ON THE STRUCTURE OF SOFT m-ARY SEMIHYPERGROUPS

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Molodtsov introduced the concept of soft set theory, which can be seen and used as a new mathematical tool for dealing with uncertainly. In this paper we introduce and initiate the study of soft m-ary semihypergroups by using the soft set theory. The notions of soft m-ary semihypergroups, soft m-ary subsemihypergroups, soft j-hyperideals, soft hyperideals, soft quasi-hyperideals and soft bi-hyperideals are introduced, and several related properties are investigated.

1. Introduction and preliminaries

Hyperstructure theory was introduced in 1934, at the eighth congress of Scandinavian Mathematicians, when F. Marty [26] defined hypergroups based on the notion of hyperoperation, began to analyze their properties and applied them to groups. In the following decades and nowadays, a number of different hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics by many mathematicians such as in fuzzy sets and rough set theory, optimization theory, theory of discrete event dynamical systems, cryptography, codes, analysis of computer programs, automata, formal language theory, combinatorics, artificial intelligence, probability, graphs and hypergraphs, geometry, lattices and binary relations, physics, chemistry, biology etc. (see [6, 7, 8, 9, 10, 15, 16, 36]).

Many complicated problems in economics, engineering, environment, social sciences, medicine and many other fields involve uncertain data. Most of the modeling, reasoning and calculations are performed by those mathematical tools which are certain or precise and deal with certain problems, so these are not useful to deal with uncertain problems. In classical mathematics, a mathematical model of an object is devised and the notion of the exact solution of this model is determined. Because of that the mathematical model is too complex, the exact solution cannot be found. There are several well-known theories developed to overcome these difficulties which

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arise due to uncertainty. For instance probability theory, fuzzy sets theory [38], rough sets theory [29] and other mathematical tools. But all these theories have their inherited difficulties as pointed out by Molodtsov [27]. In probability theory, we have to make a great deal of experiments in order to check samples. In social sciences and economics, it is not always possible to make so many experiments.

Pawlak [29] introduced the theory of rough sets in 1982. It was a significant approach for modeling vagueness. In this theory, equivalence classes are used to approximate crisp subsets by their upper and lower approximations. This theory has been applied to many problems successfully, yet has its own limitations. It is not always possible to have an equivalence relation among the elements of a given set, so we cannot have equivalence classes to get upper and lower approximations of a subset. Later, some authors [35] tried to have approximations not with the help of equivalence relation but by a relation in general.

Fuzzy set theory was developed by Zadeh [38]. It is the most appropriate approach to deal with uncertainties. However, some authors [27] think that the difficulties in fuzzy set theory are due to the inadequacy of parameterization tools of the theory.

In 1999, Molodtsov introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties that is free from the difficulties affecting existing methods. Soft set theory has rich potential for applications in several directions, few of which had been demonstrated by Molodtsov in his pioneer work [27]. Molodtsov also showed how Soft Set Theory (SST) is free from parametrization inadequacy syndrom of Fuzzy Set Theory (FST), Rough Set Theory (RST),Probability Theory, and Game Theory. SST is a very general framework. At present, research in the theory of soft sets is in progress. Many papers have appeared to discuss theoretical and application aspects of the soft set theory [25, 30, 31, 24]. Maji et al. [25] described the application of soft set theory to a decision making problem and studied several operations on the theory of soft sets. Pei et al. [30] discussed the relationship between soft sets and information systems. Roy et al. [31] described the application of fuzzy soft set theory to a decision making problem. Maji et al. [24] studied several operations on the theory of soft sets. Ali et al. [3] also studied some new notions such as the restricted intersection, the restricted union, the restricted difference, and the extended intersection of two soft sets. The algebraic structure of soft sets has been studied by several authors. For example, Aktas and Cagman [2] introduced the basic concepts of soft set theory and compared soft sets to the related concepts of fuzzy sets and rough sets. They also discussed the notion of soft groups and drove their basic properties using Molodtsov's definition of soft sets. Other applications of soft set theory in different algebraic structure can be found in [4, 19, 20, 21, 28, 22, 14, 33, 1, 32] etc. Recently, Leoreanu-Fotea and Corsini [23], then Davvaz et. al. [5, 34, 37] and Hila et. al. [18] introduced the notion of soft hyperstructure studying the notion of soft hypergroupoids, soft semihypergroups, soft polygroups, soft ternary semihypergroups which extends the notion of the hypergroupoids and semihypergroups to include the algebraic structures of soft sets.

In this paper, we introduce and initiate the study of soft m-ary semihyper-groups by using the soft set theory. The notions of soft m-ary semihypergroups,

soft m-ary subsemihypergroups, soft j-hyperideals, soft hyperideals, soft quasi-hyperideals and soft bi-hyperideals are introduced, and several related properties are investigated.

2. Algebraic hypersystems and m-ary hyperstructures

In this section we recall some known notions on what is meant by an algebraic hypersystem and m-ary hyperstructure.

Let H be a nonempty set and f be a mapping $f : H \times H \to \mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ denotes the set of all nonempty subsets of H. Then f is called a *binary (algebraic) hyperoperation* on H. In general, a mapping $f : H \times H \times ... \times H \to \mathcal{P}^*(H)$ where H appears m times, is called an *m-ary (algebraic) hyperoperation*, and m is called the *arity* of this hyperoperation. An algebraic system (H, f), where f is an m-ary hyperoperation defined on H, is called an *m-ary hypergroupoid* or an *m-ary hypersystem*. Since we identify the set $\{x\}$ with the element x, any m-ary (binary) groupoid is an m-ary (binary) hypergroupoid. (see [12, 13, 17, 11]).

Let f be an m-ary hyperoperation on H and $A_1, A_2, ..., A_m$ subsets of H. We define $f(A_1, A_2, ..., A_m) = \cup\{f(x_1, x_2, ..., x_m)|x_i \in A_i, i = 1, 2, ..., m\}$. We shall use the follwoing abbreviated notation: the sequence $x_i, x_{i+1}, ..., x_j$ will be denoted by x_i^j. For $j < i$, x_i^j is the empty symbol. In this convention, $f(x_1, ..., x_i, y_{i+1}, ..., y_j, z_{j+1}, ..., z_m)$ will be written as $f(x_1^i, y_{i+1}^j, z_{j+1}^m)$. In the case when $y_{i+1} = ... = y_j = y$, the last expression will be written in the form $f(x_1^i, \overset{(j-i)}{y}, z_{j+1}^m)$. Similarly, for subsets $A_1, A_2, ..., A_m$ of H we define

$$f(A_1^m) = f(A_1, A_2, ..., A_m) = \cup\{f(x_1^m)|x_i \in A_i, i = 1, ..., m\}.$$

An m-ary hyperoperation f is called $(i, j) - associative$ if

$$f(x_1^{i-1}, f(x_i^{m+i-1}), x_{m+i}^{2m-1}) = f(x_1^{j-1}, f(x_j^{m+j-1}), x_{m+j}^{2m-1}),$$

holds for fixed $1 \leq i < j \leq m$ and all $x_1, x_2, ..., x_{2m-1} \in H$.

