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EXPERIMENTAL EVIDENCE OF "VIBRATIONAL RESONANCE" IN VCSELs

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În lucrare sunt raportate dovezi experimentale ale „rezonanței vibraționale" într-o diodă laser de tip VCSEL și caracterizarea fenomenului. Sistemul utilizat este excitat de două semnale sumate, având frecvențe care diferă cu câteva ordine de mărime. El este studiat în două cazuri: cuasipotențial intern simetric şi asimetric. Fenomenul este pus în evidență în dinamica emisiei polarizate a laserului ca o rezonanță a răspunsului la semnalul de joasă frecvență şi atingerea unui maxim de către raportului semnal-zgomot, în funcție de amplitudinea semnalului de înaltă frecvență. Posibilitatea utilizării fenomenului pentru detecția semnalelor slabe este demonstrată experimental.

The experimental evidence and characterization of "vibrational resonance" in a VCSEL are reported. The system is driven by two summed forcings, having frequencies different by several orders of magnitude. It is studied in two cases: symmetrical and asymmetrical internal quasipotentials. The phenomenon shows up in the dynamics of the polarized laser emission as a resonance in the low-frequency response and reaching a maximum of the signal-to-noise ratio, depending on the amplitude of the high-frequency forcing. The possibility to use the phenomenon for low-level detection is experimentally demonstrated.

Key words: stochastic resonance, vibrational resonance, gain, signal-to-noise ratio, quasipotential, low level signals

1. Introduction

Bistable systems driven by a periodic signal and noise are known to display a resonance-like behaviour at the signal frequency as a function of the applied noise strength [1]. Such a phenomenon, known as "stochastic resonance" (SR), has been found in very different physical, chemical and biological systems [2]. In this context, Landa and McClintock have studied a model where a high frequency *deterministic* modulation replaced the added noise [3]. They have numerically shown, that the response to the low frequency periodic modulation (LF) passes through a maximum, depending on the amplitude of an additional

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high-frequency (HF) modulation. Such a phenomenon has been named "vibrational resonance" (VR). They have also noted a clear analogy between VR and SR, though the effect of noise on VR was not considered there. Analytical results for VR concerning a bistable oscillator were also given in [4]. More recent observations of the noise influence (which exists everywhere) in an electronic circuit with VR was presented in [5].

The importance of VR lies in the possibility to get in bistable systems outputs which are identical (except inherent scale factors) or, more precisely, synchronous with an "useful" input signal, periodical or not, having an amplitude *below the system threshold*, by summing to this signal a *deterministic* modulation, which is easier to obtain and control in comparison with noise (especially large bandwidth noise). This effect improves the signal-to-internal noise ratio, but it can be also considered as a parametric amplification near a critical point. A similar phenomenon has been shown in period-doubling systems [6]. We point out that the phenomenon of VR is *different* from the so-called "deterministic SR" where, instead of noise, has been used an external broadband chaotic signal generated by a *deterministic* system [7].

2. Experimental data and comments

In this paper, we present the experimental evidence and characterization of the phenomenon of VR in a vertical cavity surface emitting laser (VCSEL). This kind of diode laser is known to display a bistability of two orthogonal polarization states of the emitted field which can be selected by modifying the applied injection current [8]. From a theoretical point of view, dynamics of the polarization switchings in the VCSEL can be locally well described by an overdamped Langevin model with a two-well quasipotential [9], [10]. Evidence of SR was given in such a system [11]. Different cases of the occurrence of VR were investigated experimentally. Thanks to the flexibility of the system (described in [11]), we were able to observe VR for the case of both nearly symmetrical and strongly asymmetrical quasipotentials. In both cases the laser response at the low frequency passes through a maximum depending on the amplitude of the applied HF signal (either noise for SR or deterministic modulation for VR). Since noise is inevitably present in any real physical systems, we measured also the signal-to-noise ratio at the frequency of the LF signal, which displays the same behaviour.

