# EXPERIMENTAL VERIFICATION AND DOUBLE PARTICLE DIAMETER CALCULATION MODEL OF THERMAL CONDUCTIVITY OF FROZEN SOIL CONSIDERING LATENT HEAT

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*The thermal conductivity of frozen soil is more complex with the change of pore water freezing process and a problem that has not been solved satisfactorily in the theory of heat transfer of frozen soil. It should be taken into account that in the soil pore water ice phase change and significant differences of thermal conductivity between water and ice. Based on the objective facts that frozen soil is composed of four phase system includes soil particles, the pore water and pore ice and pore gas, and significantly changes of unfrozen water content in negative temperature, Double particle diameter calculation model of the thermal conductivity with temperature change is presented in freezing stage. The latent heat was received by the change of unfrozen water content, and the part of the energy was reckoned in the total energy of hybrid system caused by temperature change. On the basis of the ideal model of thermal decomposed into heat conduction of soil column, water column, icicles in parallel, frozen soil thermal conductivity model of Johansen method and thermoelectric analogy method were established. By means of comparison with the measured results and Johansen's results, the calculation results shows: the results of considering latent heat is consistent with the test results, has the same trend with the change of thermal conductivity of Johansen method. The calculation method can effectively simulate the thermal conductivity of frozen soil with the phase change latent heat, and has the characteristics of clear concept, high agreement with the test results etc..*

**Keywords:** latent heat; frozen soil; particle diameter calculation model; thermal conductivity; thermoelectric analogy method

**Symbols**:

$H$: Enthalpy of water renter into ice; kJ/kg
$n$: Number of corners; Dimensionless quantity
r: Radius of frozen column; Dimensionless quantity
R: Large soil column radius; Dimensionless quantity
S: freezing area; Dimensionless quantity
$V_m$: Volume of soil particles in frozen soil at different time; Dimensionless quantity

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$V_w$: Volume of unfrozen water in frozen soil at different time; Dimensionless quantity
$V_i$: Volume of ice in frozen soil at different time; Dimensionless quantity
Vw: Pore water volume per unit area; Dimensionless quantity
Vs: Volume of soil column per unit area; Dimensionless quantity
$\Delta V_i$: Volume of unfrozen water to ice per unit time; Dimensionless quantity
$\lambda_f$: Thermal conductivity considering unfrozen water content in frozen soil; kW/(m·K)
$\lambda_m$: Average thermal conductivity of soil minerals; kW/(m·K)
$\lambda_W$: Thermal conductivity of pore water; kW/(m·K)
$\lambda_i$: Thermal conductivity of porous ice; kW/(m·K)
$\phi$: Volume ratio of ice to unfrozen water in frozen soil; Dimensionless quantity
$\Delta\phi$: Volume ratio of unfrozen water in frozen soil; Dimensionless quantity
$\rho_i$: Density of unfrozen water, kg/m$^3$
$J_0(u)$, $J_1(u)$: The zero order and the first order functions of the first kind of Bessel functions
$Y_0(u)$, $Y_1(u)$: The zero order and the first order functions of the second kind of Bessel functions

## 1. Introduction

Thermal conductivity is a key parameter among numerous thermal parameters and mechanical parameters of frozen soil. It has a decisive influence on the thermal conduction process of frozen soil, and is a sign of the evolution process of the temperature field of frozen soil. With the large-scale development and utilization of resources and the progress of underground engineering construction, artificial freezing construction has gradually become an important means of disposing difficult geotechnical engineering problems. The corresponding freezing scheme, construction parameter calculation design and temperature field prediction control need reliable thermal conductivities to provide support. Therefore, thermal conductivity plays an important role in the thermal physical indexes of soil. In view of the thermal conductivity of frozen soil, scholars at home and abroad have done a lot of research on the methods, instruments, influencing factors, theoretical and numerical calculation of thermal conductivity. In view of the problems and shortcomings of the traditional thermal conductivity measurement methods and instruments and equipment, P P Overduin et al. [1] improved the method of measuring the thermal conductivity of frozen soil by hot wire method. Based on the basic principle and theoretical calculation method of hot wire method, the thermal thermal conductivity of frozen soil under high temperature phase change phase was obtained, which provided a new method for the test of thermal conductivity of frozen soil containing unfrozen water.

Based on one dimensional heat conduction theory, A Alrtimi et al. [2] presented another method of thermal conductivity measurement. This test method, from the technical level, avoided the heat flow loss to the maximum extent, thus the test error of the thermal conductivity was controlled in a smaller range. On the basis of transient heat source method, Abdelwaheb, Trigui, et al. [3] studied the thermal properties of composite materials with phase change characteristics. The different heat, latent heat and latent heat characteristics of these composites were obtained by experiments.