Note that (i, k)-associativity follows from (i, j)- and (j, k)-associativities.

If the above condition is satisfied for all $i, j \in \{1, 2, ..., m\}$, then we say that f is *associative*.

By an *algebraic hypersystem* $(H, f_1, f_2, ..., f_m)$ or simply H is meant a set H closed under a collection of m_i-ary hyperoperation f_i and often also satisfying a fixed set of laws, for instant, the associative law. A subset S of H constitues a *subhypersystem* iff S is closed under the same hyperoperations and satisfies the same fixed laws in H.

Let H be an algebraic hypersystem. A *j-hyperideal* $j = 1, 2, ..., m$ relative to the m-ary hyperoperation is defined to be a subhypersystem I_j such that for any $x_1, x_2, ..., x_m \in H$, if $x_j \in I_j$ then $f(x_1, x_2, ..., x_m) \subseteq I_j$. The j-hyperideal relative to f generated by an element $a \in H$ (usually called a principal j-hyperideal) is denoted by $(a)_j = f(H, H, ..., \overset{j}{a}, ..., H) \cup \{a\}$. A subhypersystem I which is a j-hyperideal for each $j = 1, ..., m$ is simply called an *hyperideal*. An m-ary hypergroupoid (H, f) will be called an *m-ary semihypergroup* if and only if f is associative.

3. Preliminaries from soft set theory

Let U be an initial universe set and E be a set of parameters. The power set of U is denoted by $\mathcal{P}(U)$ and A is a subset of E.

Definition 3.1. *A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.*

In the other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A, F(a)$ may be considered as the set of a-approximate elements of the soft set (F, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [27]. To illustrate this idea, let us consider the following example. We quote it directly from [24].

Example 3.1. [24] *Let us consider a soft set (F, E), which describes the "attractiveness of houses" that one is considering for purchase. Suppose that there are six houses in the universe U, given by $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of decision parameters, where $e_i(i = 1, 2, 3, 4, 5)$ stand for the parameters "expensive", "beautiful", "wooden", "cheap" and "in green surroundings", respectively. Consider the mapping F denoted by houses $(\cdot)$, where the dot position $(\cdot)$ is to be filled by one of the parameters $e_i \in E$. For instance, suppose that $F(e_1) = \{h_2, h_4\}, F(e_2) = \{h_1, h_3\}, F(e_3) = \{h_3, h_4, h_5\}, F(e_4) = \{h_1, h_3, h_5\}, F(e_5) = \{h_1\}$. The soft set (F, E) is a parameterized family $\{F(e_i), i = 1, ..., 5\}$ of subsets of the set U, and can be viewed as a collection of approximations: $(F, E) = \{expensive\ houses = \{h_2, h_4\},\ beautiful\ houses = \{h_1, h_3\},\ wooden\ houses = \{h_3, h_4, h_5\},\ cheap\ houses = \{h_1, h_3, h_5\},\ in\ green\ surroundings\ houses = \{h_1\}\}$. Each approximation has two parts: a predicate and an approximate value set.*

In [14], for a soft set (F, A), the set $Supp(F, A) = \{x \in A | F(x) \neq \emptyset\}$ is called the support of the soft set (F, A). If $Supp(F, A) \neq \emptyset$, then a soft set (F, A) is called non-null.

Definition 3.2. *Let (F, A) and (G, B) be two soft sets over a common universe U. The extended intersection of (F, A) and (G, B), denoted by $(F, A) \bigcap_E (G, B)$ is defined to be the soft set (K, C) satisfying the following conditions: (i) $C = A \cup B$; (ii) for all $c \in C$,*

$$K(c) = \begin{cases} F(c) & if\ c \in A \backslash B, \\ G(c) & if\ c \in B \backslash A, \\ F(c) \cap G(c) & if\ c \in A \cap B. \end{cases}$$

Definition 3.3. *Let (F, A) and (G, B) be two soft sets over a common universe U. The restricted intersection of (F, A) and (G, B), denoted by $(F, A) \bigcap_R (G, B)$, is defined to be the soft set (K, C) satisfying the following conditions: (i) $C = A \cap B$; (ii) for all $c \in C$, $K(c) = F(c) \cap G(c)$. In this case, we write $(F, A) \widetilde{\cap} (G, B) = (K, C)$.*

Definition 3.4. *Let (F, A) and (G, B) be two soft sets over a common universe U. The bi-intersection of (F, A) and (G, B) is defined to be the soft set (K, C), where $C = A \cap B$ and $K : C \to \mathcal{P}(U)$ is a mapping given by $K(x) = F(x) \cap G(x)$ for all $x \in G$. In this case, we write $(F, A) \widetilde{\cap} (G, B) = (K, C)$.*

Definition 3.5. *Let (F, A) and (G, B) be two soft sets over a common universe U. The extended union of (F, A) and (G, B), denoted by $(F, A) \bigcup_E (G, B)$ is defined to be the soft set (K, C) satisfying the following conditions: (i) $C = A \cup B$; (ii) for all $c \in C$,*

$$K(c) = \begin{cases} F(c) & \text{if } c \in A \backslash B, \\ G(c) & \text{if } c \in B \backslash A, \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

Definition 3.6. *Let (F, A) and (G, B) be two soft sets over a common universe U. The restricted union of (F, A) and (G, B), denoted by $(F, A) \bigcup_R (G, B)$, is defined to be the soft set (K, C) satisfying the following conditions: (i) $C = A \cap B$; (ii) for all $c \in C$, $K(c) = F(c) \cup G(c)$. In this case, we write $(F, A) \widetilde{\cup} (G, B) = (K, C)$.*

Definition 3.7. *Let $(F_i, A_i)_{i \in I}$ be a non-empty family of soft sets over a common universe U. The union of these soft sets is defined to be the soft set (G, B) such that $B = \bigcup_{i \in I} A_i$ and for all $x \in B$, $G(x) = \bigcup_{i \in I(x)} F_i(x)$, where $I(x) = \{i \in I | x \in A_i\}$. In this case, we write $\widetilde{\bigcup}_{i \in I} (F_i, A_i) = (G, B)$.*

Definition 3.8. *Let (F, A) and (G, B) be two soft sets over a common universe U. Then $(F, A) AND (G, B)$ denoted by $(F, A) \widetilde{\wedge} (G, B)$ is defined by $(F, A) \widetilde{\wedge} (G, B) = (K, A \times B)$, where $K(x, y) = F(x) \cap G(y)$, $\forall (x, y) \in A \times B$.*

Definition 3.9. *Let (F, A) and (G, B) be two soft sets over a common universe U. Then $(F, A) OR (G, B)$ denoted by $(F, A) \widetilde{\vee} (G, B)$ is defined by $(F, A) \widetilde{\vee} (G, B) = (K, A \times B)$, where $K(x, y) = F(x) \cup G(y)$ for all $(x, y) \in A \times B$.*

Definition 3.10. *Let (F, A) and (G, B) be two soft sets over a common universe U. We say that (F, A) is a soft subset of (G, B), denoted by $(F, A) \widetilde{\subseteq} (G, B)$, if it satisfies: (i) $A \subseteq B$; (ii) $F(a) \subseteq G(a)$ for all $a \in A$.*

4. Soft m-ary semihypergroups

Hereafter, we shall consider soft sets over a m-ary semihypergroup (H, f).

