Work dedicated to the 70th anniversary of Prof. Pier Paolo DELSANTO

EVALUATION OF THE PHYSICAL PARAMETERS OF THE PRINCIPAL GROWTH STAGES OF THE HUMAN BODY AND THEIR INTERPRETATION

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Pornind de la rezultatele obținute în cadrul studiului nostru anterior [1], lucrarea de față prezintă: a) caracteristicile specifice statisticilor existente de volum înalt ale proceselor de creştere, b) un criteriu propus pentru evaluarea compatibilității modelelor teoretice în raport cu datele experimentale studiate, c) rezultatele obținute prin analiza datelor statisticilor de volum înalt privind procesele de creştere, comparate cu rezultatele găsite cu ajutorul datelor furnizate de anumite statistici de volum restrâns privind aceste procese (de creştere).

Starting from the results obtained in the frame of our previous study [1], this paper reports the: a) specific features of the existing high-volume statistics growth data, b) proposed criterion of for the evaluation of the compatibility of theoretical models relative to the studied experimental data, c) results obtained by means of the analysis of the high-volume statistics growth data, compared with the results obtained by means of some low-volume statistics growth data.

Key words: High-volume statistics growth data, Power laws, West Universality class (U2), human body growth stages, Compatibility of theoretical models relative to the experimental data.

1. Introduction

The first part of this study [1] (accomplished for some low-volume statistics growth data [2] - [4]) was intended to the search of the best quantitative tools which allow the identification of the basic growth stages of the human body. It was found that: a) the best choice of the representation space for the study of the growth of a certain parameter x (length, height/stature, weight, and head circumference) corresponds to the pair of dynamic variables $\{y = \ln(x/x_o), \dot{y}\}$,

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b) the description of all basic types of growth stages (burst, inflation, auto-catalytic [5] with some possible oscillations, Gompertzian [6], West's type [7] slowing down, and of growth stop) is accurately provided by: (i) our modified version of West's growth model: $\dot{y} = \left(\dot{y}_o + \frac{\beta}{\gamma}\right)e^{\gamma(y-y_o)} - \frac{\beta}{\gamma}$, (1)

and (independently) by the: (ii) power law: $\dot{y} = \dot{y}_o + C(y - y_o)^n$, (2)

where $y_o, \dot{y}_o$ are the values of the used dynamic variables at beginning of the considered growth stage.

Some preliminary (due to the low confidence level of the low-volume statistics growth data) identifications of the main growth stages and of their corresponding parameters were achieved, being obtained some specific conclusions referring to the different growth stages and to the whole studied growth process. Given being the existing high-volume statistics growth data [8], [9], characterized also by a considerably better quality [the possibility to know not only the average values, but additionally the probability distribution function $f_t(P)$ (hence the percentile values and plots) for all growth times t and different growing parameters x, a considerably higher confidence level of all numerical values, etc], the analysis of the high-volume statistics growth data is necessary. Due to the huge amount of the growth data involved by these high-volume statistics [8], [9], this work will deal only with: a) the matters related to the specific applications of the previously found quantitative tools for the identification of the main growth stages, and the evaluation of their parameters, b) the study of the compatibility of the used theoretical descriptions with the existing experimental data, c) the comparison of the results obtained the high- and low-volume statistics, respectively.

2. High-volume statistics

2.1. *General Features of the High-Volume Statistics Data*

As it is known, the most important present high volume statistics referring to the human body growth are those of the organization Centers for Disease Control and prevention (CDC) [8], [9]. The monthly values indicated in the frame of CDC statistics present a high accuracy [7 characteristic figures, hence obtained starting from the registered growth data for a set of (at least) 10^8 people]. In such conditions, the most imperfections affecting the growth data corresponding to the low-volume statistics are eliminated, e.g. the most discontinuities are smoothed and the local (regional) specificity are also avoided. The evaluation of the probability function becomes possible, leading to the calculation of the percentile plots.

Given being the huge amount of data provided by the high-volume statistics sources [8], [9], the test of our proposed quantitative tools intended to the identification of the main growth stages of the human body growth has to be accomplished for a limited number of data and (of course) for some of the most important such ones. That is why our work will study mainly the data referring to the median values of the growing quantities.

It is expected then to find the continuity both of the first $\dot{y}$ and of the second derivative $\ddot{y}$ of the non-dimensional growth parameter $y = \ln(x/x_o)$ and of its related parameter – slope of the $\dot{y} = f(y)$ plot: $d\dot{y}/dy = \ddot{y}/\dot{y}$.

The usefulness of this last parameter $d\dot{y}/dy = \ddot{y}/\dot{y}$ is pointed out by its possible applications:

a) to uncover some discontinuities (and perhaps some imperfections) of the existing data sets, even of those corresponding to high-volume statistics,

b) to establish the limits of the main stages of the human body growth.