The experimental setup was the same as in [11]. It consits in a VCSEL emitting at 850 nm, thermostabilized for eliminating the thermal drift of the quasipotential shape, an accurately tunable injection current source, very stable for precisely controlling the quasipotential shape, two waveform generators (one for the "useful" LF signal and the other for the HF modulation – be it noise for SR or deterministic for VR), a bias-T for summing the two signals over the dc injection current, a quarter-wave plate for selecting the two emission orthogonal polarizations, an optical isolator for preventing optical feedback, an avalanche photodiode (APD) and a computer-interfaced high bandwidth (1 GHz) digital oscilloscope. First the quasipotential shape (symmetrical or asymmetrical) was controlled by the dc injection current value, then, after the VCSEL temperature stabilization, the "useful" LF signal was applied, together with the HF modulation, which was noise for SR or deterministic for VR. These signals summed via the bias-T modulate the injection current. The APD detects the laser emission in one of the two orthogonal polarization states. The data is acquired by the digital scope and data files are created in the computer (25,000 points / file, corresponding to 50 periods of the "useful" LF signal). For the SR scenario the LF signal was modulated by a 10 MHz bandwidth white noise, and for the VR scenario – by a rectangular 100 kHz signal with the duty factor 0.5. In both scenarios the LF "useful" signal was a 500 Hz sinusoid. This way, the analysis of the "merit" parameters could be performed at a unique frequency, since a pure sinusoid has no harmonics. For the VR scenario, we tried to use other waveforms too, different of rectangles with duty factor 0.5, for the HF deterministic modulation (ex.: sinusoid, rectangles with low duty factor, i.e. rectangular pulses short in comparison with their period); we obtained the same results, but for higher amplitudes of the HF deterministic modulation. Consequently, we preferred the rectangles with 0.5 duty factor.

The used parameters were the following: f_s = the frequency of the "useful" LF signal = 500 Hz ; $I_s(f_s)$ = the power spectral component of the VCSEL signal at the frequency f_s, in the presence of the HF noise or deterministic modulation; $I_0(f_s)$ = the same, but in the absence of the HF noise or deterministic modulation (provided the amplitude of the "useful" LF signal is high enough to switch the VCSEL by itself); $I_N(f_s)$ = the power spectral component of the whole setup noise at the frequency f_s, in the presence of the HF noise or deterministic modulation; $I_{N0}(f_s)$ = the same, but in the absence of the HF noise or deterministic modulation.

Two "merit" parameters are defined, in order to compare the two scenarios – VR and SR:

$$G = \left. \frac{I_s - I_N}{I_0 - I_{N0}} \right|_{f = f_s} = \text{the gain} \tag{1}$$

$$SNR[dB] = 10 \cdot \lg\left(\frac{I_s + I_N}{I_N}\right)\Bigg|_{f=f_s} = \text{the signal to noise ratio} \qquad (2)$$

Actually, we have further used:

$$SNR^n = \frac{SNR}{SNR(0)} = \text{the } \textit{normalized SNR} \qquad (3)$$

where $SNR(0)$ corresponds to the presence of the "useful" LF signal, with no HF modulation at all (be it noise or deterministic). The RMS amplitudes of the "useful" signal, noise (for SR) and HF deterministic modulation (for VR) are: A_{LF}, A_N and A_{HF}, respectively.

A first experiment was performed using a quasipotential nearly *symmetrical*, obtained by tuning the dc injection (pump, bias) current [11]. Various shapes of the two-well quasipotential were experimentally obtained as histograms of the polarized laser intensity for different dc injection current, as in fig. 1 [11].

