Many structural components such as hallow cylinders, thin-walled shells, pipes and thick-spheres are often subjected to the varying loads and temperatures. In these cases, the behaviour of the structure depends on the nature of loading condition. In mechanical engineering, it is important to distinguish the reasons of failure at each loading condition and develop methods of design and manufacturing to reduce the damages caused by failures. If a body is deformed elastically at the presence of variable loads, its strength is determined by the fatigue properties of the material and fracture occurs after large number of cycles. But, if the body undergoes elastic-plastic deformation, a load which is less than the limiting value, can cause the attainment of a critical state in comparatively small number of cycles. In this case the structure fails after low cycles of loading.

Mechanical behavior and failure mechanisms of commonly used materials such as steels have been studied extensively in the past and based on the results of these research studies, sophisticated design codes have been established. In recent years, new materials are used in modern industry and therefore, it is necessary to study the mechanical behavior of these materials to provide required data for designing high-tech engineering components. Functionally Graded Materials (FGMs) are among the recently developed materials which need more attention in

[1] Assoc. Prof., Dept. of Mechanical Eng, University of Tabriz, Iran, Email: f_vakili@tabrizu.ac.ir
[2] Assoc. Prof., Dept. of Mechanical Eng, University of Tabriz, Iran, Email: zehaz@tabrizu.ac.ir
[3] PhD student, Dept. of Mechanical Eng, University of Tabriz, Iran, Email: saeimi.sadigh@gmail.com

terms of material and mechanical behaviours. The properties of Functionally Graded Materials vary gradually and continuously with location within the material. In this way, the negative or undesired effects of abrupt changes of material properties, which can be observed in composite materials, are eliminated. FGMs are widely used in aerospace structures, cutting tools, pressure vessels because of their good performance under varying thermal and mechanical loads for a long period of time.

Due to the increasing application of FGMs, during the last decade, many research studies have been carried out to solve different thermo-mechanical problems for FGMs using analytical and numerical solution methods. Luts and Zimmerman developed an analytical solution method to obtain stresses distribution in spheres and cylinders made of FGM [1, 2]. They considered thick sphere and cylinder under radial thermal loads, where radially graded material with linear composition of constituent material was considered. Jabbari et al. assumed the material properties of FGM that obey a power law distribution of the volume fraction of the constituents [3, 4]. They obtained analytical solutions for one and two dimensional steady state thermo-elastic stresses in functionally graded hollow cylinder. Shao et al. obtained the analytical solution for the stress field in a functionally graded hollow cylinder with finite length under mechanical loading [5, 6]. Eslami et al. obtained an exact solution for the one dimensional steady-state thermal and mechanical stresses in a hollow thick sphere made of FGM [7, 8]. Also, modeling study of the thermal and mechanical stresses of the FGM components using Finite Element Method (FEM) is one of the main research directions [9-13]. Zhang et al. studied the distribution of the residual stresses of carbon based Sic FGM with different technical parameters using FEM method [14]. Jahromi et al. used an extension of variable material property (VMP) method to evaluate the residual stresses in an autofrettage thick vessel made of FGM [15].

In this paper, alternating plasticity of elastic-plastic state of a thick hollow sphere, made of Functionally Graded Material which is subjected to internal pressure has been studied. The internal pressure varies according to the scheme of $0 \rightarrow P \rightarrow 0$. With first loading ($0 \rightarrow P$) a zone of plastic deformation will be developed and after unloading ($P \rightarrow 0$) residual stresses will be induced in the sphere wall. So, the effect of variable elasto-plastic properties of FGM in the propagation of plastic deformation during the loading and unloading has been investigated. In addition, in each case study, maximum pressure that causes secondary plastic deformation during unloading has been obtained. In industrial application, if the sphere is loaded below the obtained maximum pressure, the dominant failure mechanism would be high cycle fatigue, whereas, above this threshold, the failure mechanism would be low cycle fatigue. Hence, obtaining this threshold pressure is important.

