U.P.B. Sci. Bull., Series A, Vol. 69, No. 1, 2007 ISSN 1223-7027

CALIBRATION PROCEDURE FOR STABILIZED LASERS USING THE METHOD OF OPTICAL BEATS MEASUREMENT UNCERTAINTY

Elena DUGHEANU[1]

Lucrarea descrie metoda de etalonare a laserelor stabilizate prin metoda absorbției saturate în cazul $^{127}I_2$ prin metoda bătăilor optice, aplicată în laboratorul Lungimi al Institutului Național de Metrologie, Bucureşti, Romania şi estimarea bugetului de incertitudine a instalației de comparare a frecvențelor laser, în conformitate cu specificațiile standardului SR ENV 13005:2003 „ Ghid pentru exprimarea incertitudinii de măsurare".

The paper describes the calibration method used for the calibration of stabilized lasers by saturated absorption in molecular iodine $^{127}I_2$ and the estimation of the uncertainty budget of the installation used for the comparison of two laser frequencies in Length Laboratory, National Institute of Metrology, Bucharest, Romania. The contribution of the author consists of the analyses of the sources of error which appear during the measurement, due to laser and to the equipments that compound the installation, according to the specifications of "Guide to the expression of uncertainty in measurements, SR ENV 13005:2003"

Key words: stabilized lasers, installation for calibration of stabilized lasers, uncertainty budget

1. Introduction

Before the invention of the laser, the infrared and visible spectrum were studied using interference techniques whose accuracy was limited by the nature of radiation source as spectral width and by method itself (errors due to diffraction, mirrors phase, etc.).

The measurement of frequencies is in principle not limited by stability and accuracy of the oscillators used. The measurement method of frequency is more accurate with several orders than the measurement of wavelength.

A laser is a monochromatic source radiation having a spectral width comparable to radio frequency and microwaves oscillator.

From the second resolution of CIPM – Comité International des Poids et Mesures adopted in 1983, the length may be defined as the interval of time in which the light passes a distance. This principle is put into practice by a

[1] Phys.; National Institute of Metrology Bucharest, ROMANIA

technique of interferential measurement method where the displacement of the mobile retroreflector of the interferometer is given by the following formula:

$$L = (k+\varepsilon)\lambda /2 \quad (1)$$

using $\lambda = \frac{c}{f}$, equation (1) becomes:

$$L = (k+\varepsilon)\cdot\frac{c}{2f} \quad (2)$$

where: k is a whole number given by numbers of interferential fringes, counted by interferometer counter, ε represents the fraction fringe given by known radiation and c is the light velocity.

The optic interferometer could be thus considered as a length measurement device, based on its measurements of the time necessary for the light to pass through a given distance. This interpretation used the light's frequency as a clock to measure the period of time, from where the length results according to equation (2). The generation and the measurement of optical frequencies were possible due to the achievement of stabilized lasers based on saturated absorption and to the development in optical range of a heterodyne technique where one or more oscillators (from which one has a known frequency) are mixed in an adapted element in order to give a difference of low frequency which can be measured directly (fig 1).

