THEORETICAL DEVELOPMENT OF A MATHEMATICAL MODEL TO EVALUATE GRAVIMETRICAL FLOW RATE OF SEEDS THROUGH ORIFICES

Tudor CĂSĂNDROIU [1], Cristian MIEILĂ [2]

Se cunosc o serie de modele matematice de evaluare a debitelor gravimetrice de curgere ale semințele de cereale prin orificii de diferite forme geometrice şi dimensiuni, dar valorile obținute la evaluările cu acestea sunt, în numeroase cazuri, semnificativ diferite şi limitate ca arii de aplicare. În lucrare se dezvoltă teoretic un model matematic al debitului gravimetric de curgere al semințelor prin orificii, luând în considerare zona de curgere liberă care se formează în numerose situații, deasupra orificiului de curgere, când acesta are anumite dimensiuni. S-a propus pentru debit un model care a fost testat cu date experimentale din literatură pentru semințe de grâu, dând satisfacție, eroarea fiind în majoritatea cazurilor între ± 5%. Modelul propus poate fi folosit într-o sferă mai largă de situații şi este util la evaluarea debitului gravitațional al semințelor prin orificii pentru diverse activități inginereşti.

A series of mathematical models for gravimetric flow rate of seeds through orifices with different geometric shapes are known, but values obtained in many cases are significantly different and have narrow actual range of application. The paper develops a theoretical mathematical model of gravimetric flow rate of seeds through orifices, taking into account the free flowing zone which is generated, in many cases, above the flow orifice, when it has certain dimensions. For the flow ratea a model has been proposed, which was tested with experimental data from scientific literature for wheat seeds. This model is satisfactory, errors compared with measured values in most cases being in the range of ± 5%. The proposed model can be used in a broader range of situations and it is useful in predicting the gravimetric flow rate of seeds through orifices for various engineering activities.

Keywords: seeds, gravimetric flow, flow rate, orifices, geometric shape.

1. Introduction

During handling operations of granular cohesionless materials, particularly of cereal seeds, gravitational flow is frequently encountered during the transfer, when the particles discharge is under the own weight influence [1, 2, 3, 4, 5, 6]. Cereal seeds transfer from storage bunkers is accomplished using the flow

[1] Professor, Chair of Biotechnical Systems, University POLITEHNICA of Bucharest, Romania, e-mail: cctu39@yahoo.com

[2] PhD. Student, Chair of Biotechnical Systems, University POLITEHNICA of Bucharest, Romania

through orifices of various geometric shapes and sizes. Aspects of the seed flow through outlets of storage bunkers are present in the systems of dosing-filling-packaging, in bunkers feeding systems, in the complex technological processes for control and/or monitoring the plants in real time etc. [2, 4, 5, 7]. Since 1900, many researchers have studied the phenomenon of granular materials flow through orifices, from the theoretical and experimental point of view, in particular to develop certain mathematical models for flow rate evaluation through mathematical calculus, as close as possible to the experimental results [1÷6, 8]. Despite current achievements, the level of sophistication required by industry demands, in many cases, a better understanding of the seeds flow behaviour and of the associated performance criteria for the design of seeds handling technical systems [1, 2, 3, 7÷14]. The seeds flow rate depend on the discharge orifice and geometrical shape of the container in which the flow occurs, on the physical properties of the granular material (internal friction, size and shape of the granular material, bulk density, moisture content), on the storage time and on the environmental conditions [2, 4, 6, 8, 10÷15].

If there are level differences between the transfer points of seeds, pipes for gravitational transport can be used [1]. A gravitational transport pipe requires a sufficient slope in order to start the operation without plugging. Generally, the transport pipes do not present convergent sections because they can impede the seeds sliding. The inclination angle for seeds transport pipes has a minimum value of 30° with respect to the horizontal [1, 4, 7, 10, 11, 12, 14]. Seeds velocities and seeds mechanical injuries increase with the slope. The seeds external coats are often abrasive and after a long period of time the seed flow may cause a significant wear and even the appearance of holes in the pipe wall impact points, albeit the pipes are made of steel [1]. It is necessary to size each exhaust orifice in order to ensure the outflow without plugging of the granular material, taking into account the product nature and properties, regarding the friction, and also the outlet building material [4, 7, 10, 11, 12, 14].