2. Theoretical model

2.1 Sphere plasticity

A thick-walled sphere, with internal and external radii of *'a'* and *'b'* respectively is assumed. By virtue of spherical symmetry ($\sigma_\phi = \sigma_\theta$), the stresses at any given stage are function of radius *"r"* and satisfy the equilibrium equation (see Eq. (1));

$$\frac{d\sigma_r}{dr} + \frac{2(\sigma_r - \sigma_\theta)}{r} = 0 \tag{1}$$

Radial stress, σ_r, and hoop stress ($\sigma_\phi = \sigma_\theta$) in elastic and plastic region through the sphere thickness are derived using the compatibility and stress-strain equations in spherical coordinate as shown in Eqs. (2)-(4).

$$\frac{d\varepsilon_\theta}{dr} + \frac{(\varepsilon_\theta - \varepsilon_r)}{r} = 0 \tag{2}$$

$$\varepsilon_r = \frac{1}{E}(\sigma_r - 2\mu\sigma_\theta) + \varepsilon_r{}^p \tag{3}$$

$$\varepsilon_\theta = \frac{1}{E}((1-\mu)\sigma_\theta - \mu\sigma_r) + \varepsilon_\theta{}^p \tag{4}$$

where E is elastic modulus, μ is Poisson' ratio, ε_r^p and ε_θ^p are radial and hoop plastic strains respectively. The amount of pressure which causes initial yielding at the inner surface of the sphere-wall, employing Von-Mises yield criteria in the non hardening metal, is calculated from Eq. (5). Also, Eq. (6) defines the limit pressure for which the sphere is entirely in yield state [16].

$$P_c = \frac{2((b/a)^3 - 1)}{3(b/a)^3}\sigma_Y \tag{5}$$

$$P_l = 2\sigma_Y Ln(b/a) \tag{6}$$

In theses equations σ_Y is yield stress. If the pressure which is applied to the sphere is between P_c and P_l, after its removal, residual stresses will be developed along the sphere thickness. By increasing the initially applied pressure to the amount, P_s ($P_c<P_s<P_l$), after the load removal, secondary plastic deformation will occur. If the sphere is loaded with alternating internal pressure as $0 \rightarrow P_s \rightarrow 0$, after few loading and unloading cycles, fracture occurs as a result of alternating plastic deformation. This is called low cycle fatigue. The threshold pressure which can cause reverse yielding is regarded as the shakedown pressure for the spheres and is introduced by Prager [16]:

$$P_s = \frac{4}{3}(1 - \frac{a^3}{b^3})\sigma_Y \tag{7}$$

The safety condition requires that the internal pressure should be confined to the shakedown region. Also, P_s should not exceed the P_l, otherwise the vessel will fail due to the plastic collapse.

In FGM sphere vessels, the mechanical and elasto-plastic properties of the material changes along the thickness. So, the stress state differs inside the sphere thickness. Hence, different amount of pressures causes reverse yielding.

2.2. Mechanical behavior of FGM spheres

In this study the thick pressurized vessel was assumed as shown in fig.1-a with internal radius "a" and thickness "*t*". The FGM metal/ceramic is assumed to be locally isotropic and yield according to Von-Mises criteria. Also, its uni-axial stress-strain curve is assumed to be as shown in Fig.1-b. As depicted in Fig. 1, metal is elastic- perfectly plastic material and ceramic is linear elastic. In all calculations presented here, the metal component of the FGM has mechanical elastic modulus of $E_m=56GPa$, hardening modulus of $H_m=0$, yield stress of $\sigma_y^m=106MPa$. As it can be seen in this figure, transfer from elastic to plastic behaviour occurs when stress level reaches yield level of σ_y. It is assumed that ceramic remains in elastic region and its elastic modulus is $E_c=80GPa$. The Poisson ratio of 0.25 is assumed to be the same and constant for both materials. The elastic modulus of the metal ceramic composite (E_{comp}), the overall flow strength of the composite corresponding to the onset of plastic yielding (σ_y^{comp}) and its tangent modulus (H_{comp}), which represent its strain hardening behavior, are obtained using the modified rule of mixture for composites [15]:

$$E_{comp} = \left[(1-f)(\frac{q+E_c}{q+E_m}) + f\right]^{-1} \times \left[(1-f)E_m(\frac{q+E_c}{q+E_m}) + fE_c\right] \tag{8}$$