Definition 4.1. *The restricted hyperproduct of soft sets $(F_1, B_1), ..., (F_m, B_m)$ over H, denoted by $\widehat{f}((F_1, B_1), ..., (F_m, B_m))$, is defined as a soft set $(K, D) = \widehat{f}((F_1, B_1), ..., (F_m, B_m))$, where $D = \bigcap \{B_i | i = 1, ..., m\} \neq \emptyset$ and $K : D \to \mathcal{P}(H)$ defined by $K(d) = f(F_1(d), ..., F_m(d))$, $\forall d \in D$.*

Definition 4.2. *Let (F, A) be a non-null soft set over H. Then, (F, A) is called a soft m-ary semihypergroup over H if $F(x)$ is a m-ary subsemihypergroup of H for all $x \in Supp(F, A)$, i.e. $\widehat{f}((F, A), ..., (F, A)) \subseteq (F, A)$.*

Proposition 4.1. *A soft set (F, A) over H is a soft m-ary semigroup over H if and only if for all $a \in A, F(a) \neq \emptyset$ is a m-ary subsemihypergroup of H.*

Proof. Let (F, A) be a soft m-ary semihypergroup over H and $a \in A$ such that $F(a) \neq \emptyset$. We have by definition $\widehat{f}((F, A), ..., (F, A)) = (K, A \cap ... \cap A) = (K, A)$, where K is defined by $K(a) = f(F(a), ..., F(a))$, $\forall a \in A$. Since $\widehat{f}((F, A), ..., (F, A)) \subseteq$

(F, A), then $(K, A) \subseteq (F, A)$. That is, $K(a) \subseteq F(a)$, $\forall a \in A$, and so $f(F(a), ..., F(a))$ $\subseteq F(a)$. This implies that $F(a)$ is a m-ary subsemihypergroup of H.

Conversely, let us assume that $F(a) \neq \emptyset$ is a m-ary semihypergroup of H for all $a \in A$. We have to show that (F, A) is a soft m-ary semihypergroup over H. By definition, we have $\widehat{f}((F, A), ..., (F, A)) = (K, A \cap ... \cap A) = (K, A)$ and $K(a) = f(F(a), ..., F(a))$ for all $a \in A$. Since $F(a)$ is a m-ary subsemihypergroup of H, we have $f(F(a), ..., F(a)) \subseteq F(a)$, that is $K(a) \subseteq F(a)$. Thus, $(K, A) \subseteq (F, A)$. This implies that $\widehat{f}((F, A), ..., (F, A)) \subseteq (F, A)$. Hence (F, A) is a soft m-ary semihypergroup over H. $\qquad \square$

Proposition 4.2. *Let (F, A) and (G, B) be two soft m-ary semihypergroups over H such that $A \cap B = \emptyset$. Then $(F, A) \bigcup_E (G, B)$ is a soft m-ary semihypergroup over H.*

Proof. Let $(K, C) = (F, A) \bigcup_E (G, B)$, where $C = A \cup B$ and $A \cap B = \emptyset$. Then $\forall c \in C$, either $c \in A \backslash B$ or $c \in B \backslash A$. If $c \in A \backslash B$, then $K(c) = F(c)$ and if $c \in B \backslash A$, then $K(c) = G(c)$. So in both cases $K(c)$ is a m-ary subsemihypergroup of H. Therefore, (K, C) is a soft m-ary semihypergroup over H. $\qquad \square$

Proposition 4.3. *Let (F, A) and (G, B) be two soft m-ary semihypergroups over H such that $A \cap B \neq \emptyset$. Then $(F, A) \bigcup_E (G, B)$ is a soft m-ary semihypergroup over H.*

Proof. Let us assume that $A \cap B \neq \emptyset$. Let $(K, C) = (F, A) \bigcup_E (G, B)$ where $K(C)$ satisfies the condition (ii) of Definition 3.5. To show that (K, C) is a soft m-ary semihypergroup over H, we have to show that $\widehat{f}((K, C), ..., (K, C)) \subseteq (K, C)$. We have $\widehat{f}((K, C), ..., (K, C)) = (P, C)$ and $P(c) = f(K(c), ..., K(c))$ for all $c \in C$. For $c \in C$, we have

$$
\begin{aligned}
P(c) \quad &= \quad f(K(c), ..., K(c)) = f((F, A) \cup_E (G, B), ..., (F, A) \cup_E (G, B)) \\
&\subseteq \quad \begin{cases} F(c) & \text{if } c \in A \backslash B, \\ G(c) & \text{if } c \in B \backslash A, \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}
\end{aligned}
$$

Then $P(c) \subseteq K(c)$ for all $c \in C$. Therefore, $(F, A) \cup_E (G, B)$ is a soft m-ary semihypergroup over H. $\qquad \square$

Proposition 4.4. *Let (F, A) and (G, B) be two soft m-ary semihypergroups over H such that $A \cap B \neq \emptyset$. Then $(F, A) \cap_R (G, B)$ is a soft m-ary semihypergroup over H.*

Proof. Let us assume that $A \cap B \neq \emptyset$. Let $(K, C) = (F, A) \cap_R (G, B)$ where $C = A \cap B \neq \emptyset$ and $K(c) = F(c) \cap G(c)$ for all $c \in C$. To show that (K, C) is a soft m-ary semihypergroup over H, we have to show that $\widehat{f}((K, C), (K, C), ..., (K, C)) \subseteq (K, C)$. We have $\widehat{f}((K, C), ..., (K, C)) = (P, C)$ and $P(c) = f(K(c), ..., K(c))$ for all $c \in C$. For $c \in C$, we have $P(c) = f(K(c), ..., K(c)) = f(F(c) \cap G(c), ..., F(c) \cap G(c)) \subseteq F(c) \cap G(c) = K(c)$. Then $P \subseteq K$ and $(F, A) \cap_R (G, B)$ is a soft m-ary semihypergroup over H. $\qquad \square$