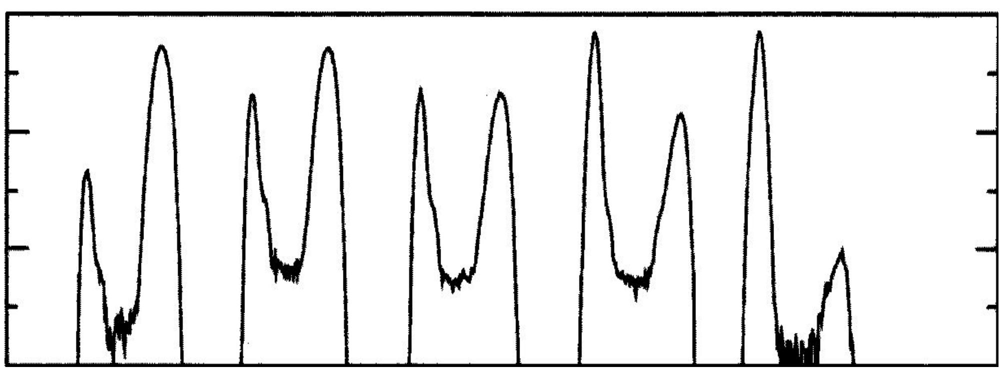
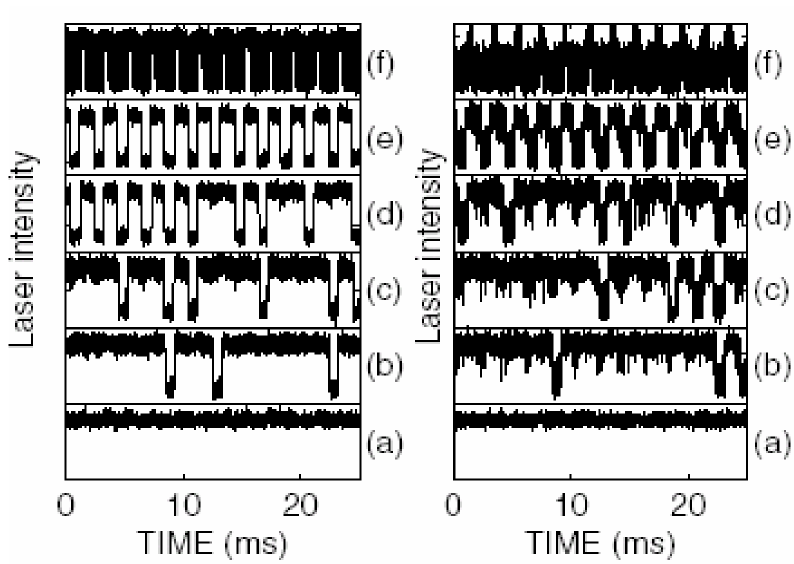


Fig. 1. Histograms *N(I)* of the polarized laser intensity *I(t)* for different pump currents (increasing from left to right)

First, the amplitude A_{LF} of the "useful" LF signal was tuned slightly *below* the threshold, such as the VCSEL output does *not* reproduce the "useful" LF signal yet, but a small increase of A_{LF} leads to its reproduction. In fig. 2 are shown temporal responses (of the VCSEL) in the VR and SR scenarios (for comparison), for various amplitudes A_{HF} of the HF deterministic modulation (for VR) and noise (for SR).

It can be readily seen that in both scenarios (VR and SR) there are optimal amplitudes of the HF deterministic modulation and noise, for which the intensity of the investigated polarization state (one of the two orthogonal) reproduces the LF "useful" signal. But the optimal situation occurs in the SR scenario for an amplitude of the noise which is *higher* than the corresponding one for the HF deterministic modulation in the VR scenario (14mV vs. 10.4 mV). Moreover, in the SR scenario the output signal is visibly more noisy than in the VR scenario (e). The gain *G* and the normalized signal to noise ratio SNR^n corresponding to fig. 2 are shown in fig. 3.



VR **SR**

Fig. 2. Temporal response for symmetric quasipotential; VR: A_{HF} = 0; 10.1; 10.2; 10.3; 10.4; 14mV and SR: A_N = 0; 10.6; 11; 11.4; 14; 20 mV (increasing from (a) to (f) waveforms)

The minima of the SNR^n curves for both scenarios – VR and SR (fig. 3) - are probably due to rare output switching induced by the internal noise of the VCSEL, which are shown in fig. 2b and c. The values of *G* and especially SNR^n are higher in the VR scenario than in the SR one.