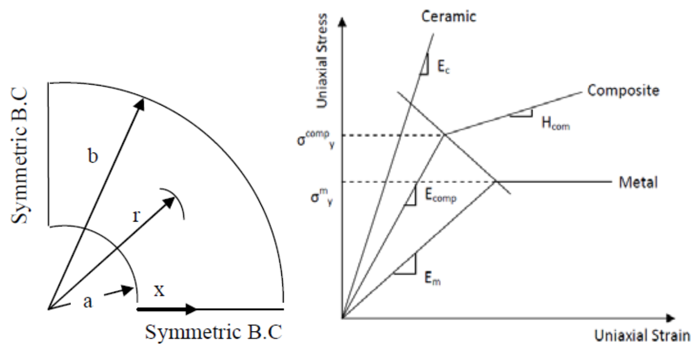
$$\sigma_{comp} = \sigma_Y^m \left[(1-f)(\frac{q+E_c}{q+E_m}) + f\right]^{-1} \times \left[(1-f)(\frac{q+E_c}{q+E_m}) + \frac{E_c}{E_m}\right] \tag{9}$$

$$H_{comp} = \left[(1-f)(\frac{q+E_c}{q+E_m}) + f\right]^{-1} \times \left[(1-f)H_m(\frac{q+E_c}{q+E_m}) + fE_c\right] \tag{10}$$

where f denotes the volume fraction of ceramic particles and q is the ratio of stress to strain transfer, which is a parameter that defines the metal/ceramic composite behavior. The value of q is 17.2MPa for this type of FGMs [15]. In this study, the ceramic particle reinforcement is assumed to have a volume fraction that varies from 0 at the inner surface of the sphere to f_o at the outer surface according to the Eq. (11).

$$f(x) = f_o(x/t)^n \tag{11}$$

where $x=(r\text{-a})$ and "r" is the local radius and "t" is the sphere thickness. By using this relationship for the volume fraction along the wall thickness, the inner surface of the vessel is all metal which provides maximum mechanical strength.



(a) (b)

Fig. 1: (a) Schematic of a thick FGM sphere vessel.
(b) Schematic of modified rule of mixtures used to estimate the behavior of ceramic.

3. Result

In this research, the effect of the variation of elasto-plastic properties due to the variation of the ceramic volume fraction, on the threshold pressure which causes reverse yielding (P_s) after unloading, has been studied. In addition, stress distribution profile along the sphere thickness has been compared for different ceramic volume fractions.

In all case studies, the inner radius of the vessel “a” is 10 mm and “*b*/a” is 3. A finite element model of the spherical vessel which is subjected to internal pressure is constructed using 2 dimensional plane-axisymmetric elements. The FE based numerical solutions have been carried out using ANSYS computer code. This type of element is known as PLANE82 in ANSYS. In order to model Functionally Graded Material of the sphere, its thickness is divided into 40 equal layers. Each layer is considered to have constant mechanical properties, which were calculated based on the volume fraction of the ceramic at the center of each layer using Eqs. (8)-(10) [15]. Mesh sensitivity analysis was performed to assure that the results are not sensitive to the mesh size; also increasing the effect of number of layers was checked so that the acceptable convergence of the results has been obtained within 5% difference.

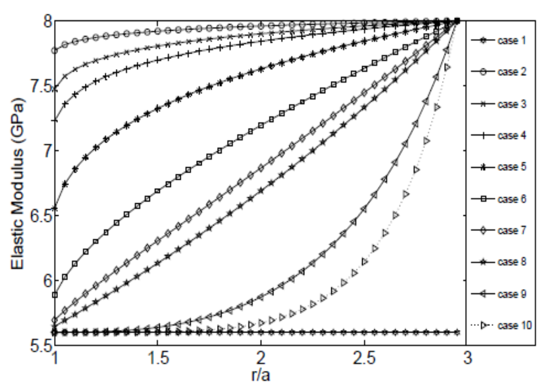
In order to investigate the effect of the increasing or decreasing the volume fraction of the ceramic, 15 different case studies have been defined. Table 1 shows the case studies with different reinforcement coefficient f_o and n. Case 1 and 11 with $f_o=0$, denotes a metal vessel and other cases are metal/ceramic FGMs. Figs. 2(a and b) illustrate the variation of the elastic modulus through the graded region for case studies 1 to 10 and 11 to 15 respectively. In case studies 1-10, the value of f_o was assumed to be the same and the value of n varied from 0.02 to 5, and in case studies 11 to 15 the value of n was set to be 0.5 and the value of f_o is changed from 0 to 1. As it can be seen in Figs. 2(a and b), case studies 2 and 15 have the largest ceramic volume fraction and case studies 10 and 12 have the lowest ceramic volume .

