TOWARDS A PREDICTING MODEL FOR HEAT TRANSFER INSIDE AN INDUSTRIAL ROBOT ARM

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Unul din factorii care afectează funcţionarea unui robot industrial este reprezentată de creşterea temperaturii în cuplele sale. Aceasta este dată de pierderile de căldură în motoarele electrice amplasate în fiecare cuplă a robotului, în special în roboţii industriali de tip "clean room", unde de obicei sunt izolate faţă de mediul ambiant prin utilizarea de carcase, fapt ce se concretizează într-o crestere a temperaturii în motorul electric şi celelalte componente din jurul său. Lucrarea de faţă îşi propune abordarea unui model teoretic pentru estimarea creşterii temperaturii în interiorul elementului robotului industrial.

One of the factors that affect the functionality of an industrial robot is the temperature rise in its joints. This is generated by the thermal losses in the electric motors placed in each joint of the robot, mainly in the case of robots used in clean room environments, where the robotic joints are encapsulated, thus increasing the temperature in the electric motor and the surrounding components. The present paper purpose is to approach a theoretical model for predicting the temperature rise in the robotic arm element.

Keywords: Robot arm, heat losses, prediction

1. Introduction

The goal of the present paper is to approach a theoretical model that helps predicting the temperature given by electric motors inside industrial robots joints. This tool can help in the design and optimisation process, mainly in choosing the electric motor for a joint. Given that the electric motor has a specified maximum temperature, it can be very useful to have a tool with certain inputs (like heat loss, motor size, etc.) that can update the data in a short period of time, instead of performing CAD modifications and running thermal analysis for each modification. This ensures a significant decrease in the time spent for the above mentioned purpose. The electric motor was taken as the main source of heat in a robotic joint, as shown by Pranchalee Poonyapak and M. John D. Hayes [1]. The case of a SCARA industrial robot was approached, given the similar shape of the rig used for the measurements. This is briefly shown in Fig. 1.

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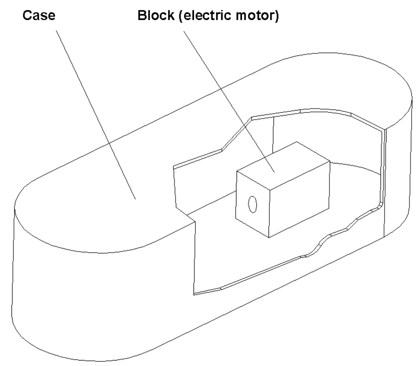

Fig. 1. Simulation rig similar with a SCARA robot element

Because of the different shapes involved in the heat transfer, different correlations were used. In parallel, measurements were performed to check the accuracy of the model. Heat transfer correlations from Teertstra, Yovanovich and Culham [2] where used for shape conduction factor for the rectangular block simulating the motor. Also other correlations from Churchill and Usagi [3], Fuji[4] and F. Kreith and R. F. Boehm [5] were used, given different shapes and orientation for the surfaces involved in the heat transfer.

2. Thermal model description

For estimating the temperature of the inner components in a SCARA industrial robot arm, a rig was built. The motor was simulated with a rectangular block which had a resistor in it, to generate the heat losses. In order to calculate the overall heat transfer from the rectangular block, air volume inside the enclosure of the robot arm and the robotic arm cover (case), a thermal resistances circuit will be used, like described in Fig 2.

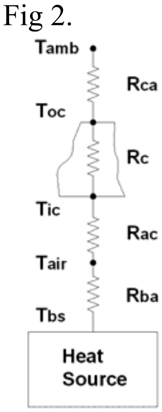

Fig. 2. Thermal resistances circuit.

In the above figure, we have:

- R_{ba} – thermal resistance in convection between the rectangular block and the inner air filling the whole volume of the robotic arm;
- R_{ac} – thermal resistance in convection between the air volume and the inner total surface of the case;
- R_c – thermal resistance in conduction of the case;

Regarding the temperatures of all the above mentioned elements, we have:

- T_{bs} – temperature of the outer surface of the rectangular block;
- T_{air} – temperature of the volume of air enclosed in the robotic arm;
- T_{ic} – temperature of the inner surface of the robotic arm case;
- T_{oc} – temperature of the outer surface of the robotic arm case;
- T_{amb} – temperature of the ambient.

For the best accuracy of the estimations, we need to take into account the thermal resistance in conduction between the resistor and the rectangular block. This thermal resistance is calculated using the thermal circuit shown in Fig. 3.