A distinctly different behaviour is observed in the case of a strongly asymmetric quasipotential, obtained by changing the dc injection current [11] – see fig. 1. Just like in the previous case (symmetric quasipotential), we have used an "useful" LF signal with an amplitude slightly tuned below the threshold. In fig. 4 is shown the temporal response of the system, in the VR scenario, for various amplitudes of the HF deterministic modulation A_{HF}.

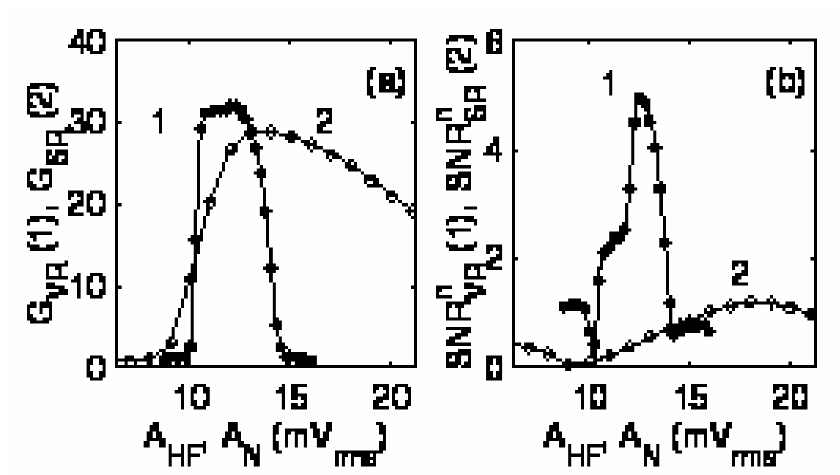


Fig. 3. The gain G and normalized signal-to- noise ratio SNR^n corresponding to fig. 1; VR – curves 1, SR – curves 2

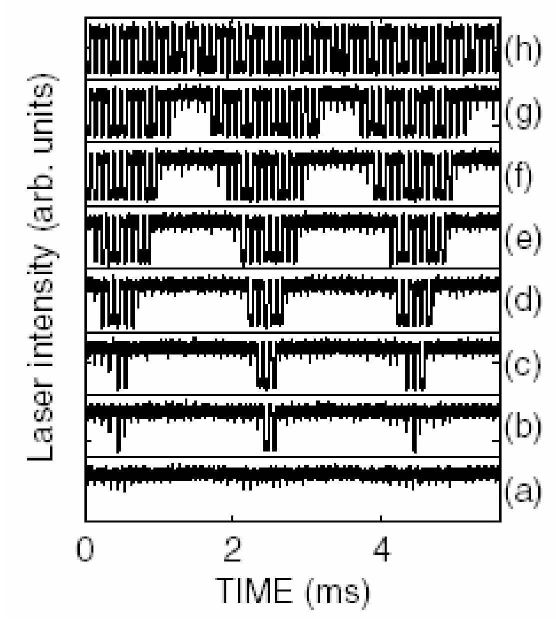


Fig. 4. Temporal response for asymmetric quasipotential; VR: A_{HF} = 0; 16.8; 17.2; 17.8; 19; 20.5; 22; 23 mV (increasing from (a) to (g) waveform)

One can notice the appearance of bursts in the output signal, due to the HF deterministic modulation and having an envelope with the frequency of the LF "useful" signal. We have used an identical LF "useful" signal in respect with the first case (i.e. a 500 Hz sinusoid), but the frequency of the HF deterministic modulation was decreased from 100 KHz to 10 kHz in order to see better the output bursts. This was possible because, using various frequencies, it was observed that VR depends weakly on the HF deterministic modulation frequency, up to a cutoff value, which is characteristic to the system used.