The most used mathematical model to predict seeds flow rate through orifices is Beverloo model, which is developed using the dimensional analysis theory applied to the phenomenon of seeds flow through orifices [4]. In the general form of Beverloo model, we take into account that the flow rate mass Q of the particles through orifices depends on the gravitational acceleration g, on a characteristic size of the outlet $R, R = \dfrac{d}{d_p}$, (where d is the diameter of the outlet, and d_p is the diameter of the particles), on the particles friction coefficient μ through coefficient $C(\mu)$ and on the particles bulk density ρ_v of. Thus, for circular shape orifices [3, 4, 9]:

$$Q = C(\mu)\rho_v \sqrt{g} R^{5/2}. \quad (1)$$

For rectangular type orifices, a dependence of the flow rate with the area of the outlet is considered as $Q \sim A^{3/2}$, where A represents the area of the discharge orifice [3, 4, 10]. The Beverloo flow model is certified for large size orifices. For discharge orifices where $R \to 1$, an empirical relationship has been established [3]:

$$Q = C(\mu)\rho_v \sqrt{g}(R-k)^{5/2}, \text{ where } k \in (1,3). \tag{2}$$

According to the British Code of Practice [9, 12], based on Beverloo model, the granular materials flow rate through funnels orifice of undefined shape is given by a relatively simple formula [4, 12]:

$$Q = 0{,}58 \cdot \rho_v \cdot \sqrt{g}\left(d - K_p d_p\right)^{5/2}. \tag{3}$$

where Q is measured by g/sec, ρ_v by g/cm^3, d by cm, d_p by cm and where the shape factor K_p has been introduced. The shape factor is determined by the expression:

$$K_p = \tan(\beta)^{-0.35} \text{ for } \beta < 45^{\circ}, \; K_p = 1 \text{ for } \beta > 45^{\circ}. \tag{4}$$

The effect of the friction coefficient between the funnel wall and the granular material was described by Williams model [9] which leads to valid results for discharge orifices of funnels smaller than 20 mm. This model does not lead to a unique value of discharged flow rate but it ranges the interval of mass flow variation. Maximum limit of flow rate discharged corresponds to the value of the zero friction with the wall. Minimum limit of the flow rate discharged can be estimated from the equation:

$$Q = K_p \cdot \rho_v \cdot g^{1/2} \cdot D_e^{5/2}. \tag{5}$$

In equation (5), for the discharged flow rate of granular materials through the funnel orifices, in this case, the form factor values *(K_p)* are fixed, as follows: for circular orifice K_p = 1.6, for rectangular orifice K_p = 2.4, without taking into account the funnel walls slope [9].

Beverloo et al. [9, 3, 10, 12] evaluated flow rate effectively by placing the term that contains the porosity influence ε of the material and by fixing the form factor at the value K_p = 1.5; therefore, the form factor K_p reframe the effective size of evacuation orifice, depending on the size of the particles taken into account through d_p, such as:

$$Q = 0{,}58 \cdot (1-\varepsilon) \cdot \rho_v \cdot \sqrt{g}\left(D_e - 1{,}5 \cdot d_p\right)^{5/2} \tag{6}$$

This distinction between the British Code of Practice model and Beverloo model indicates that the form factor K_p has in its original meaning proposed by Beverloo the signification of the flow reduction area through an evacuation orifice of some form, and in the case of the British Code of Practice K_p contains the practical influence of the funnel evacuation shape, taking into account the inclination angle of funnel wall.