The factors affecting thermal conductivity are diverse and complex. The thermal conductivity is closely related to the salt content and the salt content of the soil, whether it is positive or negative temperature [4]. Based on the principle of transient heat source method, Zhang Ting et al. [5] studied the relationship between thermal conductivity and moisture content and dry density of clay and silt in shallow soil of Nanjing at normal temperature and -10℃. Similarly, using transient heat source method, Hong Tao et al. [6] studied the relationship between thermal conductivity and dry density, water content and temperature of three different rocks in the source area of the Yellow River, and found that the correlation was strong.

In the theoretical model of thermal conductivity, experts have proposed a model for calculating the thermal conductivity of soil according to the composition characteristics of soil and the volume fraction of each component. In essence, the key to this idea is the mode in which effect size of component thermal conductivity combines with each other. Generally speaking, it basically includes two kinds, namely shunt or series. Of course, there are also mixed models, and the type of model is large. In this regard, the model created by Xiao Henglin, Yuan Xizhong et al [7， 8] according to the particle composition, pore ratio, dry density, water content and soil water form and the model created by Yuan Xizhong on the basis of the thermal conductivity of a fully dry and fully saturated state were typical.

On the theoretical basis of the dual phase lag model, K. Q. Hu et al. [9] studied the transient heat conduction process with a crack plane, and obtained a plane transient temperature field with cracks. Liu Weimin et al. [10] analyzed the influencing factors of thermal conductivity in detail, studied the effect of each factor, and established another method of thermal conductivity estimation. According to the test results of temperature field evolution of freezing wall model test, Wang Renhe et al. [11] established a variable numerical model for the inverse analysis of thermal conductivity. The nonlinear thermal conductivity of the frozen wall was obtained by using the finite element software, the selection method and the inverse analysis method.

In recent years, more and more scholars have begun to pay attention to the physical properties and engineering properties of lunar soil and other star soil. For example, F Gori et al. [12] studied Martian soil analogs, investigated the thermal conductivity characteristics and temperature changes during the drying and freezing conditions, and predicted and estimated the thermal conductivity of Martian soil. Wan et al. [13] took tunnel engineering in cold area as the object of study, presented a thermal conductivity model for rock and soil medium in cold region. The prediction model was related to the pore ratio, density, saturation and temperature of soil. It was a reasonable calculation model of thermal conductivity.

According to the unfrozen water content, the latent heat change is calculated, and the latent heat energy is combined with the energy of water and particle to total energy. The calculation model of thermal parameters in freezing stage is deduced. Based on laboratory tests, the relationship between soil moisture content, temperature and thermal conductivity is studied; based on the soil column model with double particle size, a theoretical model of thermal conductivity at freezing stage with temperature is established; the thermal conductivity calculation model of soil in melting stage, freezing stage and frozen stage is established. The model provides a basis for the superposition calculation of thermal conductivity of multi granular soil.

## 2. Theoretical model of frozen soil thermal conductivity double particle diameter heat transfer

### 2.1 Model assumption

In order to reflect the influence of nonuniformity of particle size on the thermal conductivity of soil, a thermal conductivity calculation model consisting of two particle diameter soil columns is presented based on the one-dimensional steady heat conduction model. According to the characteristics of the most closely packed soil particles, the heat transfer model composed of double grain soil columns is composed of a large soil column with equal radius and a small diameter soil column embedded in a large soil column, as shown in figure 1.

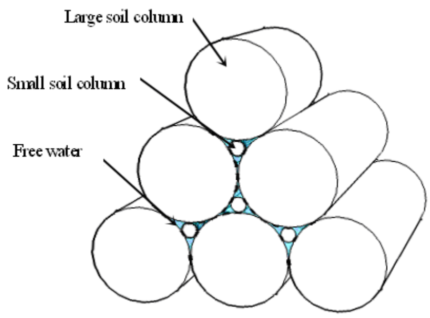



Fig.1 Double particle diameter calculation model of thermal conductivity

### 2.2 Calculation of phase proportion in frozen soil at initial stage

Figure 2 is a model section of the soil columns tangent to each other, wherein the circle O, the circle A, the circle B and the circle C are tangent respectively, and the shadow part is filled with free water. It is not difficult to tell that freezing begins at the center of inscribed circle of the corner area enclosed by any tangent circle according to objective fact the freezing occurs first away from the soil particle surface.