2.2. ***Identification of the Discontinuities of Growth Plots given by the High-Volume Statistics Data***

As it will be found below, due to its high non-linearity – the West's descriptions are extremely sensitive to any discontinuity of the growth plots, the corresponding iterative evaluation procedures leading quickly to instabilities. For this reason, the identification of each such discontinuity is important.

Given being the huge amount of information about the human body growth given by the high-volume statistics data (of the magnitude order of almost 10^5 numbers and 10^6 figures, e.g. [9]), the appearance of some discontinuities of the $\dot{y} = f(y)$ plots is not surprising. The strongest tool for the identification of these discontinuities is given by the values of the slope $d\dot{y}/dy = \ddot{y}/\dot{y}$ of these plots. This criterion allowed us to find some discontinuities of the *median* plots referring to the: a) baby boys length growth around 2 years, b) boys height/stature growth around of 3 years, and finally find that the differences of the monthly averages of the boys lengths and statures, respectively, for all 12 months between the 24th and the 35th one appear as exactly equal to 0.80000 cm (see the web pages ZLENAGEINF and ZSTATAGE [9]), which seems to be an artifact.

2.3. ***Evaluation of the limits of the basic growth stages***

The extreme values of the slope $d\dot{y}/dy = \ddot{y}/\dot{y}$ indicate the inflexion points of the $\dot{y} = f(y)$ plots, hence the limits of some growth domains. E. g., the maximum value (corresponding to the middle of the 145th month) of the slope $d\dot{y}/dy$ of the median plot inside the interval 127…159 months of adolescents stature growth indicates that the burst growth acts between 127 and 145 months, while the following inflation stage is located between 145 and 159 months.

3. Study of the compatibility of the theoretical models with the experimental data on the human body growth

In order to simplify the study of the compatibility of theoretical models $\dot{y}_{theor} = f(y_{\exp})$ relative to the existing experimental data $\dot{y}_{\exp}$, we will study the linear correlation between these experimental data $t_i \equiv \dot{y}_{\exp,i}$ $(i = 1,2,...N)$ and the corresponding theoretical values $u_i \equiv \dot{y}_{theor,i}$ $(i = 1,2,...N)$. Taking into account that the frequent very near to 1 values of the modulus $|r|$ of the correlation coefficient:

$$r = \frac{Cov(t,u)}{\sqrt{V(t) \cdot V(u)}} \tag{3}$$

do not correspond always to the theory/experiment compatibility, we defined [10] the compatibility criterion λ as: $\lambda = \frac{V(r)}{(1-|r|)^2}$, (4)

where *V(r)* is the variance of the correlation coefficient *r*.

The accomplished calculations led to the following detailed expression of the compatibility criterion λ:

$$\lambda = \frac{(1-|r|)^{-2}}{V(t) \cdot V(u)} \cdot \left[N^2 \left\{(t - s \cdot u - c)^2 \cdot s_u^2 + (u - s' t - c')^2 \cdot s_t^2\right\}\right], \tag{5}$$

where: a) the square brackets are permanently reserved in this section to the Gaussian symbol of the sum: $[f(U,V,...Z)] = \sum_{i=1}^{N} f(U_i, V_i,...Z_i)$, (6)

f(*U, V,… Z*) being an arbitrary function of the parameters *U, V, … Z*,

b) s_u and s_t stand for the square mean (standard) errors of the uniqueness *u* and of the test *t* parameters, respectively, in the state *i*,

c) the covariance of certain (arbitrary) *t* and *u* parameters, and the variance of the *u* parameter are given by the following expressions:

$$Cov(t,u) = [N \cdot t \cdot u] - [N \cdot t] \cdot [N \cdot u] \ , \quad V(u) = Cov(u,u) \ , \tag{7}$$

where N_i $(i = 1, 2, ... N)$ are the normalized weights, which can be expressed by means of the non-normalized weights W_i as it follows: $N_i = \frac{W_i}{[W]}$, (8)

c) *s, c* and *s', c'*, respectively are the slopes and the coordinates of the crossing points with the ordinate axis of regression straight lines (least squares fits) *t* = *f*(*u*) and *u* = *F*(*t*), respectively:

$$s = \frac{Cov(t,u)}{V(u)}, \quad c = \frac{Cov(u, t \cdot u) - Cov(t, u^2)}{V(u)} \ , \tag{9}$$

and – accomplishing the permutation between *u* and *t* – the corresponding expressions of the parameters *s'* and *c'*.