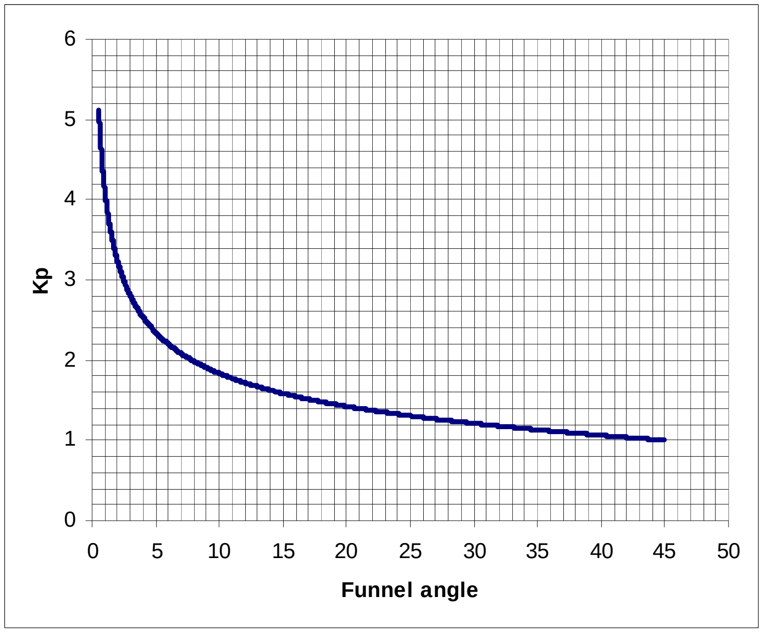

Fig.1. The evaluation of form factor K_p, K_p=(tan β)$^{-0.35}$ for the funnel angle β, β < 45^o , using the British Code of Practice [9] recommendations.

We admit that through proposed values of British Code of Practice for form factor K_p, for circular and square orifices form, in conjunction with Williams model, that the influence of geometrical shape of the orifice on flow rate must take into account both the orifice geometrical shape itself, and the manner in which the outlet is placed in the considered flow system, by considering additional, e.g. of geometrical shape of funnel. It requires such a completed model flow rate through orifices with a coefficient of geometrical shape of the flow system.

The objectives of this paper is to develop a mathematical model to predict flow rate gravitational of seeds through orifices located in a horizontal plane, and to test the model data with technical and scientific literature.

2. Theoretical considerations

In a convergent funnel, granular material is subjected to a stress decrease and it is completely uncompressed to the bottom [10, 12, 16]. It is fully possible that, if a small enough orifice is cut from the bottom of the funnel, the material will not flow until around tensions are not sufficiently small. This is a fundamental difference over the behaviour of fluids whose pressure increases continuously - following hydrostatic distribution of tensions (reaching maximum at the bottom) [10].

To discharge the granular material from a silo it is necessary to provide a sufficient opening for the outlet located at the bottom funnel - to ensure the flow. Jenike's method for funnels design is based on that remark and the fact that the material is forming arches that block the flow, so they need to be continuously broken in order to obtain the desired flow [10, 12]. The model for seed flow through orifice is achieved using the concept of instable arches forming and disintegration, arches of granular material settled above the outlet. From theoretical bases of particles flow the movement of bodies in fluids and the fall speed limit concept are used. The flow is considered continuous if the conditions of forming granular material stable arches are not satisfied. The flow velocity at outlet depends only on the falling height which is given by the granular material arches height profile. Free flowing zone (or free falling zone) of granular material through funnel outlets is supposed to be situated inside a dilated zone, confined by an unstable arch, (see Fig. 2) [15].

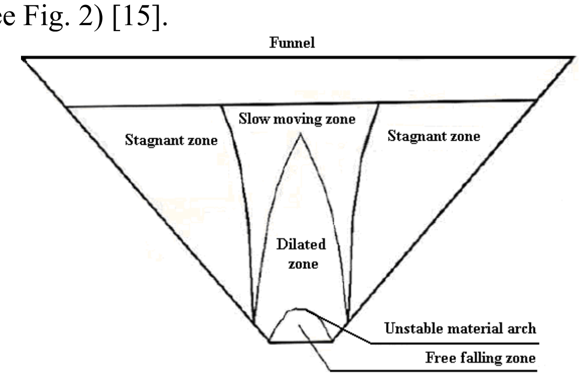

Fig. 2. Diagram of flow zones for granular material discharge from funnel outlets [15]

The existence of a surface above outlet, where the normal stress is zero is admitted. The structure of granular material in these conditions is unstable, yielding a surface of flow called unstable material arch. Under this region, the flow conditions are those with the free fall regardless of height of the material found out over this region, and these conditions are constant during the discharge process.

For modelling the flow through the outlet, the following assumptions are used [5]:

- the arching effect acts at all times on the flowing granular material through outlet and the arches can be stable or instable; this effect is responsible for strangle of flow through the discharge orifice;

- the properties of instable arches are similar to stable arches regarding the shape and stresses;

- the shape of a dome can be described as paraboloid. Its shape depends on the properties of seeds, on the shape of funnel and on the friction of seeds on the wall.

Free flowing zone for the outlet corresponding to a funnel, is alleged to be placed over the entire area of the discharge orifice and bounded by a rotation paraboloid having the form of a granular material unstable arch, (Fig.3) [5].