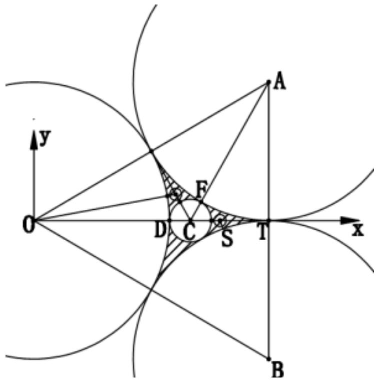


Fig. 2 The calculation model of small soil column inscribed in soil column

When the freezing column and soil column are disjoint or tangent in cross section, freezing area $S = \pi r^2$. Then

$$r \le \frac{2-\sqrt{3}}{6-\sqrt{3}} R \tag{1}$$

The area of free water of each single corner part is calculated based on Fig. 3

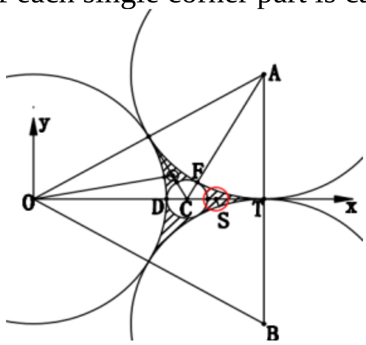


Fig.3 The calculation model when radius changing

$$S = 2\left(S_{\Delta ACT} - S_{SCEF} - S_{SAFT}\right) = \frac{(17-8\sqrt{3})\pi + 6\sqrt{3}}{18} R^2 \tag{2}$$

### 2.3 Calculation of phase proportion in frozen soil during contact stage

When the freeze develops further, icicle and soil column will intersect. Before icicle and adjacent side of small soil column is frozen, icicle intersects with soil column. When r<SF, as shown in figure 3. Infinitesimal element is taken to establish a coordinate system as shown in figure 4. According to figures 3 and 4, the area of the intersection of the circle S and the circle C can be calculated

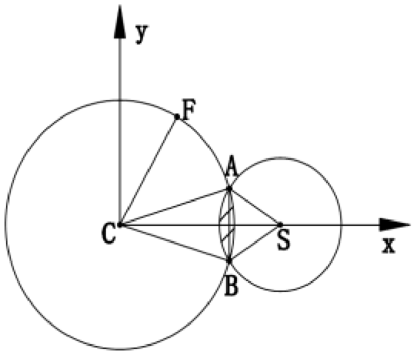



Fig.4 Calculation diagram of intersecting region of circular S and circle C

$$S = S_{SACB} - S_{\Delta ACB} + S_{SASB} - S_{\Delta ASBT}$$
$$= \frac{7-4\sqrt{3}}{540} R^2 arctg \frac{y}{x} + \frac{\pi r^2}{180} arctg \frac{y}{a-x} - ay \tag{3}$$

In formula (5), it shall meet

$$\begin{cases} \frac{2-\sqrt{3}}{6-\sqrt{3}} R < r \le \frac{\sqrt{20\sqrt{3}-13.25}}{6-\sqrt{3}} R \\ a = \frac{4\sqrt{3}-6}{6-\sqrt{3}} R \\ x = \frac{1722-991\sqrt{3}}{456\sqrt{3}-756} R - \frac{6-\sqrt{3}}{8\sqrt{3}-12} \times \frac{r^2}{R} \\ y = \sqrt{\left(\frac{2\sqrt{3}-3}{3} R\right)^2 - x^2} \end{cases} \tag{4}$$

In figure 3, when r<SF, the coordinate system shown in figure 5 is established. Then the intersection area between soil column S and soil column A is

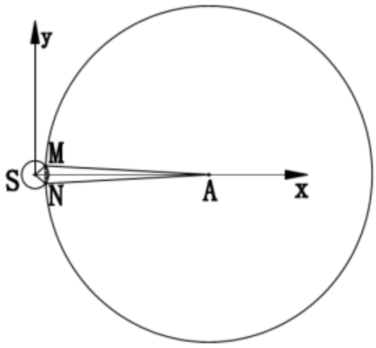



Fig.5 Intersection region of circle S and second phase of circle A

$$S = S_{SMSN} - S_{\Delta MSN} + S_{SMAN} - S_{\Delta MAN} = \frac{\pi r^2}{180} arctg \frac{y}{x} + \frac{\pi R^2}{180} arctg \frac{y}{b-x} - by \quad (5)$$