Proposition 4.5. *Let (F, A) and (G, B) be two soft m-ary semihypergroups over H. Then $(F, A) \wedge (G, B)$ is a soft m-ary semihypergroup over H.*

Proof. Let $(F, A) \wedge (G, B) = (K, A \times B) = (K, C)$ where $K(a, b) = F(a) \cap G(b)$, $\forall (a, b) \in A \times B$. We have to show that $(K, A \times B)$ is a soft m-ary semihypergroup over H. We have $(P, C) = \widehat{f}((K, A \times B), ..., (K, A \times B))$, where $C = A \times B$ and $P(a, b) = f(K(a, b), ..., K(a, b))$, $\forall (a, b) \in C$. For $(a, b) \in C$, we have $P(a, b) = f(K(a, b), ..., K(a, b)) = f(F(a) \cap G(b), ..., F(a) \cap G(b)) \subseteq F(a) \cap G(b) = K(a, b)$. Then $P \subseteq K$ and $(F, A) \wedge (G, B)$ is a soft m-ary semihypergroup over H. $\square$

Definition 4.3. *Let* $(F_1, A_1), ..., (F_m, A_m)$ *be* m *soft sets over* H. *Define* $f^*((F_1, A_1),$ $(F_2, A_2), ..., (F_m, A_m)) = (K, A_1 \times A_2 \times ... \times A_m)$ *be a soft set where* $K(a_1, a_2, ..., a_m) = f((F_1(a_1), F_2(a_2), ..., F_m(a_m))$.

Proposition 4.6. *Let* (H, f) *be a commutative* m-*ary semihypergroup. If* (F_1, A_1), $(F_2, A_2), ..., (F_m, A_m)$ *are soft* m-*ary semihypergroup over* H, *then* $f^*((F_1, A_1), (F_2, A_2), ..., (F_m, A_m)$ *is a soft* m-*ary semihypergroup over* H.

Proof. Let $f^*((F_1, A_1), ..., (F_m, A_m)) = (K, A_1 \times ... \times A_m)$ where $K(a_1, ..., a_m) = f(F_1(a_1), ..., F_m(a_m))$, $\forall (a_1, ..., a_m) \in A_1 \times ... \times A_m$. We have $(P, C) = \widehat{f}((K, A_1 \times ... \times A_m), ..., (K, A_1 \times ... \times A_m))$, where $C = A_1 \times ... \times A_m$ and $P(a_1, ..., a_m) = f(K(a_1, ..., a_m), ..., K(a_1, ..., a_m))$, $\forall (a_1, ..., a_m) \in C$. For $(a_1, ..., a_m) \in C$, since H is commutative, we have $P(a_1, ..., a_m) = f(K(a_1, ..., a_m), ..., K(a_1, ..., a_m))$ $= f(f(F_1(a_1), ..., F_m(a_m)), ..., f(F_1(a_1), ..., F_m(a_m))) \subseteq f(F_1(a_1), ..., F_m(a_m)) = K(a_1, ..., a_m)$. Then $P \subseteq K$. This completes the proof. $\square$

5. **Soft** m-**ary subsemihypergroups and soft hyperideals**

Definition 5.1. *Let* (F, A) *and* (G, B) *be two soft sets over* H *such that* (F, A) *is a* m-*ary semihypergroup and* $(G, B) \subseteq (F, A)$. *Then* (G, B) *is called a soft* m-*ary subsemihypergroup(hyperideal) of* (F, A) *if* $G(b)$ *is a* m-*ary subsemihyper-group(hyperideal) of* $F(b)$ *for all* $b \in B$.

Proposition 5.1. *Let* (F, A) *be a soft* m-*ary semihypergroup over* H *and consider the set* $\{(K_i, A_i) | i \in I\}$ *be a non-empty family of soft* m-*ary subsemihypergoups of* (F, A). *Then the following hold:*

(1) $\cap_{R_{i \in I}} (K_i, A_i)$ *is a soft* m-*ary subsemihypergroup of* (F, A).
(2) $\wedge_{i \in I} (K_i, A_i)$ *is a soft* m-*ary subsemihypergroup of* $\wedge_{i \in I} (F, A)$.
(3) $\cup_{E_{i \in I}} (K_i, A_i)$ *is a soft* m-*ary subsemihypergroup of* (F, A) *if the set* $\{A_i | i \in I\}$ *is pairwise disjoint.*

Definition 5.2. *A soft set* (F, A) *over* H *is called a soft* j-*hyperideal over* H, $j = 1, 2, ..., m$ *if* $\widehat{f}((H, E), ..., (\overset{j}{F, A}), ..., (H, E)) \subseteq (F, A)$. *A soft set* (F, A) *over* H *is called a soft hyperideal over* H, *if it is a soft* j-*hyperideal over* H *for each* $j = 1, 2, ..., m$.

The soft hyperideal over H defined above is different from the soft hyperideal of a soft m-ary semihypergroup.

Proposition 5.2. *A soft set* (F, A) *over* H *is a soft* j-*hyperideal over* H *where* $j \in \{1, 2, ..., m\}$, *if and only if for all* $a \in A$, $F(a) \neq \emptyset$ *is a* j-*hyperideal of* H.

Proof. Let us suppose that (F, A) is a soft j-hyperideal over H. We show that $F(a) \neq \emptyset$ is a j-hyperideal of H. By the definition we have

$$\widehat{f}((H, E), ..., (\overset{j}{F}, A), ..., (H, E)) = (K, E \cap ... \cap A ... \cap E) = (K, A), \text{ where}$$

$f(H(a), ..., \overset{j}{F}(a), ..., H(a)) = K(a), \forall a \in A$. That is, $f(H, ..., F(a), ..., H) = K(a)$. Since $\widehat{f}((H, E), ..., (F, A), ..., (H, E)) \subseteq (F, A)$, where $(K, A) \subseteq (F, A)$. Thus we have, $(K(a) \subseteq F(a), \forall a \in A$. Therefore, $f(H, ..., F(a), ..., H) \subseteq F(a)$. This shows that $F(a)$ is a j-hyperideal of H.