From the definition (4) of the compatibility criterion λ, it results that if $\lambda << 1$ the linear correlation is incompatible with the analyzed growth data, while for $\lambda >> 1$ (or even for $\lambda \geq 1$) – this correlation is compatible with the studied experimental data. We have used 3 different types of non-normalized weights:

(i) *the E type of equal weights:* W_i = const., hence: $N_i = \frac{1}{N}$, (10)

was used when there are not known all the square mean (standard) errors corresponding to the individual values u_i, t_i (i = 1, 2, … N) and the values t_1, t_2, … t_N are located in a narrow interval (hence: $V(t) << [N \cdot t]^2$),

(ii) *the M type of weights* (of "magnitude"): $W_i = 1/t_i^2$, (11)

was used when there are not known all the square mean (standard) errors corresponding to the individual values u_i, t_i (i = 1, 2, … N) and the values t_1, t_2, … t_N are considerably different (hence: $V(t) \sim [N \cdot t]^2$),

(iii) *the A type of weights* (of "accuracy"): $W_i = 1/s_{ti}^2$, (12)

was used when all the square mean (standard) errors s_{ti} corresponding to the individual values of the "test" parameter t in the state i were known.

For the E type of weights, the relation (5) reaches its simplest expression:

$$\lambda = \frac{1+|r|}{1-|r|} \cdot \frac{1}{N} \left\{ \frac{s_t^2}{V(t)} + \frac{s_u^2}{V(u)} \right\} , \tag{13}$$

where s_t^2 and s_u^2 are the estimated average (over all studied states i) values of the s_{ti}^2 and s_{ui}^2 values, respectively.

Taking into account that the numerical value of the growing quantity x (length, height/stature, weight) is expressed as: $x = \sum_{n=-m}^{p} c_n \cdot 10^n$, (14)

where c_n are figures, and m, p are integers ≥ 0, it results that the square mean (standard) error of non-dimensional variable $y = \ln(x/x_o)$ can be evaluated as:

$$s_y = \frac{1}{x} s_x \text{, where the magnitude order of } s_x \text{ is: } s_x = \frac{1}{2} \cdot 10^{-m} . \tag{15}$$

Starting from the Finite Differences expressions of the first and third order derivatives (see e.g. [10]): $\dot{y}(0) = \frac{y(\tau) - y(-\tau)}{2\tau} - \frac{1}{3!} \dddot{y}(0) \cdot \tau^2 + \dots$, (16)

and: $$\dddot{y}(0) = \frac{y(2\tau) - 2y(\tau) + 2y(-\tau) - y(-2\tau)}{2\tau^3} , \tag{17}$$

it results that the magnitude order of the square mean (standard) error affecting the parameter $\dot{y}$ is: $$s_{\dot{y}} = \frac{y(2\tau) - 2y(\tau) + 2y(-\tau) - y(-2\tau)}{12\tau} . \tag{18}$$

Finally, if the definition (4) [and the implied expressions (5), (13)] are used for the linearized correlation $\dot{y} = f(\dot{y}_c)$ [where $\dot{y}_c$ are the calculated values of the time derivative $\dot{y}$, calculated by means of the evaluated parameters (by means of the gradient method) of West's or power law type correlation], then the estimated square mean error (over all studied states) of the parameter $u \equiv \dot{y}_c$ will be:

$$s_u \equiv s_{\dot{y}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\dot{y}_{ci} - \dot{y}_i)^2} \quad . \qquad (19)$$

It results that the value of the compatibility criterion can be calculated by means of relations (5) or (13) [depending of the type of used weights], (18) - always, and (15) for a linear correlation (as the Gompertz's one) or (19) for a linearized relation of the type $\dot{y} = f(\dot{y}_c)$.

4. Numerical Results corresponding to the high-volume Statistics Data and Comparison with those of the low-volume Statistics

Excepting the values of the criterion λ of compatibility of the studied theoretical models relative to the existing high-volume statistics data, which where calculated (for simplicity) by means of the relation (13) corresponding to equal weights, all other values from the following Table 1 were obtained using the *M* type (of magnitude) of weights, given by the relation (11). The iterative procedure of the classical gradient method [10], [11] was used for the evaluation of the specific parameters of the different growth stages.

Taking into account that the low-volume statistics growth data [2], distinguish only the burst length growth stage in adolescence instead of the 2 successive burst and inflation phases, we have introduced in Table 1 the obtained numerical values of the growth parameters of the burst phase [2] between those corresponding to the burst growth stage and the inflation one, respectively.

For the interpretation of the numerical results from Table 1, we will remember that according to the newly proposed version of West's model (1), the growth process is the resultant of 2 competitive pairs of antagonist processes: the amplification/damping one (of coefficient β) and the generation/annihilation process (of microscopic growth centers, of coefficient γ), while the power laws parameters are related to the growth rate rise (parameter C) and to the growth acceleration (for $n > 1$) or deceleration (for $n > 1$).