Wherein, it shall meet

$$\begin{cases} \frac{2-\sqrt{3}}{6-\sqrt{3}} R < r \le \frac{\sqrt{20\sqrt{3}-13.25}}{6-\sqrt{3}} R \\ b = \frac{2\sqrt{19-8\sqrt{3}}}{6-\sqrt{3}} R \\ x = \frac{282-157\sqrt{3}}{(156-48\sqrt{3})\sqrt{19-8\sqrt{3}}} R - \frac{r^2}{2b} \\ y = \sqrt{r^2 - x^2} \end{cases} \quad (6)$$

When the corner between icicle and small soil column is frozen, there is still unfrozen water in the corner between big soil column and icicle, as shown in figure 6. As the freeze continues, the frozen ice will continue to increase, as shown in shadow of figure 7. At this point, the area of the shadow can be calculated in the following formula

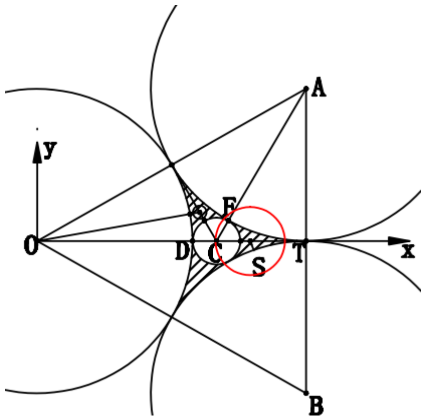


Fig.6 Calculation diagram when ice column intersecting with three soil columns

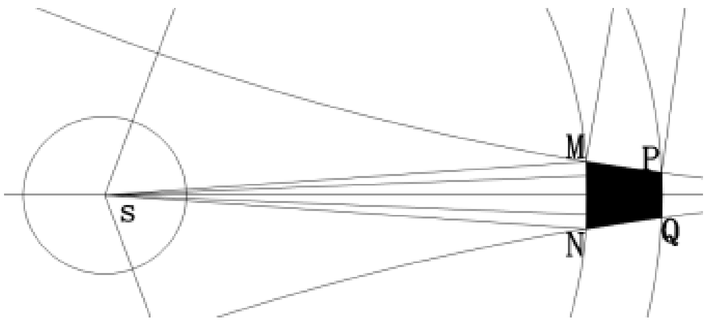


Fig.7 Intersecting region of ice column with soil column A

$$S_{SH} = S_{TMNQP} - (S_{SMSN} - S_{\Delta MSN}) + (S_{SPSN} - S_{\Delta PSN}) - 2(S_{SMAP} - S_{\Delta MAP}) \tag{7}$$

$$\begin{aligned}
&S_{TMNQP} = 0.5 \times (x + x_0)(y - y_0) \\
&S_{SMSN} - S_{\Delta MSN} = \frac{\pi r_0^2}{180} \arcsin \frac{y_0}{r_0} - y_0 \sqrt{r_0^2 - y_0^2} \\
&S_{SPSN} - S_{\Delta PSN} = \frac{\pi r^2}{180} \arcsin \frac{y}{r} - y \sqrt{r^2 - y^2} \\
&S_{SMAP} = \frac{\pi r^2}{180} \arcsin \frac{\sqrt{(x - x_0)^2 + (y - y_0)^2}}{2R} \\
&S_{\Delta MAP} = \frac{1}{2} \sqrt{(x - x_0)^2 + (y - y_0)^2} \times \\
&\sqrt{\left[\sqrt{3}R - 0.5 \times (x - x_0)\right]^2 + \left[R - 0.5 \times (y - y_0)\right]^2}
\end{aligned} \tag{8}$$

Then, wherein

$$\begin{cases}
\dfrac{\sqrt{20\sqrt{3} - 13.25}}{6 - \sqrt{3}} R < r \le \dfrac{5 - 2\sqrt{3}}{6 - \sqrt{3}} R \\
b = \dfrac{2\sqrt{19 - 8\sqrt{3}}}{6 - \sqrt{3}} R \\
x = \dfrac{282 - 157\sqrt{3}}{(156 - 48\sqrt{3})\sqrt{19 - 8\sqrt{3}}} R + \dfrac{r^2}{2b} \\
y = \sqrt{r^2 - x^2} \\
x_0 = \left[\dfrac{282 - 157\sqrt{3}}{(156 - 48\sqrt{3})} + \dfrac{20\sqrt{3} - 13.25}{4(6 - \sqrt{3})}\right] \times \dfrac{R}{\sqrt{19 - 8\sqrt{3}}} \\
y_0 = \sqrt{\dfrac{20\sqrt{3} - 13.25}{39 - 12\sqrt{3}} R^2 - x_0^2}
\end{cases} \tag{9}$$

For saturated soil, namely soil column corners have no pore gas. According to the freezing process, the double particle diameter thermal conductivity model can be divided into 3 stages. In the first stage, the corner between columns began to freeze, the icicle and soil column were disjoint, the limit state was inscribed. In the second stage, the icicle and soil column intersected, and then development of icicle tended to slow. In the third stage, corners between small and large soil columns were in a frozen state, and only corners between two large soil columns had unfrozen water. Then soil would be frozen completely if icicle continued developing.