Conversely, let us assume that $F(a) \neq \emptyset$ is a j-hyperideal of H. We have $\widehat{f}((H, E), ..., (F, A), ..., (H, E)) = (K, E \cap ... \cap A .... \cap E) = (K, A)$
where $f(H(a), ..., F(a), ..., H(a)) = K(a), \forall a \in A$. That is,
$f(H, ..., F(a), ..., H) = K(a)$. But $f(H, ..., F(a), ..., H) \subseteq F(a)$. This implies $K(a) \subseteq F(a)$ and so $(K, A) \subseteq (F, A)$. Thus $\widehat{f}((H, E), ..., (F, A), ..., (H, E))$
$\subseteq (F, A)$. Hence, (F, A) is a soft j-hyperideal over H. $\qquad \square$

Proposition 5.3. *Let (F, A) and (G, B) be any two soft hyperideals over a m-ary semihypergroup H, with $A \cap B \neq \emptyset$. Then $(F, A) \cap_R (G, B)$ is a soft hyperideal over H contained in both (F, A) and (G, B).*

Proposition 5.4. *Let (F, A) and (G, B) be any two soft hyperideals over H. Then $(F, A) \cup_E (G, B)$ is a soft hyperideal over H containing (F, A) and (G, B).*

Proof. Let $(K, C) = (F, A) \cup_E (G, B)$, where $C = A \cup B$ for all $c \in C = A \cup B$, either $c \in A \backslash B$ or $c \in B \backslash A$ or $c \in A \cap B$. If $c \in A \backslash B$, then $K(c) = F(c)$ if $c \in B \backslash A$, then $K(c) = G(c)$, and if $c \in A \cap B$, then $K(c) = F(c) \cup G(c)$, in all the cases $K(c)$ is a hyperideal of H. Hence, (K, C) is soft hyperideal over H. Since $A \subseteq A \cup B, B \subseteq A \cup B$ and $F(c) \subseteq K(c), G(c) \subseteq K(c)$ for all $c \in C$. Therefore, $(F, A) \subseteq (K, C)$ and $(G, B) \subseteq (K, C)$. $\qquad \square$

Proposition 5.5. *Let (F, A) and (G, B) be any two soft hyperideals over a m-ary semihypergroup H. Then $(F, A) \wedge (G, B)$ and $(F, A) \vee (G, B)$ are soft hyperideals over H.*

Proposition 5.6. *Let (F, A) be a soft m-ary semihypergroup over H and the set $\{(K_i, A_i)|i \in I\}$ be a non-empty family of soft hyperideals of (F, A). Then the following hold:*

(1) $\cap_{R_{i \in I}}(K_i, A_i)$ *is a soft hyperideal of (F, A).*
(2) $\wedge_{i \in I}(K_i, A_i)$ *is a soft hyperideal of $\wedge_{i \in I}(F, A)$.*
(3) $\cup_{R_{i \in I}}(K_i, A_i)$ *is a soft hyperideal of (F, A).*
(4) $\vee_{i \in I}(K_i, A_i)$ *is a soft hyperideal of $\vee_{i \in I}(F, A)$.*

6. Soft quasi-hyperideals over m-ary semihypergroups

Definition 6.1. *A soft set (F, A) over a m-ary semihypergroup H is called a soft quasi-hyperideal over H if*

(1) $\widehat{f}((F, A), (H, E), ..., (H, E)) \cap_R \widehat{f}((H, E), (F, A), (H, E), ..., (H, E)) \cap_R ..... \cap_R$
$\widehat{f}((H, E), (H, E), ..., (F, A)) \subseteq (F, A)$.

(2) $\widehat{f}((F,A),(H,E),....,(H,E)) \cap_R .... \cap_R$

$\underbrace{\widehat{f}((H,E),...,(H,E),(\overset{j}{F},A),(H,E),....,(H,E))}_{2m-1} \cap_R ..... \subseteq (F,A).$

where (H,E) is the absolute soft set over H and $3 \leq j \leq 2m-3$.

Proposition 6.1. *A soft set (F,A) over H is a soft quasi-hyperideal over H if and only if $\forall a \in A, F(a) \neq \emptyset$ is a quasi-hyperideal of H.*

Proof. Let us suppose that a soft set (F,A) over H is a soft quasi-hyperideal over H. We show that $F(a)$ is a quasi-hyperideal of H. We have

$\widehat{f}((F,A),(H,E),....,(H,E)) = (G, A \cap E \cap .... \cap E) = (G,A)$ 　　　　(1)

$\widehat{f}((H,E),(F,A),....,(H,E)) = (S, E \cap A \cap .... \cap E) = (S,A)$ 　　　　(2)

. . . .

$\widehat{f}((H,E),....,(H,E),(F,A)) = (I, E \cap E \cap .... \cap A) = (I,A)$ 　　　　(m)

$\widehat{f}(\underbrace{(H,E),...,(H,E),(\overset{j}{F},A),(H,E),....,(H,E)}_{2m-1}) =$

$= (J, E \cap E \cap ... \cap A \cap E... \cap E) = (J,A)$ 　　　　(m+1)

The equations (1), (2),...,(m) imply that

$\widehat{f}((F,A),(H,E),....,(H,E)) \cap_R \widehat{f}((H,E),(F,A),(H,E),....,(H,E)) \cap_R .....\cap_R$

$\widehat{f}((H,E),(H,E),....,(F,A)) = (G,A) \cap_R (S,A) \cap_R .... \cap_R (I,A)$ 　　(m+2)

$\widehat{f}((F,A),(H,E),....,(H,E)) \cap_R ... \cap_R \widehat{f}((H,E),....,(F,A)) = (K,A)$ (m+3)

But (F,A) is a soft quasi-hyperideal over H. Thus

$\widehat{f}((F,A),(H,E),....,(H,E)) \cap_R ... \cap_R \widehat{f}((H,E),(H,E),....,(F,A)) \subseteq (F,A)$. From the equation (m+3), $(K,A) \subseteq (F,A)$, that is, $K(a) \subseteq F(a), \forall a \in A$. Again the equation (m+3) implies that $\forall a \in A$, $f(F(a), H(a), ..., H(a)) \cap ... \cap f(H(a), ..., H(a), F(a)) = K(a)$. This implies that $\forall a \in A$,

$f(F(a), H(a), ..., H(a)) \cap ... \cap f(H(a), ..., H(a), F(a)) \subseteq F(a)$ 　　　　(m+4)

Smilarly, from the equations (1), (m), (m+1), we have

$\widehat{f}((F,A),(H,E),....,(H,E)) \cap_R .... \cap_R$

$\widehat{f}(\underbrace{(H,E),(H,E),...,(H,E),(\overset{j}{F},A),(H,E),....,(H,E),(H,E)}_{2m-1}) \cap_R ..... \cap_R$

$\widehat{f}((H,E),....,(F,A)) = (P,A)$ 　　　　(m+5)

where $3 \leq j \leq 2m-3$. Since (F,A) is a soft quasi-hyperideal over H, we have

$\widehat{f}((F,A),(H,E),....,(H,E)) \cap_R .... \cap_R$

$\widehat{f}(\underbrace{(H,E),...,(H,E),(\overset{j}{F},A),(H,E),....,(H,E)}_{2m-1}) \cap_R ... \cap_R$

$\widehat{f}((H,E),....,(F,A)) \subseteq (F,A)$. Therefore, $(P,A) \subseteq (F,A)$, that is, for all $a \in A$, $P(a) \subseteq F(a)$. From the equation (m+5), we have for all $a \in A$,

$f(F(a), H(a), ..., H(a)) \cap ... \cap f(\underbrace{H(a), ..., H(a), \overset{j}{F}(a), H(a), ..., H(a)}_{2m-1}) \cap ... \cap f(H(a), ...,$

$H(a), F(a)) = P(a)$, where $3 \leq j \leq 2m-3$. This implies that

$f(F(a), H(a), ..., H(a)) \cap ... \cap f(\underbrace{H(a), ..., H(a), \overset{j}{F}(a), H(a), ..., H(a)}_{2m-1}) \cap ... \cap f(H(a), ...,$

$H(a), F(a)) \subseteq F(a)$ (m+6)

From (m+4) and (m+6), it is clear that $F(a)$ is a quasi-hyperideal of H.