Table 1

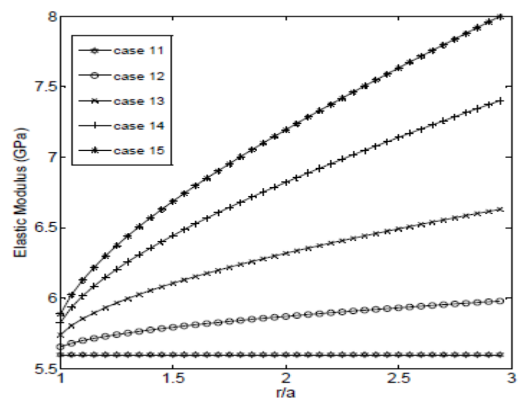
The case studies with different reinforcement coefficient f_o and n.

Case studies	f_o	n
1	0	0
2	1	0.02
3	1	0.05
4	1	0.08
5	1	0.2
6	1	0.5
7	1	0.8
8	1	1
9	1	3
10	1	5
11	0	0.5
12	0.2	0.5
13	0.5	0.5
1	0.8	0.5
15	1	0.5

To obtain threshold pressure P_s, a trial and error method has been used. For this purpose a computer code has been developed which carries out elasto-plastic solution of the vessel. First, an arbitrary internal pressure is applied to the vessel and then it is removed. The residual stresses are calculated along the vessel thickness, if the effective stresses along the whole wall thickness are below the yield level, the shake down has not occurred yet, so the pre-selected pressure is increased until the amount of the residual effective stress reaches yield level at a point or node along the wall thickness. This point is known as the critical point and the associated pressure is known as the critical pressure, P_s. At this pressure or beyond this level reverse yielding will occur.



Figs. 2(a): The variation of the elastic modulus through the graded region (case studies 1-10)



Figs 2(b): The variation of the elastic modulus through the graded region(case studies 11-15)

According to Eq. (7), for the metal vessel, if the internal pressure reaches to the value of *136.3MPa* or higher, unloading will induce reverse yielding through the sphere thickness. If this vessel undergoes a cyclic loading with *P≥136.3MPa*, fracture will occur as a result of alternating plasticity which is called low cycle fatigue.

In order to study the effect of using FGM with different volume fractions on the value of P_s, vessels with different ceramic volume fractions were studied. Fig. 3 shows the value of P_s that causes reverse yielding in each case study. As it can be seen, among the cases in which the n changes from 0 to 5 (cases 1-10), the maximum P_s is obtained for case number 2 with $n=0.02$ which has the largest ceramic volume fraction. Comparing the cases number 11 to 15 which have different f_o value, case study 15 has the largest P_s, which again has the largest ceramic fraction. Therefore, it can be concluded that by increasing the ceramic volume fraction through the sphere thickness, the load carrying capacity of the sphere increases which leads to the lower risk of low cycle fatigue failure.

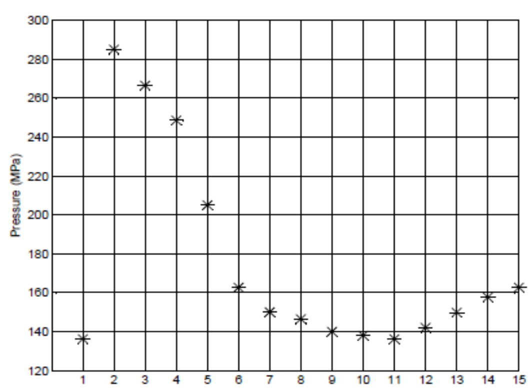


Fig. 3: The threshold pressure that causes reverse yielding

In order to compare the stress distribution in the sphere thickness under the action of P_s (the load that causes reverse yielding in each case study) Figs 4 and 5 have been presented. Figs. 4 and 5 illustrate the radial and hoop stresses distribution along the thickness of the sphere for case studies 1 to 7, respectively. In addition, the residual radial and hoop stresses through the sphere thickness, which are induced after the removal of the internal pressure at the first cycle, have been shown in Figs 6 and 7. This has been observed, although the higher stresses have been induced in vessels with greater ceramic volume fraction (see Figs. 4 and 5), after load removal, lower residual stresses are developed through the sphere thickness for these models (see Figs. 6 and 7). For instance, hoop stress at a distance of 0.2 from the inner radius of sphere in case 1 (metal sphere) is 10MPa and in case 2 this stress reaches to 100MPa, however residual hoop stress at this point is -40MPa for case 1 and -2MPa for case 2. The stress distribution along the sphere thickness for other case studies (8-15) show the same trend, so they are not included in the figures to prevent confusion.