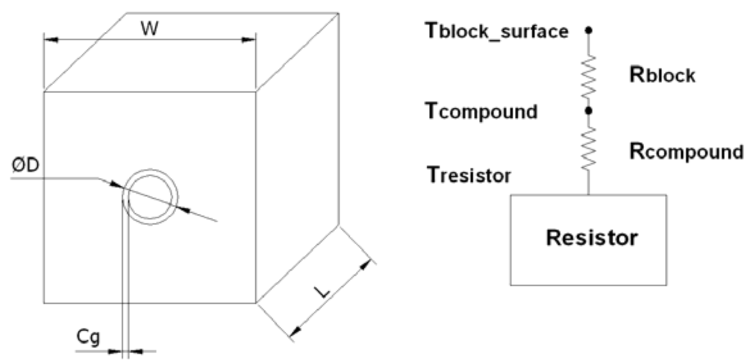

Fig. 3. Rectangular bloc with a resistor placed in the center.

In Fig. 3 we have:

- $R_{compound}$ – thermal resistance in conduction of the thermal compound used. The gap "Cg" represents the radial gap given by the loose tolerance between the resistor diameter (D-2·Cg) and the diameter of the rectangular block hole (D). The value of this radial gap is 100 micrometers. The thermal resistance of the compound is given by the following equation:

$$R_{compound} = \frac{\ln(r_2 / r_1)}{2 \cdot \pi \cdot k \cdot L_{res}} \tag{1}$$

where: r_2 – radius of the hole, is 0.003675 m;
r_1 – radius of the resistor, is 0.003575 m;
k – thermal conductivity of the thermal compound, is 1 W/m·K;
L_{res} – length of the resistor; is 0.016 m.

The value of the above thermal resistance is 0.274 W/m·K.

- R_{block} – thermal resistance in conduction of the rectangular block. For calculating this thermal resistance, we use the shape conduction factor for a square circular cylinder of length L centered in a square solid of equal length. The shape conduction factor will be evaluated with the following relation Teertstra, Yovanovich and Culham [2]:

$$S = \frac{2 \cdot \pi \cdot L_{hole}}{\ln(1.08 \cdot W / D)} \tag{2}$$

In the equation above, we have:

W - value of the square segment, is 0.038 m;

L_{hole} - length of the block hole, is 0.06m;

D – diameter of the center hole of , is 0.0715m.

The expression for the conduction thermal resistance through the rectangular block is given by:

$$R_{block} = \frac{1}{k \cdot S} \tag{3}$$

The calculated value for S was 0.219 m. The value for R_{block} was 0.018 K/W, given the thermal conductivity of 250 W/m·K for the block material (Al).

The total thermal resistance in conduction between the resistor and the outside surface of the block is equal to the sum of the thermal resistances $R_{compound}$ and R_{block}. This is because of the analogy in the way of calculating the total thermal resistance between the thermal circuits and the electrical circuits. For the thermal circuits, the thermal resistances calculations follows the same rule like for the electrical resistances, stating that if the resistances are connected in series, the value of the total resistance equals the sum of all the resistances; if the resistances are connected in parallel, than the inverse of the total resistance equals the sum of all the inverses of the individual resistances in the circuit. We use Fourier's law of conduction which yields the following relation:

$$q = \frac{\Delta T}{R} \tag{4}$$

The above relation states that the total temperature difference (ΔT) depends on the heat input (q) and the value of the total thermal resistance (R). By following this rule, the calculated temperature difference between the resistor and the faces of the block was 1.639K.

Having the conduction thermal resistances evaluated, the next step of the mathematical model is to evaluate the heat transfer in convection from the block to the air volume filling the arm element. The nature of this transfer is through free convection, so we select and use the appropriate empirical equations that fit the setup of an electric servomotor mounted inside an industrial robot arm which is enclosed. In Fig. 3 we can observe that the block, which is basically representing an electric servomotor with a simplified shape, has six surfaces. Most of the servomotors have a similar shape, but some of them can have a cylindrical shape. The actual model is aiming at providing a temperature prediction tool for the types of servomotors characterized by flat surfaces. In order to estimate the free convection heat transfer between the surfaces and air, we characterize each face individually, depending on its orientation, shape and surface area. For estimating the convection heat transfer coefficient, which together with the surface area yield the values for the convection thermal resistances for the above mentioned areas, we need to use the empirical correlations for evaluating the Nusselt (Nu) number:

$$Nu = \frac{h_c \cdot L}{k} = \frac{q \cdot L}{A_s \cdot \overline{\Delta T} \cdot k}; \quad R_{conv} = \frac{1}{h_c \cdot A_s} \tag{5}$$