Conversely, let $F(a) \neq \emptyset$ be a quasi-hyperideal of H for all $a \in A$. From the equations (1), (2), ..., (m) we have

$\widehat{f}((F, A), (H, E), ..., (H, E)) \cap_R \widehat{f}((H, E), (F, A), (H, E), ..., (H, E)) \cap_R ..... \cap_R$

$\widehat{f}((H, E), (H, E), ..., (F, A)) = (G, A) \cap_R (S, A) \cap_R ... \cap_R (I, A) = (K, A).$

By the definition, $\forall a \in A$, we have $f(F(a), H(a), ..., H(a)) \cap ... \cap$

$f(H(a), ..., H(a), F(a)) = K(a)$. But $F(a)$ is a quasi-hyperideal of H. Therefore, $K(a) = f(F(a), H(a), ..., H(a)) \cap ... \cap f(H(a), ..., H(a), F(a)) \subseteq F(a)$ and so $(K, A) \subseteq (F, A)$. Thus

$\widehat{f}((F, A), (H, E), ..., (H, E)) \cap_R ... \cap_R \widehat{f}((H, E), ..., (F, A)) \subseteq (F, A)$ (m+7)

From the equations (1), (m), and (m+1) we have for all $a \in A$,

$\widehat{f}((F, A), (H, E), ..., (H, E)) \cap_R ... \cap_R$

$\widehat{f}(\underbrace{(H, E), (H, E), ..., (H, E), (\overset{j}{F}, A), (H, E), ..., (H, E), (H, E)}_{2m-1}) \cap_R ... \cap_R$

$\widehat{f}((H, E), ..., (F, A)) = (G, A) \cap_R (I, A) \cap_R ... \cap_R (J, A) = (P, A)$

and also $f(F(a), H(a), ..., H(a)) \cap f(\underbrace{H(a), ..., H(a), \overset{j}{F}(a), H(a), ..., H(a)}_{2m-1}) \cap$

$f(H(a), ..., H(a), F(a)) = P(a)$, where $3 \leq j \leq m$. Since $F(a)$ is a quasi-hyperideal of H, so we have $P(a) = f(F(a), H(a), ..., H(a)) \cap$

$f(\underbrace{H(a), ..., H(a), \overset{j}{F}(a), H(a), ..., H(a)}_{2m-1}) \cap f(H(a), ..., H(a), F(a)) \subseteq F(a)$

and thus $(P, A) \subseteq (F, A)$. Hence we have $\widehat{f}((F, A), (H, E), ..., (H, E)) \cap_R .... \cap_R$

$\widehat{f}(\underbrace{(H, E), ..., (H, E), (\overset{j}{F}, A), (H, E), ..., (H, E)}_{2m-1}) \cap_R ..... \subseteq (F, A)$.   (m+8)

From (m+7) and (m+8), (F, A) is a soft quasi-hyperideal over H. $\square$

Proposition 6.2. *Let (F_j, A_j) be soft j-hyperideals for $j = 1, 2, ..., m$ respectively over H, respectively. Then $(F_1, A_1) \cap_R (F_2, A_2) \cap_R .... \cap_R (F_m, A_m)$ is a soft quasi-hyperideal over H.*

Proposition 6.3. *Let (F_j, A_j) be soft j-hyperideals for $j = 1, 2, ..., m$ respectively over H, respectively such that $A_1 \cap A_2 \cap ... \cap A_m = \emptyset$. Then $(F_1, A_1) \cap_E (F_2, A_2) \cap_E .... \cap_E (F_m, A_m)$ is a soft quasi-hyperideal over H.*

Proof. By the definition, $(S, A) = (F_1, A_1) \cap_E (F_2, A_2) \cap_E .... \cap_E (F_m, A_m)$, where $A = A_1 \cup A_2 \cup .... \cup A_m, A_1 \cap A_2 \cap ... \cap A_m = \emptyset$, and $\forall a \in A$

$$S(a) = \begin{cases} F_1(a) & \text{if } a \in A_1 \backslash A_2 \cap ... \cap A_m, \\ ... \\ F_j(a) & \text{if } a \in A_j \backslash A_1 \cap ... \cap A_{j-1} \cap A_{j+1} \cap ... \cap A_m, \\ ... \\ F_m(a) & \text{if } a \in A_m \backslash A_1 \cap ... \cap A_{m-1}. \end{cases}$$

In each case, $S(a)$ is a quasi-hyperideal of H. Since every j-hyperideal ($j = 1, 2, ..., m$) of H is a quasi-hyperideal of H, thus, by the definition, $(S, A) = (F_1, A_1) \cap_E .... \cap_E (F_m, A_m)$ is a soft quasi-hyperideal over H. $\square$

Proposition 6.4. *Every soft j-hyperideal ($j = 1, 2, ..., m$) over a m-ary semihypergroup H is a soft quasi-hyperideal over H.*

Proof. Let (F_j, A_j) be a soft j-hyperideal ($j \in \{1, 2, ..., m\}$) over H. Then $F_j(a)$ is a j-hyperideal of H. Since each j-hyperideal of H is a quasi-hyperideal of H, therefore $F_j(a)$ is a quasi-hyperideal of H. Hence (F_j, A_j) is a soft quasi-hyperideal over H. $\square$

Proposition 6.5. *Every soft j-hyperideal ($j = 1, 2, ..., m$) over H is a soft m-ary semihypergroup over H and every soft quasi-hyperideal is a soft m-ary semihypergroup over H.*

Proposition 6.6. *Let (F_j, A_j) be soft j-hyperideals for $j = 1, 2, ..., m$ respectively over H, respectively. Then $(F_1, A_1) \wedge .... \wedge (F_m, A_m)$ is a soft quasi-hyperideal over H.*