To predict the flow rate of seeds through the circular orifice of diameter *d*, in terms of above assumptions, we assess seeds average velocity through the discharge orifice. The surface of the revolution paraboloid is generated by its generator parabola axis of symmetry rotation. With respect to the system of axes XOY from Fig. 3, the equation of the generator parabola is:

$$y = h\left[1-\left(\frac{2x}{d}\right)^2\right], \tag{7}$$

where: *h* represent the height dome of granular material; *d* represents the diameter of the outlet, $\delta = h/d$ means the rate of the height arch and the diameter of outlet for a granular material with data from paper [5], see Table 1.

In a vacuum, all bodies left to free fall they are moving uniformly accelerated, with a speed *v* which depending on the height of the fall, and being independent of size, shape and density of the body [4]. For the arch of granular material being above the outlet is results, in the plane XOY:

$$v(x) = \sqrt{2gy}\,. \tag{8}$$

In these conditions, the discharged seeds velocity field of the circular shape orifice with diameter *d*, taking into account equation (7), can be assessed by:

$$v(x) = \sqrt{2g\delta d\left(1-\left(\frac{2x}{d}\right)^2\right)}\,. \tag{9}$$

Considering the speed of flow being constant on circular coronae of radius *x* and width d*x* and expressed by equation (9), the average velocity will be:

$$v_m = \frac{\int_0^{d/2} 2\pi \cdot x \cdot v(x)dx}{\pi \frac{d^2}{4}}\,. \tag{10}$$

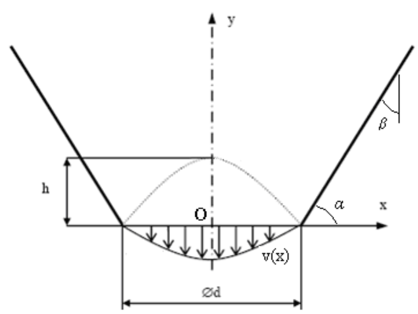

Fig. 3. Free flowing zone for a funnel discharge orifice [5]

Table 1

The rate of arch height and diameter of outlet for certain cultivation plant seeds, δ=h/d [5]

Material:	Wheat seeds	Maize seeds	Poppy seeds	Oat seeds
δ:	0,4	0,3	0,46	0,3

Substituting in this equation *v(x)* through equation (9), after carrying out these calculations, one obtains:

$$v_m = \frac{2}{3}\sqrt{2g\delta d}\,. \tag{11}$$

If constant bulk density of seeds ρ_v near the outlet is assumed, the mass flow rate, *Q* [g/s], can be expressed as:

$$Q = \rho_v \cdot \frac{\pi \cdot d^2}{4} \cdot v_m, \tag{12}$$

Taking into account the equation (11), the mass flow rate becomes:

$$Q = \frac{\pi\sqrt{2g}}{6}\sqrt{\delta}\rho_v \cdot d^{\frac{5}{2}}. \tag{13}$$

The seeds are brought to the funnels outlet through the angle of inclination α of the walls vs. horizontal, Fig. 3. Assimilating the funnels walls with inclined planes with α angle, the seeds friction displacement speed on the walls for a height of dropping *h*, admitting that they begin moving from the state of rest is [7]:

$$v = \sqrt{2g \cdot h(1 - \mu \cdot ctg\alpha)}, \tag{14}$$

where μ is the seeds friction coefficient on the funnel wall.

For inclined funnel walls with $\alpha = 45^o$ vs. horizontal, the seeds velocity on the walls at the lower end of these is:

$$v = \sqrt{2gh} \cdot \sqrt{1 - \mu}\,. \tag{15}$$

This equation can be used in the following form:

$$v = \sqrt{2gh} \cdot (1 - tg\varphi)^{1/2} . \quad (16)$$

where, μ = tgφ, in which φ is the friction angle of the material on the funnel walls. The equation (16) is suggesting that, if we take into account the fact that the flow rate of the seeds through the orifices is influenced by the flow through the funnel, the placement of $(1 - tg\varphi)^{1/2}$ as correction factor in the mathematical flow rate assessment relationship is justified, and equation (13) becomes:

$$Q = \frac{\pi\sqrt{2}}{6}\sqrt{\delta \cdot g \cdot \rho_V} \cdot d^{5/2}(1 - tg\varphi)^{1/2} . \quad (17)$$

Equation (17) represents a theoretical fundament for the mathematical model of the seeds gravimetrical flow rate, seeds which are flowing through a circular orifice, and gives also a fundament for the model found by Beverloo, eq. (3), based on the applied dimensional analyses theory to the study of the phenomenon of seeds flow through orifices .