As equations 5 show, the Nu number yield the value of the convection heat transfer coefficient, knowing the thermal conductivity of the air and the height of the vertical surface. The variables in the above equations are as follows:

- h_c – heat transfer coefficient, in K/W·m;
- L – height of the vertical surface, in m;
- k – thermal conductivity of air, in W/m·K;
- q – heat, in W; A_s – surface of the area involved in heat transfer, in m^2;
- ΔT – temperature difference between the surface and the ambient, in K.

The Nu number actually depends on all the variables form the following relation:

$$Nu = f(Ra, \mathrm{Pr}, \text{geometric shape}, \text{boundary conditions}) \tag{6}$$

Equation 6 states that for given boundary conditions, and a given shape of the body, the Nu number depends only of the geometry of the Rayleigh (also written as “Ra”) number and Prandtl number. The Rayleigh number for a fluid is a dimensionless number associated with buoyancy driven flow (also known as free convection or natural convection). When the Rayleigh number is below the critical value for that fluid, heat transfer is primarily in the form of conduction; when it exceeds the critical value, heat transfer is primarily in the form of convection. The expression for Rayleigh number is given depending on the type of surface involved in the heat transfer, with respect to the position and orientation of the surface. The Prandtl (Pr) number is a dimensionless number approximating the ratio of momentum diffusivity (kinematic viscosity) and thermal diffusivity. The Rayleigh and Prandtl numbers are evaluated at T_f, which is the average temperature calculated between the wall temperature and the ambient temperature. Pr number is given by the following expression:

$$\mathrm{Pr} = \frac{c_p \cdot \mu}{k} \tag{7}$$

The considered value for Pr number was 0.71, and it was taken from the table bellow

Table 1

Physical properties for air at different temperatures

Temperature	Density	Specific heat	Thermal conductivity	Kynematic viscosity	Thermal expansion coefficient	Prandtl number
(^oC)	(kg/m3)	(kJ/kg K)	(W/m K)	(m2/s) x 10^{-6}	(1/K) x 10^{-3}	
-150	2.793	1.026	0.0116	3.08	8.21	0.76
-100	1.98	1.009	0.016	5.95	5.82	0.74
-50	1.534	1.005	0.0204	9.55	4.51	0.725
0	1.293	1.005	0.0243	13.3	3.67	0.715
20	1.205	1.005	0.0257	15.11	3.43	0.713
40	1.127	1.005	0.0271	16.97	3.2	0.711
60	1.067	1.009	0.0285	18.9	3	0.709
80	1	1.009	0.0299	20.94	2.83	0.708

For each surface type, some of the empirical equations for Nusselt and Rayleigh numbers are different:

1) For the two vertical surfaces of the block, with length L and width W, the Rayleigh number is given by the expression:

$$Ra = g\beta\overline{\Delta T}L^3 / \nu\alpha \quad (8)$$

In equations 7 and 8 we have the following variables:

g – acceleration due to gravity, is 9.81 m/s^2;

β - thermal expansion coefficient, in (K^{-1});

ν - kinematic viscosity, in Stokes; α - thermal diffusivity, in m^2/s;

c_p – specific heat, in J/kg·K; μ – dynamic viscosity, in Pa·s;

k – thermal conductivity, in W/m·K.

For estimating the convection heat transfer, according to Churchill and Usagi [3], the Pr dependent functions from equations (9), (10) and (11) is necessary. For a laminar flow regime, equation (9) was written:

$$\overline{C_l} = 0.671/\left(1 + (0.492/\mathrm{Pr})^{9/16}\right)^{4/9} \quad (9)$$

For accounting the turbulent flow regime, the following two equations are used:

$$C_t^V = 0.13\,\mathrm{Pr}^{0.22}/\left(1 + 0.61\,\mathrm{Pr}^{0.81}\right)^{0.42} \quad (10)$$

$$C_t^H = 0.14\,\mathrm{Pr}\left(\frac{1 + 0.107\,\mathrm{Pr}}{1 + 0.01\,\mathrm{Pr}}\right) \quad (11)$$