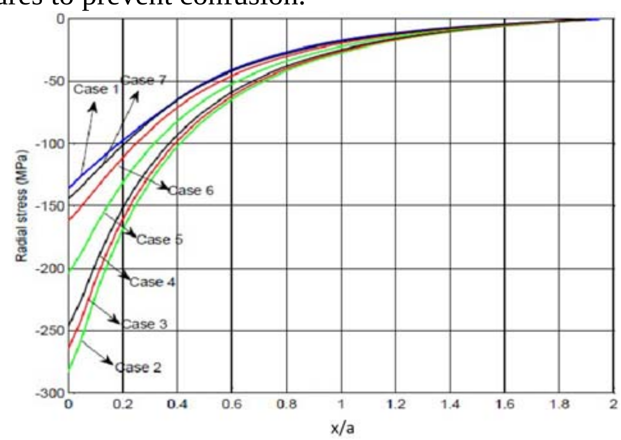


Fig. 4: Radial stresses distribution along the thickness of case studies 1to 7.

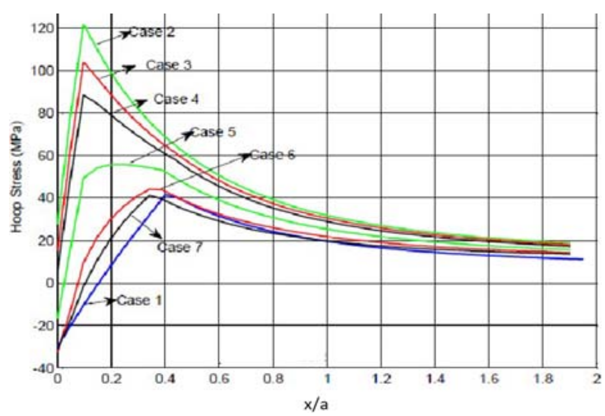


Fig. 5: Hoop stresses distribution along the thickness of case studies 1to 7.

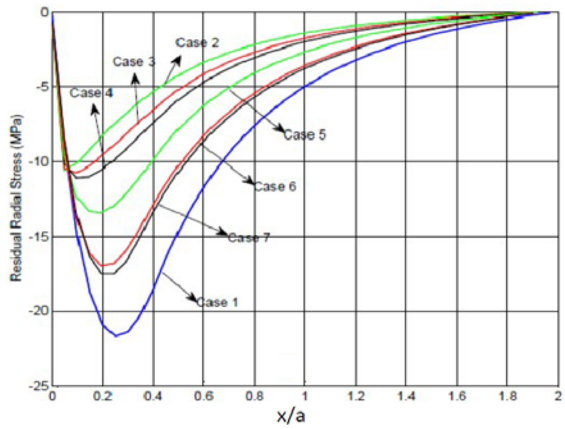


Fig. 6: Residual radial stresses distribution along the thickness of case studies 1to 7.

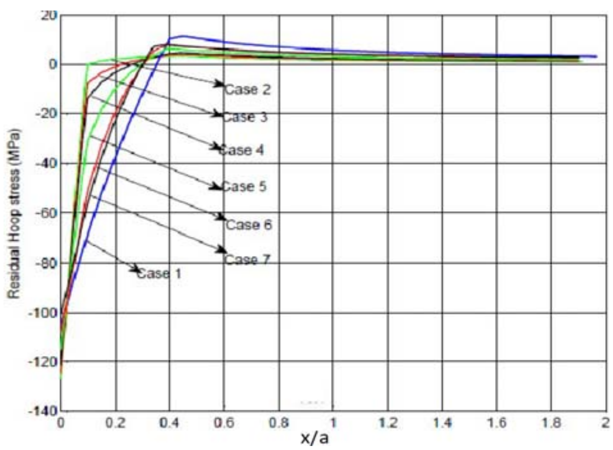


Fig. 7: Residual hoop stresses distribution along the thickness of case studies 1to 7.

4. Conclusion

A Finite Element based numerical solution method has been used to study the mechanical behaviour and stress distribution of a metal/ceramic FGM sphere which is subjected to variable internal pressure. The threshold pressure which causes reverse yielding has been obtained for different case studies which have different ceramic volume fraction. The results of this study show that using metal-ceramic FGM with increasing ceramic volume fraction through the thickness, increases the load carrying capacity and reduces the risk of low cycle fatigue failure of the spherical vessel. Although this work does not provide an optimum material configuration for the FGM, however the results signify the reinforcement effect of the use of FGM in thick spherical vessels and provide an insight for selecting near optimum material configuration.