Prediction of the flow rate using eq. (13) and (17) was tested with our experimental data and with experimental data available in scientific literature [5].

3. Methods, procedures and discussions.

To test the proposed mathematical models for flow rate through orifices, we used data obtained from experiments with wheat seeds, for circular orifices in range of 20 mm ÷ 60 mm [13], presented in Table 3.

The flow rate model for seeds, based on instable arches disintegration, is experimentally validated for circular outlets within 20 mm ÷ 60 mm, where the hopper effect for hopper walls with 45º slope, attached to a bin with 100 mm diameter is relieved. Five different measurements were undertaken using wheat seed samples of 5 kg, with total flow time keeping. Physical characteristics of the seeds utilised in experiments are given in Table 2.

Table 2

Seeds and hopper characteristics utilized in experiments

Wheat seeds		*Hopper fitted with outlet orifice*	
Particle size (mm) (l × b × c)	5.2×3.3×2.8 (average for 20 seeds)	Geometrical shape	circular
Bulk density of stationary and free material (kg/m^3)	760	Diameter of discharge orifice, (mm)	20...60
Bulk density of compacted material (kg/m^3)	843	Wall angle with the vertical θ, (˚)	45
Moisture content, (% w.b)	12.7	Sliding coefficient of friction	0.36

In our previous work [11], we proposed as mathematical model to evaluate flow rate of seeds through orifices, the following power-type relationship:

$$Q = K \cdot D_h^{\,n}, \qquad (18)$$

where D_h represents the orifice hydraulic diameter, and K and n are constant coefficients determined by nonlinear regression. Using data from Table 3, values n=2.67 and K=0.0155 for units D_h [mm] and Q [g/s], for a correlation coefficient R^2>0.99 were found for n and K.

On the graph, the curve corresponding to equations (18) compared with the experimental data is presented in Fig. 4. A good agreement between values evaluated by the proposed model equation and experimental data can be noticed. The drawback of this mathematical model is the limited actual range, dependent on working conditions; working conditions is modifying the values for constant K [11]. Odal [5] showed that for values of discharge orifice diameter of 90 mm ÷ 100mm, arching capacity is reduced or nil.

The consideration of hopper angle inside Johansson flow rate model [12] brings corrections for values assessed by Beverloo model. The Beverloo model does not include similar parameters – it is based on bulk density, outlet diameter, particle diameter and empirical constants (see eq. (6)).

Examining the data from Table 3 we see that flow rates assessed using equation (13), for orifice diameters in range of 20 mm ÷ 60 mm, is significantly higher than those measured, deviations being within +10.4% ÷ +37.6%, while the data evaluated with equation (17), are significantly closer to the measured values, with deviations boundaries of −11.7% ÷ +10.1%; in many cases the deviation is within ± 5%.

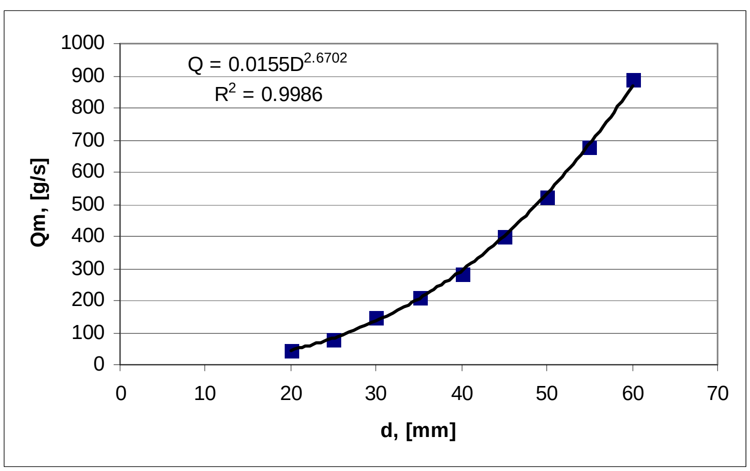

Fig. 4. The regression curve for flow rate comparing to measured flow rate (Q_m) function of discharge orifice diameter (d), for wheat seeds

Table 3

Measured* and calculated data ** for wheat seeds flow rate versus orifice diameter and the relative errors