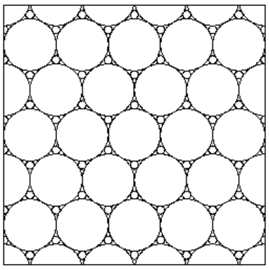


Fig.8 Distribution diagram of Soil column at a small section unit

Figure 8 is a close packed model for large soil column spaces filled with small soil columns. The calculation model of single particle size (figure 8) shows that the corners in single particle model are small, and soil column is divided into three equal smaller corners. It's not difficult to figure out in the model in figure 8, there is a total of 2×(1000/2R+1)×1000/2R×3 corners in the unit volume. The area of a single complete enclosed area S can be obtained by formula (4). At this time, the number of the corner n is

$$n = \frac{1500000}{R^2} + \frac{3000}{R} \quad (10)$$

The volume of pore water in the unit area is $V_w = 1000nS$; In the unit area, the volume of soil column is $V_s = 1\times10^9 - V_w$. In the first stage of freezing, icicle volume $V_i = 1000n\pi r^2$. Then, r meets formula (1). In the second stage of freezing, volume of icicle Vi is

$$V_i = 1000n\left(\pi r^2 - A - 2B\right) \quad (11)$$

Then, r shall meet formula (6). In formula (11)

$$A = \frac{7-4\sqrt{3}}{540}\pi R^2 arctg\frac{y}{x} + \frac{\pi r^2}{180} arctg\frac{y}{a-x} - ay$$
$$a = \frac{4\sqrt{3}-6}{6-\sqrt{3}}R \quad (12)$$

$$B = \pi r^2 arctg\frac{y}{x} + \frac{\pi R^2}{180} arctg\frac{y}{b-x} - by$$
$$b = \frac{2\sqrt{19-8\sqrt{3}}}{6-\sqrt{3}}R \quad (13)$$

In formula (14), x, y are calculated based on formula (6). While in formula (13), x, y shall be calculated according to formula (7). In the third stage of freezing, increment of icicle volume △V is

$$\Delta V = 2000n[S_{TMNQP} - (S_{SMSN} - S_{\Delta MSN}) + (S_{SPSN} - S_{\Delta PSN}) - 2(S_{SMAP} - S_{\Delta MAP})] \quad (14)$$

Then, r shall meet formula (13). Substitute into formula (17), namely

$$r = \sqrt{\frac{20\sqrt{3} - 13.25}{6 - \sqrt{3}}} R \tag{15}$$

In the first stage of freezing, icicle volume $V_i = 1000 n \pi r^2$. Summation is made with formula (15), then volume of icicle in the third stage of freezing is obtained. Volume fraction of soil particle, pore ice and pore water in each freezing stage of each component can be calculated by pore water volume in unit area, soil column volume in unit area and formula (14).

## 3. Thermal conductivity calculation model considering latent heat of phase change

Calculation of thermal conductivity based on Johansen [14] thermal conductivity calculation method considering latent heat of phase change.

### 3.1 Johansen hypothesis

Johansen proposed the generalized geometric mean method to estimate the thermal conductivity of solid particles in soil ks. Based on the Johansen estimation method, Xu Xuezu [15] proposed a method for calculating the thermal conductivity λ f considering unfrozen water content in frozen soil, that is

$$\lambda_f = 2.22^{\phi - \Delta\phi} \times 0.55^{\Delta\phi} \lambda_m^{1-\phi} \tag{16}$$

Formula (16) is the empirical formula of thermal conductivity of composite materials. Where λm is the average thermal conductivity of soil minerals, assigning value according to the mineral composition of different soils following standard method; ϕ is the volume ratio of ice and unfrozen water in the frozen soil; Δϕ is the volume ratio of unfrozen water in the frozen soil.

The thermal conductivity of the three materials is shown in table 1.

*Table 1.*

**Coefficient of thermal conductivity of different materials**

| Type of material | Soil | Water | Ice |
|---|---|---|---|
| Thermal conductivity (W/m.k) | $\lambda_m$ | 0.55 | 2.22 |

Furthermore, the thermal conductivity of frozen soil in each freezing stage can be calculated according to formula (16) and table 1.