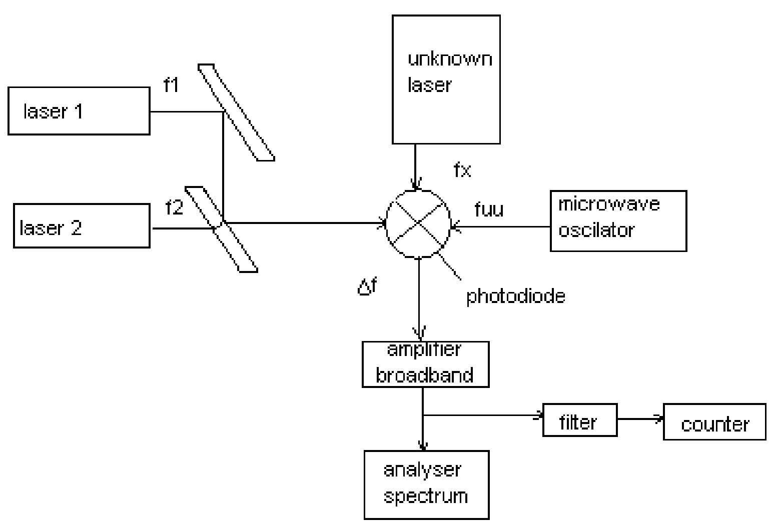

Fig. 1: Schematic of principle of heterodyne technique

Known frequency can be synthesized as an exact harmonic closed by the frequency that is mixed. This harmonic is mixed with unknown frequency, in an avalanche photo diode, so the frequencies of the heterodyne beats are close to MHz order. In this case, the unknown frequency f_x is given by formula [3]:

$$\Delta f = f_x - lf_1 - mf_2 - nf_2 - nf_{\mu u} \quad (3)$$

where: Δf are the frequency beats, f_x is the unknown frequency, f_1, f_2 are the known frequencies of the two lasers and $f_{\mu u}$ is the frequency of microwaves oscillator.

In this case the mix order is given by formula:

$$p = l + m + n \quad (4)$$

In a particular case in which a laser having a known frequency is compared with a laser having an unknown frequency, the following formula can be used:

$$f_x = kf_e \pm \Delta f \quad (5)$$

where k is a harmonic number that can be determined by measuring of a known length.

2. Measuring of the laser frequencies using the method of beats frequencies

A known method of measuring lasers frequencies is the method of optical beats. Due to the very high stability and very fine spectral width of radiations the method permits the measurement of the beats between two different lasers with closed frequencies, f_1 şi f_2.

The method consist**s** of the simultaneous illumination of the same avalanche photo diode and two laser beams using a beam splitter and several optical elements for alignment. The illumination of the fotodetector and the output current are modulated with beats frequency $\Delta f = f_1 - f_2$. Modulation frequency Δf of the current is measured by usual radio electronic methods.

This installation consists of (fig:2): 2 lasers (I and II) between which is measured the difference in frequency, optical elements for beam lasers alignment (1 and 3), mirror for the reflection of laser beam with the possibility of rotation in two rectangular axes (2), beam splitter with the possibility of rotation in two rectangular axes (4), lens with a focal distance of 1 cm, mounted on a magnetic support and provided with a sliding bar (5), avalanche photo diode mounted on a support adjustable on x, y, z axes (6), impedance adaptor, decade attenuator (7), broadband amplifier (1 GHz, G = 20) (8), frequency meter 1,3 GHz with low level of automatic trigger of counting (9), 1GHz band spectrum analyzer (10) and computer (11).

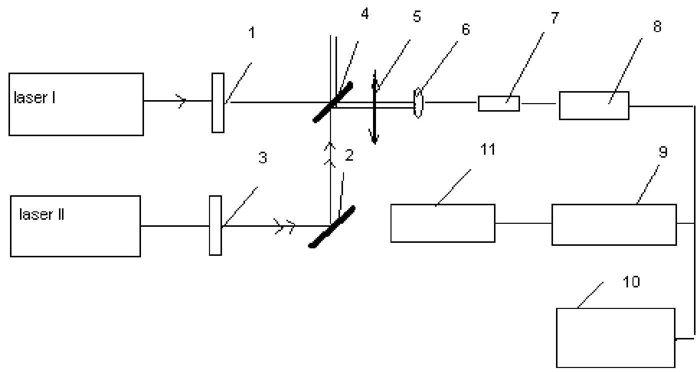

Fig. 2: Installation for measuring the laser frequency using the optical beat method

The whole installation is mounted on a stable metal base equipped with pneumatic suspension to reduce the level of vibrations under 10 Hz.
This method permits the comparison of two closed laser frequencies with a maxim difference of 1Ghz. The ranges of frequency, which are measured in case of stabilized lasers in frequency on the same nominal value, are between 0.1 MHz and 1 GHz.