Concerning the gain G and the signal-to-noise ratio SNR^n, the same behaviour can be seen in both cases – symmetric and asymmetric quasipotential – i.e. both mentioned parameters show maxima for a certain (optimal) amplitude of the HF deterministic modulation. This fact is presented in fig. 5. But the SNR^n curves shape is different: for symmetric quasipotential it is sharp, while for a strongly asymmetrical one it is more round, bell-shaped.

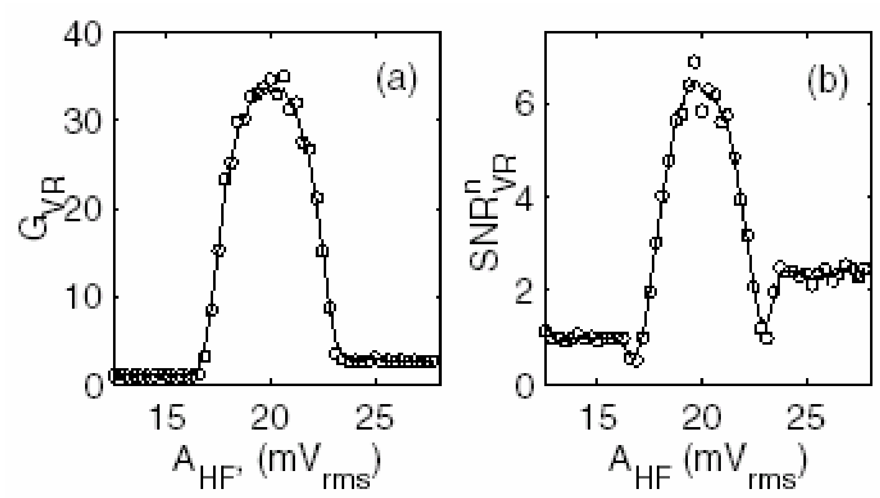
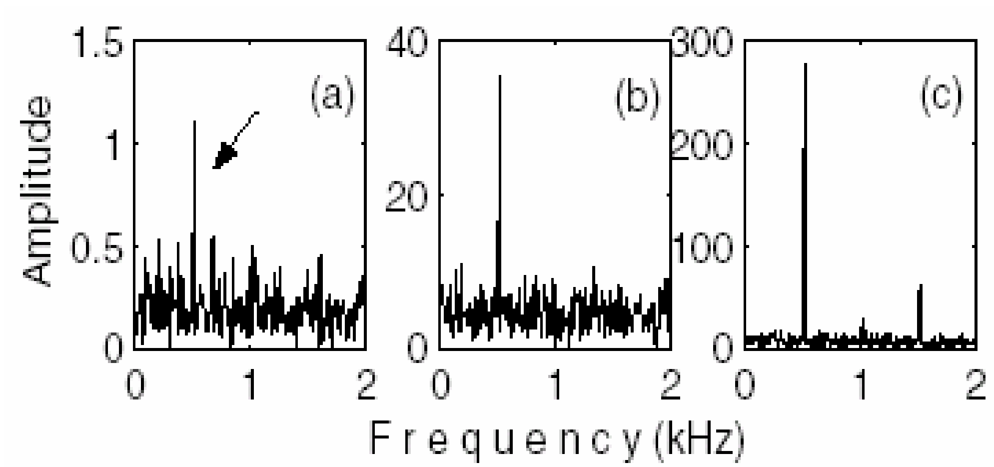


Fig. 5. Response of the system for strongly asymmetric quasipotential;

(a) G_{VR} , (b) SNR^n_{VR} versus A_{HF}

Bistable and / or threshold systems are often used as detectors of low level signals. In this context, large efforts have been devoted during the last years to improve the performance of such systems using the concept of SR [12]-[15]. We shall further show in this paper that VR leads to results which are *clearly better* for these applications, at least when using VCSELs. To this purpose we used (by appropriate VCSEL biasing) a nearly symmetric quasipotential and a "useful" LF sinusoidal signal with the same frequency of 500 Hz, but having the amplitude