### 3.2 Calculation of thermal conductivity of frozen soil based on latent heat of phase change

The frozen soil is a multiphase body composed of soil particles, pore water and pore ice. Pore water and pore ice vary with temperature, and both maintain dynamic equilibrium all the time [4]. As the water solidifies into ice, it gives off a great deal of heat, which increases the temperature of the frozen earth. The coefficient of thermal conductivity is essentially a system composed of parallel or mixed conductors. Based on this, a theoretical model of thermal conductivity of frozen soil with latent heat of phase change is proposed, as shown in Fig. 9.

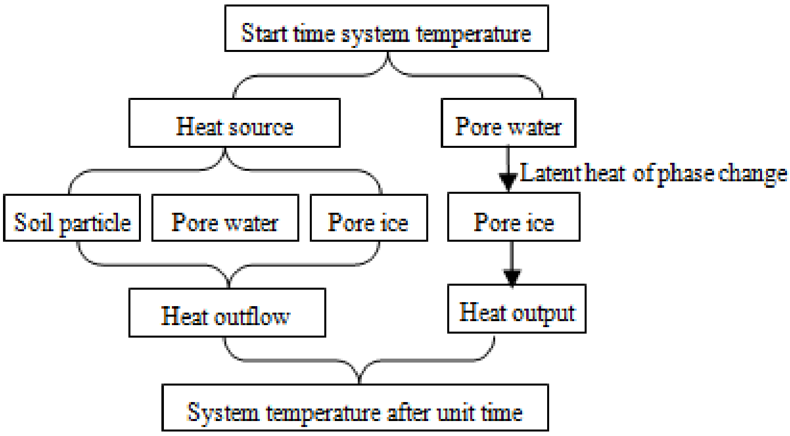



Fig.9 A analogy theory model based on latent heat of phase change

As shown in figure 9, based on the latent heat of the model, because of the different composition of each phase, the heat transfer of the adjacent section size is different. As the pore water turns into pore ice, it continuously releases heat. Under the premise of consistent transfer length, the different components in the unit volume frozen soil are divided into different sections. Considering the influence of latent heat of phase change, the thermal conductivity of frozen soil is calculated based on cross section ratio superposition according to the change of system temperature per unit time. The specific formula is

$$\lambda_f = \frac{\lambda_m V_m + \lambda_w V_w + \lambda_i V_i + \rho_i H \Delta V_i}{V_m + V_w + V_i} \tag{17}$$

In formula (17), λm is the average thermal conductivity of soil minerals; $\lambda_W$, $\lambda_i$ are thermal conductivity of pore water and pore ice, whose values are as shown in Table 1; $V_m$, $V_w$ and Vi are the volume of soil particles, unfrozen water and ice bodies in frozen soil at different times; $\rho_i$ is the density of unfrozen water; is the enthalpy of water forming ice; $\Delta V_i$ is the volume of unfrozen water turned into ice per unit time.

## 4 Test of thermal conductivity of soil

The method used in this paper is transient probe method in unsteady state method. The basic principle is that the temperature probe is embedded in the soil sample to be tested, usually the soil sample center. Record the change of temperature with time. After the temperature is stable, electrify to heat, until the temperature is stable again, save the data. Assuming that the heat flow rate of thermal probe per unit length temperature is q=I2R/l, and the numerical value remains constant due to the constant power of the probe. Compared with the probe, the volume of the soil sample is infinite. It is assumed that the soil sample to be measured is a homogeneous infinite space body. According to the differential equation of heat conduction, there must be r~r0 at the surface of the probe.

$$T(r_0,\tau)-T_0=\frac{2q\omega^2}{\pi^3\lambda}\int\frac{1-\exp\left(-\frac{\alpha\tau}{r_0^2}u^2\right)}{u^3\Delta(u,\omega)} \tag{18}$$

Wherein $\omega = 2\rho c_p / \rho_w c_{pw}$

$$\Delta(u,\omega)=[uJ_0(u)-\omega J_1(u)]^2+[uY_0(u)-\omega Y_1(u)]^2$$

Where, T0 is initial equilibrium temperature; $\omega = 2\rho c_p / \rho_w c_{pw}$ is two times the ratio of the heat capacity of the sample to the temperature probe. J0(u), J1(u) are the zeroth order and the first order function of the first kind of Bessel function; Y0(u), Y1(u) are the zeroth order and the first order function of the second kind of Bessel function; u is an integral variable. So we can get

$$\lambda=\frac{I^2R}{4\pi l}\cdot\frac{d(\ln\tau)}{d\theta} \tag{19}$$

Wherein, I is the current passing through the temperature probe, R is resistance.