REFERENCES

[1] *M.P. Lutz and R.W. Zimmerman*, “Thermal stress and effective thermal expansion coefficient of a functionally graded sphere”, in J. Thermal stress., v**ol.** 19, no.1, 1996, pp 39-54.

[2] *R.W. Zimmerman and M.P. Lutz*, “Thermal stress and effective thermal expansion in a uniformly heated functionally grade cylinder”, in J. Thermal stress., v**ol.** 22, no.2, 1999, pp 177-188.

[3] *M. Jabbari , S. Sohrabpour and M.R. Eslami,* “Mechanical and thermal stresses in functionally graded hollow cylinder due to radially symmetric loads”, in Int. J. Press. Vessels Pip. v**ol.** 79, no.7, 2002, pp 493–497.

[4] *M. Jabbari, S. Sohrabpour and M.R. Eslami*, “General solution for mechanical and thermal stresses in a functionally graded hollow cylinder due to nonaxisymmetric steady-state loads”, in J. Appl. Mech., v**ol.** 70, no.1, 2003, pp 111–118.

[5] *Z.S. Shao, L.F. Fan and T.J. Wang*, “Analytical solutions of stresses in functionally graded circular hollow cylinder with finite length”, in Key Engineering Materials, v**ol.** 261–263, 2004, pp 651–656.

[6] Z.S. Shao, “Mechanical and thermal stresses of a functionally graded circular hollow cylinder with finite length”, in Int. J. Press. Vessels Pip. v**ol.** 82, no.3, 2005, pp155–163.

[7] *M.R. Eslami, M.H. Babaei and R.Poultangari*, “Thermal and mechanical stress in a functionally graded thick sphere”, in Int. J. Press. Vessels Pip., v**ol.** 82, no.7, 2005, pp 522-527.

[8] *R. Poultangari, M. Jabbari and M.R. Eslami*, “Functionally graded hollow spheres under non-axisymmetric thermo-mechanical loads”, in Int. J. Press. Vessels Pip., v**ol.** 85, no.5, 2008, pp 295-305.

[9] *C.S. Lee, S.-H. Ahn and L.C. DeJonghe*, “Effect of functionally graded material (FGM) layers on the residual stress of polytypoidally joined Si3N4–Al2O3”, in Mater. Sci. Eng. A., v**ol.** 434, no.1–2, 2006, pp160–165.

[10] *L.P. Zhou and D.W. Tang*, “A functionally graded structural design of mirrors for reducing their thermal deformations in high-power laser systems by finite element method”, in Opt. Laser Technol., v**ol.** 39, no.5, 2007, pp 980–986.

[11] *F. Chen and W. Jie*, “Finite element design of MgO/Ni system functionally graded materials”, in J. Mater. Process. Technol., v**ol.** 182, no.1–3, 2007, pp181–184.

[12] *K. Pietrzak, D. Kali´nski and M. Chmielewski*, “Interlayer of Al2O3–Cr functionally graded material for reduction of thermal stresses in alumina–heat resisting steel joints”, in J. Eur. Ceram. Soc., v**ol.** 27 no.2–3, 2007, pp1281–1286.

[13] *O. Sarikaya, E. Celik*, “Effects of residual stress on thickness and interlayer of thermal barrier ceramic MgO–ZrO2 coatings on Ni and AlSi substrates using finite element method”, in Mater. Des., v**ol.** 23, no.7, 2002, pp645–650.

[14] *G. Zhang, Q. Guo, K. Wang, H. Zhang, Y. Song*, J. Shi and L. Liu, “Finite element design of SiC/C functionally graded materials for ablation resistance application”, in Mater. Sci. Eng., A., v**ol.**. 488, no.1-2, 2008, pp45–49.

[15] *B. Haghpanah Jahromi, G.H. Farrahi, M. Maleki, H. Nayeb-Hashemi and A. Vaziri,* “Residual stresses in autofrettaged vessel made of functionally graded material”, in Eng. Struct., v**ol.** 31, no.1, 2009, pp2930-2935.

[16] *J.Chakrabarti*, Theory of plasticity, third edition, Texas A & M University System, USA, 2006.