The unknown frequency f_x which depends on standard frequency f_e is given by the formula:

$$f_x = f_e \pm \Delta f \qquad (6)$$

The photo diode behaviour is in avalanche mode, for a high frequency (manufactured by RCA – Electro-Optics, Canada, type C 30902 E) and has the following characteristics:

- spectral range of measurement: λ = (400…1000) nm;
- small responding time: 0,5 ns
- current density at 22 ^{0}C: 5 mA/mm^2
- photosensitive layer: circular surface equal to 0.2 mm^2;
- dark current: ($1.5\cdot10^{-8}\ldots3\cdot10^{-8}$) A;
- noise current: : ($2.3\cdot10^{-13}\ldots5\cdot10^{-13}$) A/Hz$^{1/2}$
- power supply: (180 … 250) V.

A schematic of the photodiode is showed in fig. 3. The electronic components have the following values: R_1 = 2.7 kΩ, R_2 = 510 Ω, C_1 = 4.7nF/500V, C_2 = 4.7nF/500V, C_3 = 4.7nF/500V, FD photodiode C 30902 E, RCA.

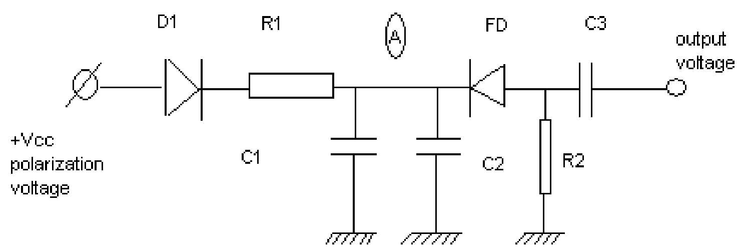

Fig. 3: Schematic of the photodiode

3. Program for the measuring of laser frequencies

The program used for measuring laser frequencies and processing the experimental data is a computer based program which has the following set of instructions:

a) initialize the HP-IB interface

b) order HP 5328 B frequency meter to set the data (resolution, interval between two readings, etc)

c) permit the user to set up the following parameters:

- sampling interval;
- "n" number of readings at a time "*t*";
- determination of average value of frequency "*f*"; standard deviation of frequency "*f*"; Allan *s*(2,*t*) variance;
- displaying of results;
- printing the results.

d) processing of the experimental data permit**s** the determination of the following characteristics:

- determination of f_x frequency of laser which is calibrated according to formula (6):
- standard deviation (*s*) for one measurement for an establish sample of time "*t*" and number of readings "*n*":

$$s = \sqrt{\frac{\sum_{i=1}^{n}(\Delta f_i - \bar{\Delta f})^2}{n-1}} \tag{7}$$

where:

$$\bar{\Delta f} = \frac{\sum_{i=1}^{n}\Delta f_i}{n} \tag{8}$$

is the frequencies average. The stability of frequency s (2,t), is determined using Allan's variance formula:

$$s(2,t) = \frac{1}{\sqrt{2}}\sqrt{\frac{\sum_{i=2}^{N}(\Delta f_i - \Delta f_{i-1})^2}{n-1}} \tag{9}$$

where: t is the sample time that be between (0.1…1000) s, Δf_i, Δf_{i-1} represent the measurements of successive differences in frequency between the two frequency between the two lasers and n is the number of readings.

The relative value of stability of frequency is determined by formula:

$$s(2,t)_{rel} = \frac{s(2,t)}{f_x} \tag{10}$$

where f_x represents the frequency of the laser which is tested.

4. Measurement uncertainty of installation for the measuring of the laser frequencies

Uncertainty measurement u is an estimator that establishes the limits of the result values. That includes, with a probability, the true value of measurand. The measurement uncertainty consists of several components. One of them (u_A component) can be estimated from statistic distributions of values and can be characterized from experimental standard deviation; other ones (u_B components) are estimated from previous experiments or from other information.

The determination of uncertainty of measurement of laser frequency using the special installation for measuring the beats frequencies is based on the evaluation of type A component for repeatability of measurement of laser stabilized frequency and of type B component, achieved by evaluating the measurement uncertainty of equipments which compound the installation.