A_{LF} lower 8 times in comparison with the previously presented measurements. In fig. 6 are shown the power spectra of the VCSEL output, with no SR or VR – (a), then with SR – (b), and last with VR – (c). In both SR and VR scenarios the amplitudes of the HF modulations were optimal (i.e. those for which the VCSEL output reproduces the “useful” LF signal, like in fig. 2e). The increase of the spectral line of 500 Hz in the SR and VR scenarios versus no SR or VR is evident. In the VR scenario this line is about 10 times stronger than in the SR one. This kind of analysis was then performed for various amplitudes A_{HF} (for VR) and A_N (for SR), in order to see the dependence of the gain G and signal-to-noise ratio SNR^n versus these amplitudes in the two scenarios (SR and VR). The results are presented in fig. 7.



No SR, VR **SR** **VR**

Fig. 6. Detection of low level signals. The arrow indicates the spectral line of the “useful” signal at 500 Hz.

It can be observed here too (fig. 7) that G and SNR^n are much higher for VR than for SR. An explanation of the low SNR^n for SR (approx. 1.3) consists in the low level of the “useful” LF signal [15]-[17]. Another fact which is worth to mention is the narrow shape of the G and SNR^n curves in the VR scenario, in comparison with the SR one. This is a drawback of VR versus SR, as the optimum (i.e. the maxima of the curves) can be obtained by *fine* tuning the amplitude of the HF deterministic modulation [18].

Every point in figures 3, 5, 6 and 7 was obtained by averaging over 50 time series.

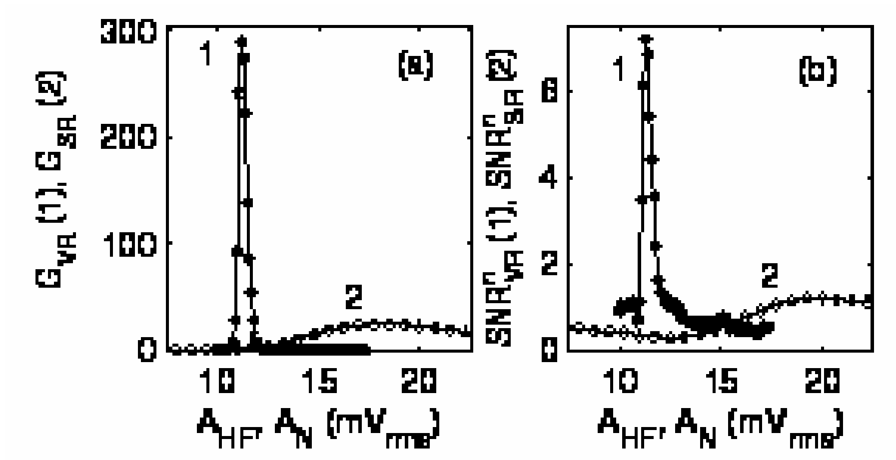


Fig. 7. Gain and signal-to-noise ratio in the scenarios SR (curves 1) and VR (curves 2) with the amplitudes of the "useful" signal far below threshold

3. Conclusions

We have reported on the experimental observation of the phenomenon of "vibrational resonance" in a VCSEL operating in the regime of polarization bistability. We have demonstrated the occurrence of VR for both the case of nearly-symmetric and strongly-asymmetric double-well quasipotentials. The experiments demonstrated the similar behaviour of a VCSEL in both SR and VR scenarios, but the gains and signal-to-noise ratios are clearly better for VR. Besides, we have also experimentally shown the possibility to apply these phenomena for an improvement of low-level detection. The SR scenario leads to a moderate increase of the gain and signal-to-noise ratio (at least for VCSELs), while VR leads to a much higher increase of these "merit" parameters, but the optimum (i.e. their maxima) can be obtained more difficult (by fine tuning the amplitude of the HF deterministic modulation).

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