Soil is a multiphase medium composed of soil particle framework, soil water, soil ice, gas and organic matter. The size and direction of soil thermal conductivity depend mainly on the composition of soil, ice content, water content, density and temperature, and are related to the microstructure characteristics of soil.

### 4.1 Relationship between soil thermal conductivity and water content

The greater the dry density of soil, the more the proportion of soil particle skeleton in the unit volume will be, the less the corresponding pores will be. Because the thermal conductivity of soil skeleton is much larger than that of air, Thermal conductivity coefficient of soil increases with the increase of soil dry density and the decrease of soil porosity. The influence of water content on thermal conductivity and its influence on freezing temperature have similar mechanism and tendency. Generally speaking, as the water content increases, the thermal conductivity increases gradually, as shown in Fig. 10.

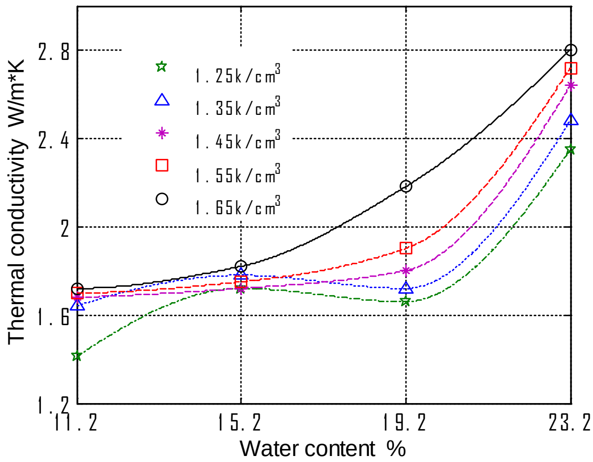


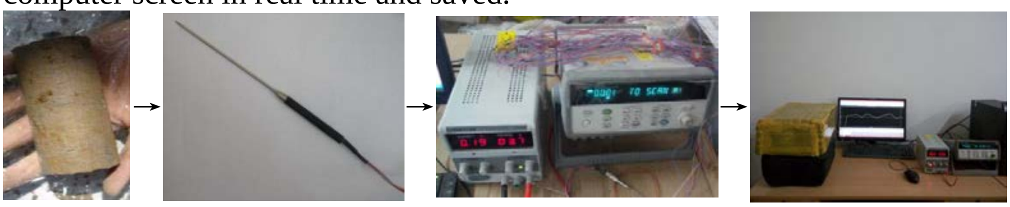

Fig.10 Change curves of thermal conductivity with water content

In addition, the thermal conductivity is also dependent on temperature. The prevailing view is that the thermal conductivity of frozen soil increases slightly with the decrease of negative temperature, but increases very little. When the temperature changes 1℃, the change rate of thermal conductivity is generally less than 5‰. Therefore, in some frozen soil engineering or some freezing method construction, it is common only to consider the influence of thermal conductivity and unfrozen water content and dry density, and ignore the influence of temperature change.

### 4.2 Study on relationship between soil thermal conductivity and temperature

Remolded soil samples are prepared and grouped into numbers to test the thermal conductivity. During the test, the soil sample is bound by a cylinder and sealed with tape to simulate field constraints. The test instruments used are shown in figure 11.

The test device on the left is a heater and on the right is Agilent whose main function is to record the real-time temperature of the temperature probe. Connect the probe, the heater, the Agilent through the wire and connect the Agilent to the computer. The temperature data measured by the probe can be displayed on the computer screen in real time and saved.



Samples Probe Test devices PC

Fig. 11 Test system of thermal conductivity of soil

The test procedure is as follows: first, the soil samples were made and put in the freezing and thawing box for more than 24h until the samples reached the setting temperatures (the freezing time is determined by the temperature sensor placed in the parallel test). Second, inserting the probe into the soil sample and monitor the temperature change of the soil sample through the probe until the sample temperature is stable. Third, turn on the heater and record the change of sample temperature during the heating process until the soil sample reachs a stable again. By using the equation (19), the thermal conductivity of samples can be tested.