Proof. Let $(S, A) = (F_1, A_1) \wedge .... \wedge (F_m, A_m)$, where $A = A_1 \times .... \times A_m$, and $\forall (a_1, ..., a_m) \in A_1 \times .... \times A_m, S(a_1, ..., a_m) = A_1(a_1) \cap ... \cap A_m(a_m)$ is a quasi-hyperideal of H. Since the intersection of j-hyperideals for $j = 1, 2, ..., m$, is a quasi-hyperideal of H, then $(F_1, A_1) \wedge .... \wedge (F_m, A_m)$ is a soft quasi-hyperideal over H. $\square$

Proposition 6.7. *Let (F, A) and (G, B) be two soft quasi-hyperideals over a m-ary semihypergroup H. Then the following hold:*

(1) $(F, A) \cap_R (G, B)$ *is a soft quasi-hyperideal over H.*
(2) $(F, A) \cap_E (G, B)$ *is a soft quasi-hyperideal over H.*
(3) $(F, A) \wedge (G, B)$ *is a soft quasi-hyperideal over H.*
(4) $(F, A) \cup_E (G, B)$ *is a soft quasi-hyperideal over H, whenever $A \cap B = \emptyset$.*

Proposition 6.8. *Let (F, A) be a soft quasi-hyperideal and (G, B) a soft m-ary semihypergroup over H. Then $(F, A) \cap_R (G, B)$ is a soft quasi-hyperideal of (G, B).*

Proof. Let $(S, C) = (F, A) \cap_R (G, B)$, where $C = A \cap B \neq \emptyset$ and $S(c) = F(c) \cap G(c)$ for all $c \in C$, since $S(c) \subseteq F(c)$ and $S(c) \subseteq G(c)$. Since $S(c) \subseteq G(c)$, $f(S(c), G(c), ..., G(c)) \cap ... \cap f(G(c), ..., G(c), S(c)) \subseteq f(G(c), ..., G(c)) \cap ... \cap f(G(c), ..., G(c)) \subseteq f(G(c), ..., G(c)) \subseteq G(c)$. This implies that

$$f(S(c), G(c), ..., G(c)) \cap ... \cap f(G(c), ..., G(c), S(c)) \subseteq G(c) \tag{1}$$

Also $S(c) \subseteq F(c)$. So $f(S(c), G(c), ..., G(c)) \cap ... \cap f(G(c), ..., G(c), S(c))$
$\subseteq f(F(c), G(c), ..., G(c)) \cap ... \cap f(G(c), ..., G(c), F(c))$
$\subseteq f(F(c), H(c), ..., H(c)) \cap ... \cap f(H(c), ..., H(c), F(c)) \subseteq F(c)$. Thus
$$f(S(c), G(c), ..., G(c)) \cap ... \cap f(G(c), ..., G(c), S(c)) \subseteq F(c) \qquad (2)$$
From the equations (1) and (2), we have
$$f(S(c), G(c), ..., G(c)) \cap ... \cap f(G(c), ..., G(c), S(c)) \subseteq F(c) \cap G(c) = S(c). \quad (3)$$
Similarly, we can show that $f(S(c), G(c), ..., G(c)) \cap$

$$f(\underbrace{G(c), ..., G(c), \overset{j}{S}(c), G(c), ..., G(c)}_{2m-1}) \cap f(G(c), G(c), ..., S(c)) \subseteq S(c). \qquad (4)$$

where $3 \le j \le 2m - 3$. From the equation (3) and (4), $S(c)$ is a quasi-hyperideal of $G(c)$. Thus $(F, A) \cap_R (G, B)$ is a soft quasi-hyperideal of (G, B). $\qquad \square$

7. Soft bi-hyperideals over m-ary semihypergroups

Definition 7.1. *A soft set (F, A) over H is called a soft bi-hyperideal over H if*

(1) *(F, A) is a soft m-ary semihypergroup over H.*
(2) *$\widehat{f}(\underbrace{(F, A), (H, E), (F, A), ..., (H, E), (F, A)}_{2m-1}) \subseteq (F, A)$ where (H, E) is the absolute soft set over H.*

Proposition 7.1. *A soft set (F, A) over a m-ary semihypergroup H is a soft bi-hyperideal over H if and only if $\forall a \in A$, $F(a) \ne \emptyset$ is a bi-hyperideal of H.*

Proof. Let (F, A) be a soft bi-hyperideal over a m-ary semihypergroup H. Then by the definition, (F, A) is a soft m-ary semihypergroup over H. By the Proposition 4.1, for any $a \in A, F(a) \ne \emptyset$ is a m-ary subsemihypergroup of H. Moreover, since (F, A) is a soft bi-hyperideal over H, we have
$\widehat{f}(\underbrace{(F, A), (H, E), (F, A), ..., (H, E), (F, A)}_{2m-1}) \subseteq (F, A)$ where (H, E) is the
absolute soft set over H. It follows that $\widehat{f}(\underbrace{F(a), H, F(a), ..., H, F(a)}_{2m-1}) \subseteq F(a)$, which
shows that $F(a)$ is a bi-hyperideal of H.

Conversely, let us suppose that (F, A) is a soft set over H such that $\forall a \in A, F(a)$ is a bi-hyperideal of H, whenever $F(a) \ne \emptyset$. Then it is clear that each $F(a) \ne \emptyset$ is a m-ary subsemihypergroup of H. Hence, by the Proposition 4.1, (F, A) is a soft m-ary semihypergroup over H. Furthermore, since $F(a) \ne \emptyset$ is a bi-hyperideal of H, then for all $a \in A$, $\widehat{f}(\underbrace{F(a), H, F(a), ..., H, F(a)}_{2m-1}) \subseteq F(a)$. Hence
we conclude that $\widehat{f}(\underbrace{(F, A), (H, E), (F, A), ..., (H, E), (F, A)}_{2m-1}) \subseteq (F, A)$. This shows
that (F, A) is a soft bi-hyperideal over H. $\qquad \square$

Proposition 7.2. *Every soft quasi-hyperideal over H is a soft bi-hyperideal over H.*

Proposition 7.3. *Let (F, A) be a soft bi-hyperideal over H and (G, B) a soft m-ary semihypergroup over H. Then $(F, A) \cap_R (G, B)$ is a soft bi-hyperideal of (G, B).*

Proof. By the definition, $(S, C) = (F, A) \cap_R (G, B)$ where $C = A \cap B \neq \emptyset$, and S is defined by $S(c) = F(c) \cap G(c)$, $\forall c \in C$. We have $\widehat{f}((S, C), (S, C), ..., (S, C)) =$
$= \widehat{f}(((F, A) \cap_R (G, B)), ..., ((F, A) \cap_R (G, B))) \subseteq \widehat{f}((F, A), ..., (F, A)) \subseteq (F, A)$. This implies that $\widehat{f}((S, C), (S, C), ..., (S, C)) \subseteq (F, A)$ \hfill (1)