4.1 Value of type A uncertainty, determined from repeatability of measurement of laser stabilized frequency is calculated by formula:

$$u_A = \frac{s}{\sqrt{n}} \tag{11}$$

From experimental data using a standard frequency generator, results u_A = 0,4 kHz

4.2 Value of type B uncertainty is given by: the uncertainty of frequency meter, u_{E1}, the uncertainty of avalanche photo diode, u_{E2}, the uncertainty of absolute laser frequency, according the BIPM certificate no. 64 / August 23 2005, u_l (f_f) = 0,2 kHz, for a frequency: f_s = 473 612 353 612.7 kHz.

a) The frequency meter error (E_1), based on the technical documentation of apparatus is given by the following formula:

$$E_1 = \pm \text{resolution} \pm \text{time base error} \cdot \text{frequency} \quad (12)$$

Using the following data: the resolution = 1 Hz, the time base error = 10^{-7}, the frequency, $f \cong 5 \cdot 10^5$ kHz one obtains $E_1 = \pm 0,051$ kHz.
For a rectangular distribution, the value of uncertainty is $u_{E1} = 0.029$ kHz.

b) the error determined by the noise of avalanche photo diode is, based on the documentation, equal to $E_2 = \pm 2 \cdot 10^{-3}$ kHz. (13)

For a rectangular distribution, the value of uncertainty is $u_{E2} = 0,001$ kHz. The B component of uncertainty is:

$$u_B = \sqrt{u^2(f_f) + u^2(E_1) + u^2(E_2)} = 0,202 \text{ kHz} \quad (14)$$

4.3 Composed uncertainty is:

$$u_C = \sqrt{u_A^2 + u_B^2} = 0.45 \text{ kHz} \quad (15)$$

The relative uncertainty of the installation is given by formula:

$$u_r = \frac{u_c}{f_l} \quad (16)$$

where: $u_r = \frac{0.45}{473612353612.7} = 0.95 \cdot 10^{-12}$.

4.4 Extended uncertainty
4.5

For an extended factor $k = 2$, the relative extended uncertainty U of the installation is given by formula:

$$U_r = 2 \cdot u_r = 1,90 \cdot 10^{-12} \quad (17)$$

5. Conclusions

In Romania, the transmission of unit of length to secondary standards is made using a stabilized laser by saturated absorption in molecular iodine $^{127}I_2$ as national standard of length in association with a special installation for calibration of others lasers He-Ne, as secondary standards of length.

The installation which is unique in our country offers the following advantages: transfers the measurement unit, the wavelength of He-Ne laser stabilized in frequency, directly from the national standard of length as He-Ne laser stabilized in frequency which is connected to the cesium standard of frequency and the light velocity in vacuum, has a high reproducibility of measurement unit ($10^{-10} \ldots 10^{-8}$).

The value of the measurement uncertainty of the installation is a very important parameter in determining the metrological characteristics of stabilized laser in frequency and assures the dissemination of length unit from the National Standard of Length to all secondary and reference standards of length with high accuracy, at a very closed level provided by metrological laboratories of length from developed countries, as it is showed in recent international key comparison BIPM.L – K11/2005.

R E F E R E N C E S

[1] Guide to the expression of uncertainty in measurements, SR ENV 13005:2003
[2] *I.M. Popescu, A.M. Preda, G.F. Cone* - "Aplicații ale laserilor", Ed. Tehnică 1979
[3] *D. Boiciuc*, Teza de doctorat:"Laserul în metrologia lungimilor", 1996
[4] *C. S. Edwards, G. P. Barwood, P. Gill and W. R. C. Rowley*, "A 633 nm iodine-stabilized diode-laser frequency standard", Metrologia 36, 41 (1999).
[5*] J.A.Stone, S.D.Phillips and G.A.Mandolfo*, Corrections for Wavelength Variation in Precision Interferometric Displacement Measurements, J.Res.Natl.Inst.Std and Technol. (US) pg. 101 (**5**) (1996)
[6] Journal of Research of the National Institute of Standards and Technology, Volume 104, Number 3, May-June 1999
[7] *A. Duta, D. Boiciuc, D.Georgescu*, Results from the CI-2005 campaign at the BIPM of the BIPM.L-K11 ongoing key comparison.