Orifice diameter d, [mm]	Measured flow rate Q_m, [g/s]	Calculated flow rates, Q_c, [g/s]		Relative errors (%), $100(Q_c - Q_m)/Q_m$	
		Eq. (13)	Eq. (17)	Eq. (13)	Eq. (17)
20	46	63.03	50.42	37.02	9.61
25	80	110.11	88.09	37.64	10.11
30	147	173.69	138.95	18.16	-5.48
35	210	255.35	204.28	21.60	-2.72
40	285	356.55	285.24	25.11	0.08
45	400	478.63	382.91	19.66	-4.27
50	520	622.87	498.29	19.78	-4.18
55	680	790.46	632.37	16.24	-7.00
60	890	982.54	786.03	10.40	-11.68

* average for 5 tests with seeds weight of 5 kg/batch
** the measured and calculated data is corresponding to the following values [5]: δ=0.4 (see Table 1).; and ρ_v=760 kg/m^3; tgφ=0.36, (see Table 2).

Based on the data in Table 3, in Fig. 5, were represented the variation curves of wheat seeds flow rate assessed using equation (13) and equation (17), compared with experimental data. This figure is emphasizing, more suggestively, the above conclusion. Hence we conclude that equation (17) allows more accurate assessment of wheat seeds flow rate through circular orifices with diameters ranged from 20 mm to 60 mm.

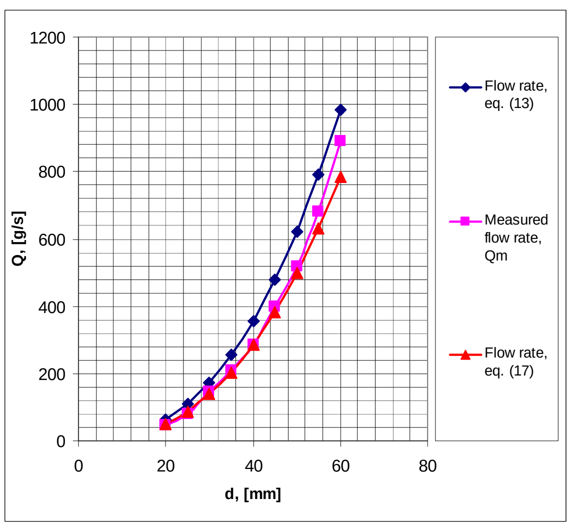

Fig. 5. Measured flow rate (Q_m) and flow rate assessed using equation (13) and equation (17), versus discharge orifice diameter (d), for wheat seeds

Equation (17) explicitly includes the influence on flow rate of the main physical and geometrical characteristics of pair orifice-granular material through parameters: d, δ, ρ_v, and φ, which broadens the scope of use in terms of valuations and other condition parameters listed, related to other types of seeds.

The shape of material arches evaluated by the coefficient δ, is dependent on the wall friction coefficient and hopper angle. Therefore, the conclusion that the shape of arches is dependent on the shape of the wall where the discharge orifice is connected. The present paper does not analyse the physical phenomenon of arches forming and disintegrating above the discharge orifice with diameter d, arches located at height h. This phenomenon requires further research.

The corrected flow rate model, equation (17), proposed in the present paper take into account the influence of arch shape (and thus the hopper geometrical shape influence) and leads to maximum errors up to 12%, indicating a better method of assessment for flow rate through the considered orifices, compared with the model given by equation (13).

4. Conclusions

A mathematical model of seeds gravimetric flow rate through orifices, based on disintegration of the arches formed above the orifice, was proposed in the form of equation (17).

The seeds flow rate model based on arches disintegration was validated with experimental data from [13], for discharge orifices within 20mm ÷ 60 mm. To estimate the arch profile of material, data from [13] was used, where $\delta=h/d$, with $\delta=0.4$, for wheat seeds. The gravimetric flow rate model, expressed by equation (17), proposed in this paper, takes into account the geometric shape of the hopper (considering the wall friction) and lead to maximum errors up to 12%, compared to the measured flow rate.

The accuracy of the proposed model for flow rate for cohesionless granular materials, particularly for seeds, expressed through equation (17), enable us to consider the assumptions concerning the arch formation and disintegration above the discharge orifice as correct. The flow model described by equation (17), has a broader range of use because it includes explicitly the main parameters of pair orifice-seeds, and it is useful in engineering design work, adjustments and control activities regarding the seeds flow rate through orifices.

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