The soil samples are placed in the freezing and thawing box, and the temperature is set at 10℃, 5℃, 3℃, 1℃, 0.5℃, 0℃, -0.5℃, -1℃, -3℃,-5℃, -10℃, respectively. When the temperature of the freezing and thawing box is set, continue to keep the temperature unchanged for 4h. After that, the temperature probe is inserted into the reserved hole of the soil sample to ensure good contact. Connect the heater and the wires and communication lines between Agilent and the computer. Agilent is turned on to record the real-time temperature of the probe and sample. When the temperature displayed on the computer is stable, the heater is turned on to heat the probe. When the temperature is stabilized again, turn off the heater, save the data and calculate the thermal conductivity of the soil. The test results of the thermal conductivity of eight specimens with temperature are shown in Fig. 12.

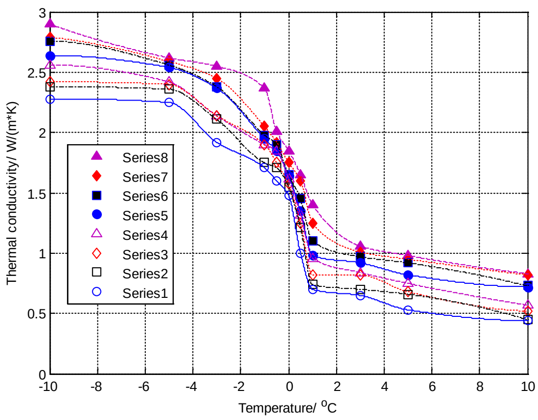



Fig.12 Relationship between thermal conductivity coefficients and temperature of soil

Among them, series 1, series 2, series 3, series 4, series 5, series 6, series 7 and series 8 show that the water content of 8 samples are 12%, 16%, 20%, 25%, 30%, 35%, 40% and 45% respectively. Variation of thermal conductivity with temperature shows that the thermal conductivity of soil increases gradually with the decrease of temperature. In the vicinity of the freezing of pore water, the thermal conductivity increases remarkably. This shows that the thermal conductivity of frozen soil is larger than that of corresponding soil. The reason for the increase of thermal conductivity of frozen soil is the formation of pore ice during freezing.

### 5. Validity analysis of the model

Based on the calculation method of the analogy model considering the latent heat of phase change, combined with the double particle size model of the thermal conductivity of frozen soil, the data obtained from the test of water content of 20% are compared and analyzed. Taking soil column radius of 1 as an example, the theoretical model of thermal conductivity is calculated, and comparison of the thermal conductivity curve using analogy method considering the latent heat of phase change and thermal conductivity curve based on thermal conductivity model using Johansen method are as shown in figure 13.

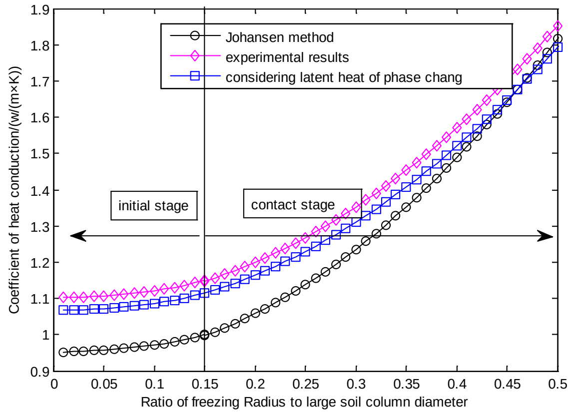



Fig.13 Comparison of considering latent heat method , Johansen method and experimental results

As can be seen from figure 13, the calculation considering the latent heat of the phase change is consistent with the experimental results and the variation trend of the thermal conductivity of the Johansen method, which is due to the linear variation of the proposed thermal conductivity model. When the particle column radius is small, Coefficient of heat conduction considering latent heat is larger, because unfrozen water emits a lot of heat when it is turned into ice; when the particle column radius is large, the two are relatively close, because the latent heat is less when the unfrozen water decreases gradually. But the two decrease with the temperature of frozen soil, and the thermal conductivity of frozen soil rises, which are consistent with the actual situation.

### 6. Conclusion

Based on the ideal two-particle cylinder model and the corneal pore water assumption, a thermal conductivity calculation model under ideal conditions is established. The thermal conductivity is calculated based on the model. The calculation method of thermal conductivity of frozen soil related to soil particle size, freezing time and temperature is obtained. Compared with the experimental data, the validity of the method is demonstrated. The results show that with the decrease of unfrozen water content in saturated frozen soil, the thermal conductivity of frozen soil increases gradually. With the decrease of soil particle size, the thermal conductivity of frozen soil decreases with temperature, and the change becomes smaller. The double particle size heat conduction model provides a theoretical basis for the study of multi-particle heat conduction in soil. At the same time, based on the research results of this model, it provides the research foundation for the superposition calculation of thermal conductivity of multi-particle group soil.

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