Equation (10) is used for accounting the turbulent heat transfer from vertical plates and equation 11 is for horizontal plates. The Total Nu number is obtained by determining firstly the Nu_l and Nu_t, two components for laminar and turbulent heat transfer. Having these two values, relation (12) gives the total value for Nu number:

$$Nu = \left((Nu_l)^m + (Nu_t)^m \right)^{1/m} \quad (12)$$

The blending parameter "m" depends on the body shape and orientation. For the actual situation, we have two vertical faces with identical shape, size and orientation. For evaluating the heat transfer from the two faces, we write the following equations:

$$Nu^T = \overline{C_l} Ra^{1/4}, \; Nu_l = \frac{2.0}{\ln\left(1 + 2.0 / Nu^T\right)}$$
$$Nu_t = C_t^V Ra^{1/3} / \left(1 + 1.4 \times 10^9 \, \mathrm{Pr} / Ra\right) \quad (13)$$

For evaluating the total Nu number, equation 12 is used by replacing m=6. For evaluating the heat transfer coefficient which yields the convection thermal resistance value for each of the two surfaces, equation (5) is used. For calculating the Ra number, the unit value for a ΔT of 1K is considered for now. The value obtained in the case of these vertical faces was 33.46 K/W for each one.

2) For the other two vertical faces of the rectangular block, the same calculation method as in case 1 was used. The only difference in this case is the size of the surface, both length and width having the value of 0.038m. The convection thermal resistance value obtained in the case of these vertical faces was 52.83 K/W for each one.

3) For the surface on the top of the rectangular block, the Ra and Nu numbers expressions change:

$$L^* = A_S / P, \quad Nu = qL^* / A_S \Delta T, \; Ra = g\beta \overline{\Delta T} L^{*3} / \nu\alpha \quad (14)$$

P – perimeter of the rectangle, in m.

In the situation of horizontal surfaces, in order to calculate the Nu and Ra numbers, we need to use L^* instead of L, like relations 14 are showing. Given the results obtained with the above relation, for estimating the heat transfer coefficient and thermal resistance we use the equations for a horizontal plate, in which the heat transfer is occurring upwards; according with Goldstein [3] the following relations are used:

$$Nu^T = 0.835 \overline{C_l} Ra^{1/4} \qquad Nu_l = \frac{2.0}{\ln\left(1 + 1.4 / Nu^T\right)} \qquad Nu_t = C_t^H Ra^{1/3} \quad (15)$$

After estimating both components of Nu number, equation (12) is also used to obtain the total Nu number by replacing m=10. The Nu number is obtained by the mathematical equality of equation (12) and equation (14) referring to Nu number. The convection thermal resistance value obtained in this situation was 19.85 K/W.

4) In the case of the surface situated on the bottom of the rectangular block, the main buoyancy force will be into the plate, so there is only a small

force driving the fluid along the plate. For this reason, only laminar heat transfer is accounted in this case. The following equations apply for Ra<10^{10} and Pr≥0.7:

$$Nu^T = H_l Ra^{1/5} \qquad H_l = \frac{0.527}{\left[1+(1.9/\mathrm{Pr})^{9/10}\right]^{2/9}} \qquad Nu = \frac{2.45}{\ln\left(1+2.45/Nu^T\right)} \tag{16}$$

H_l is a Pr dependent function released by Fuji [4]. In this situation the Nu number is directly determined by using the mathematical expressions from (16). After determining Nu number, the usual path explained on the above cases, of calculating the heat transfer coefficient and thermal resistance in convection was taken. The thermal resistance value yielded by the above method was 43.51 K/W.

After going through all the calculations presented on all the above situations, we know all the convection thermal resistances values for all the surfaces of the block. Given these values, the total convection thermal resistance between the block and the air volume filling the enclosure, R_{ba}, is determined by considering the inner volume of the block connected in a parallel circuit with each one of the faces, like in the expression bellow:

$$\frac{1}{R_{ba}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6} \tag{17}$$

In the above relation, R_1, R_2 … R_6 are the convection thermal resistances for each one of the six surfaces of the rectangular block. The calculated value for R_{ba} was 5.84 K/W .