Also, $\widehat{f}((S, C), ..., (S, C)) = \widehat{f}(((F, A) \cap_R (G, B)), ..., ((F, A) \cap_R (G, B))) \subseteq \widehat{f}((G, B), ..., (G, B)) \subseteq (G, B)$ because (G, B) is a soft m-ary semihypergroup over H. This implies that $\widehat{f}((S, C), (S, C), ..., (S, C)) \subseteq (G, B)$ \hfill (2)

From the equations (1) and (2), we have $\widehat{f}((S, C), ..., (S, C)) \subseteq (F, A) \cap_R (G, B) = (S, C)$. This implies that (S, C) is a soft m-ary semihypergroup over H. Also $\widehat{f}(\underbrace{(S, C), (G, B), (S, C), ..., (S, C)}_{2m-1})$

$= \widehat{f}(((F, A) \cap_R (G, B)), (G, B), ((F, A) \cap_R (G, B)), (G, B), ..., ((F, A) \cap_R (G, B)))$
$\subseteq \widehat{f}(\underbrace{(G, B), (G, B), ..., (G, B)}_{2m-1}) \subseteq \widehat{f}(\underbrace{(G, B), (G, B), ..., (G, B)}_{m}) \subseteq (G, B)$, and so

$\widehat{f}(\underbrace{(S, C), (G, B), (S, C), ..., (S, C)}_{2m-1}) \subseteq (G, B)$. \hfill (3)

Again, $\widehat{f}(\underbrace{(S, C), (G, B), (S, C), ..., (S, C)}_{2m-1})$

$= \widehat{f}(\underbrace{((F, A) \cap_R (G, B)), (G, B), ((F, A) \cap_R (G, B)), (G, B), ..., ((F, A) \cap_R (G, B))}_{2m-1})$

$\subseteq \widehat{f}(\underbrace{(F, A), (H, E), ..., (F, A)}_{2m-1}) \subseteq (F, A)$, because (F, A) is a soft bi-hyperideal over

H. This implies that $\widehat{f}(\underbrace{(S, C), (G, B), (S, C), ..., (S, C)}_{2m-1}) \subseteq (F, A)$. \hfill (4) From

the equations (3) and (4), we have
$\widehat{f}(\underbrace{(S, C), (G, B), (S, C), ..., (S, C)}_{2m-1}) \subseteq (F, A) \cap_R (G, B) = (S, C)$.

Henece (S, C) is a soft bi-hyperideal of (G, B). \hfill $\square$

Definition 7.2. *A soft hyperideal* (F, A) *over* H *is soft idempotent if*
$\widehat{f}((F, A), ..., (F, A)) = (F, A)$.

Theorem 7.1. *Let* (F, A) *be a soft bi-hyperideal over* H *and* (G, B) *a soft bi-hyperideal of* (F, A) *such that* $\widehat{f}((G, B), ..., (G, B)) = (G, B)$. *Then* (G, B) *is a soft bi-hyperideal over* H.

Proof. By the condition, we have $\widehat{f}((G, B), ..., (G, B)) \subseteq (G, B)$. This implies that (G, B) is a soft m-ary semihypergroup over H. Since $(G, B) \subseteq (F, A)$ and $\widehat{f}((G, B), ..., (G, B)) = (G, B)$, we have
$\widehat{f}(\underbrace{(G, B), (H, E), (G, B), (H, E), ..., (G, B)}_{2m-1})$

$= \widehat{f}(\widehat{f}((G, B), ..., (G, B)), (H, E), (G, B), (H, E), ..., \widehat{f}((G, B), ..., (G, B)))$
$= \widehat{f}((G, B), ..., \widehat{f}(\underbrace{(G, B)), (H, E), (G, B), (H, E), ...(G, B)}_{2m-1}), ..., (G, B))$

$$\subseteq \widehat{f}((G,B),...,\widehat{f}(\underbrace{(F,A)),(H,E),(F,A),(H,E),...(F,A)}_{2m-1}),...,(G,B))$$

$$\subseteq \widehat{f}((G,B),(G,B),...,(F,A),...,(G,B),(G,B))$$

$$= \widehat{f}((G,B),(G,B),...,(F,A),...,(G,B),\widehat{f}((G,B),...,(G,B)))$$

$$= \widehat{f}((G,B),\widehat{f}((G,B),...,(F,A),...,(G,B),...,(G,B)),(G,B))$$

$$\subseteq \widehat{f}((G,B),...,(G,B)) = (G,B). \text{ This implies that}$$

$$\widehat{f}(\underbrace{(G,B),(H,E),(G,B),(H,E),...,(G,B)}_{2m-1}) \subseteq (G,B). \text{ Hence } (G,B) \text{ is a soft bi-}$$

hyperideal over H. $\square$

Proposition 7.4. *Let (F,A) be a non-empty soft subset over H. If (G_j,B_j) are soft j-hyperideals resp. $(j \in \{1,2,...,m\})$, over H resp. such that*

$$\widehat{f}((G_1,B_1),....,(G_j,B_j),...,(G_m,B_m)) \subseteq (F,A) \subseteq (G_1,B_1) \cap_R ... \cap_R (G_m,B_m),$$

then (F,A) is a soft bi-hyperideal over H.

Proof. By the definition $\widehat{f}((F,A),...,(F,A)) \subseteq \widehat{f}((G_1,B_1) \cap_R ... \cap_R (G_m,B_m),...,$
$(G_1,B_1) \cap_R ... \cap_R (G_m,B_m)) \subseteq \widehat{f}((G_1,B_1),....,(G_j,B_j),...,(G_m,B_m)) \subseteq (F,A)$. This implies that $\widehat{f}((F,A),....,(F,A)) \subseteq (F,A)$. Thus (F,A) is a soft m-ary semihypergroup over H. Let us consider again
$$\widehat{f}(\underbrace{(F,A),(H,E),(F,A),(H,E),...,(F,A)}_{2m-1}) \subseteq \widehat{f}((G_1,B_1) \cap_R ... \cap_R (G_m,B_m),$$
$(H,E),(G_1,B_1) \cap_R ... \cap_R (G_m,B_m),(H,E),...,(G_1,B_1) \cap_R ... \cap_R (G_m,B_m)) \subseteq \widehat{f}((G_1,B_1),$
$(H,E),(G_2,B_2),....,(G_j,B_j),...,(H,E),(G_m,B_m)) \subseteq$
$\widehat{f}((G_1,B_1),....,(G_j,B_j),...,(G_m,B_m)) \subseteq (F,A)$. This implies that
$\widehat{f}((F,A),(H,E),(F,A),...,(H,E),(F,A)) \subseteq (F,A)$. Hence (F,A) is a soft bi-hyperideal over H. $\square$

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