In the same manner the total thermal resistance was evaluated for the “Case” from Fig. 1. The plastic material used had a thermal conductivity of 0.19W/m·K. As we can see in Fig. 1, the case has five surfaces, excluding the bottom plate, which was a 10 mm thick plate. The heat transfer through the bottom plate was neglected, because the rectangular block was suspended inside the case. The five surfaces are: two rectangular surfaces, with a length of 260 mm and a height of 100 mm; two semi-cylindrical surfaces with an external radius of 67 mm and the same mentioned height of 100 mm and a top surface composed from three elementary areas, one rectangle and two semi-disc surfaces, with radius of 67 mm and distance between the centers of 260 mm. For the two vertical flat surfaces and the top surface the same path of determining the convection thermal resistances was used. All the relations presented in the cases of vertical and horizontal surfaces where used as well in this case. The only new situation was for the two semi-cylindrical surfaces, where we use equation (1) for conduction, but for determining the convection heat transfer, we need to use the following relation from F. Kreith and R. F. Boehm [5]:

$$Nu_l = \frac{0.9\xi Nu_p}{\ln(1+0.9\xi)} \qquad \xi = \frac{2L/D}{Nu_p^T} \tag{18}$$

L and D are the length and diameter of the cylindrical shape. Nu_p and Nu^T_p are actually Nu_l and Nu^T from 13. This is because the vertical cylinder situation is quite close to the vertical flat plate in terms of heat transfer estimations, but for a better accuracy the amendment presented in equation (18) was made. By finding out Nu_l value from equation (18) and using Nu_t from equation (13), we can find out the total Nu number value by using equation (12) with m=10. For the conduction thermal resistance through a plane wall, the following equation is used:

$$R_{cond} = \frac{L}{k \cdot A} \tag{19}$$

In the above relation, L represents the plane wall thickness (0.003m), k is the thermal conductivity of the wall material and A represents the longitudinal wall section area. This equation is used to calculate the conduction thermal resistance through the walls of the plastic case. To calculate the total thermal resistance value for the whole case, is use an equation similar with equation (17), but just with five members, given the five walls of the case involved in the heat transfer. Each individual member, from R_1 to R_5 is actually the sum between the conduction and the convection thermal resistance through that wall. The calculated value represents the sum of two thermal resistances presented in Fig. 2, R_c and R_{ca}, due to the series type connection. This represents the total thermal resistance of the case, and it has a value of 0.839 K/W. To calculate the whole thermal resistance of the entire system, we have made only the sum between the thermal resistances of the rectangular block in conduction and the thermal resistances presented in Fig. 2. Thermal resistance R_{ac} (convection between the air volume and the inner surface of the case) is considered 0, because of the same temperature of the air volume and the inner surface inside the case.

The total thermal resistance of the system was calculated as being 7.091 K/W. The temperature difference yielded by this value is of 39.709 K. Given the present setup that is considered, this is the temperature difference between the resistor surface and the ambient. The temperature at the surfaces of the block is of 38.07 K, given that the temperature difference between the resistor surface and the surface of the block.

Measurements have been made to check the accuracy of the model. The temperature was measured at the surface of the block. The ambient temperature was recorded as well. The temperature difference between the block surfaces and the ambient given by the measurements was 42 K. This shows an error of 10% of the theoretical model.

A second set of measurements was made. This time the rig was moving on a linear actuator vertically, once per second. The test was made to check if moving the rig in a similar way with an industrial robot would show any difference in the temperature levels. The test showed no difference whatsoever.

This fact confirms that the empirical equations used in the model would remain valid in the case on a moving rig, like an industrial robot.

3. Conclusions

The thermal model showed an accuracy of 90%. Also another important step was to check if the model remains valid in the case of a moving rig. The temperature measurements showed no difference, which gives the confidence needed to use the present theoretical model made on a static rig for a moving one. The initial temperature difference between the walls of the block and the ambient was assumed to be 1K, in order to obtain a value for the Rayleigh number. Changing this value for a scale from 1 to 30 K showed very small variations (<0.5K) in the final calculated temperature difference yielded by the thermal model. This model can be used in order to estimate temperature levels in an electric servomotor of an industrial robot. The losses in the motor need to be known or deduced with specific methods, like creating a rig with the same shape with the motor and a heater inside, and correlate the surface temperatures of the motor with the surface temperatures in the rig.

Placing the rig on a linear actuator and moving it once per second with a stroke of 100 mm showed no difference in temperature level, which gives confidence in using this model for a industrial robot motor temperature estimation. Temperature is one of the important factors when choosing a servomotor in the design process, due to the maximum temperature levels given by the manufacturers. Using a theoretical model instead of modifying the CAD models and import them in FEA software for thermal analysis gives a way shorter path to find the temperature value of the servomotor.

R E F E R E N C E S

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