Additionally, from relations (1) and (2), one obtains:

$$\dot{y}' \equiv \frac{d\dot{y}}{dy} = (\beta + \gamma \cdot \dot{y}_o) \cdot e^{\gamma(y-y_o)}, \quad \dot{y}'' \equiv \frac{d^2\dot{y}}{dy^2} = \gamma(\beta + \gamma \cdot \dot{y}_o) \cdot e^{\gamma(y-y_o)}, \qquad (20)$$

and:

$$\dot{y}' \equiv \frac{d\dot{y}}{dy} = C \cdot n(y - y_o)^{n-1} \;, \quad \dot{y}'' \equiv \frac{d^2\dot{y}}{dy^2} = C \cdot n(n-1) \cdot (y - y_o)^{n-2} \;, \qquad (21)$$

hence: $$\gamma = \dot{y}''/\dot{y}' \ , \quad \frac{1}{n} = 1 - \frac{(\dot{y} - \dot{y}_o)\cdot \dot{y}''}{\dot{y}'^2} \ , \qquad (22)$$

and: $$\beta = \dot{y}'_o - \gamma \cdot \dot{y}_o = \dot{y}' \cdot e^{-\gamma(y - y_o)} - \gamma \cdot \dot{y}_o \ , \quad C = (\dot{y} - \dot{y}_o)\cdot (y - y_o)^{-n} \ . \qquad (23)$$

Table 1

Quantitative Features of the Main Height Growth Stages of Human Beings

Correlation Type	Source / Parameter	CDC [9] Baby Length Slowing down	CDC [9] Boy Stature Slowing down	CDC [9] Adolescent Burst Growth Stage	Hartung [2] Adolescent Burst Growth Stage	CDC [9] Adolescent Inflation Growth Stage	CDC [9] Height/ Stature Growth Stop
West	Time int.	0..33month	2 ... 8years	127...145ms	11...15 years	145...157ms	159...225ms
	$\dot{y}_o$, yr-1	1.25	0.11	0.0343	0.0286	0.0424	0.0488
	Number of data pairs	11 (one per trimester)	12 (one per semester)	9 (one at 2 months)	5 (one per year)	12 (one per month)	12 (one per semester)
	β, %/yr	0.0362	0.2122	- 1.035	2.909	1.326	- 0.6538
	γ	-3.776	-5.278	31.86	2.825	-24.713	7.713
	Standard Deviation	0.769%	1.595%	0.404%	2.89%	0.1827%	4.559%
West Linear $\dot{y} = f(\dot{y}_{calc})$	Slope	0.9985	0.9789	1.017	1.035	1.011	1.042
	Crossing Coordin.	$7.4\cdot10^{-5}$	$1.22\cdot10^{-3}$	$-6.79\cdot10^{-4}$	- 0.155	$-5.3\cdot10^{-4}$	$-1.46\cdot10^{-4}$
	Standard Deviation	0.8042%	1.697%	0.346%	1.971%	0.18826%	5.169%
	Correlation coeff.	0.99998	0.9983	0.9984	0.9794	0.9993	0.9988
	Compatibility Criter. λ	131.597	4.070	4.197	4.374	4.189	4.395
Power Law	Time interval	0 ... 3 years	2 ... 10 years	127...144 months	11 ... 15 years	145...159 months	159...239 months
	Number of data pairs	35 (one per month)	95 (one per month)	18 (one per month)	5 (one per year)	14 (one per month)	80 (one per month)
	C, %/yr	- 0.947	- 0.1097	1.191	21.88	0.056	- 1.548
	n	0.3395	0.5622	1.697	1.169	0.6794	1.56665
	Standard deviation	7.14%	3.066%	0.1001%	2.681%	0.67498%	6.363%
Power Law Linear $\dot{y} = f(\dot{y}_{calc})$	Slope	1.093	1.0037	1.0005	1.011	1.0182	1.0382
	Crossing Coordin.	- 0.0110	$-1.1\cdot10^{-4}$	$-1.97\cdot10^{-5}$	- 0.0497	$-8.8\cdot10^{-4}$	$-5.4\cdot10^{-5}$
	Standard deviation	14.59%	9.46%	0.1338%	1.875%	0.7853%	6.311%
	Correlation coeff.	0.98717	0.9947	0.99985	0.9804	0.9898	0.999266
	Compatibility Criter. λ	4.875	4.051	3.3443	4.1698	4.221	3.961

Note: In the first 2 lines of Table 1 and in following are used the abbreviations “Time int.” for the time interval and “ms” for months, “yr” for year(s).

One finds so that besides their physical meanings, the West parameters *β, γ* and those (*C, n*) of the power laws have some geometrical meaning related to the first 2 indirect [by means of the time function *y*(*t*)] time derivatives: $\dot{y}' = \frac{d\dot{y}}{dy} = \frac{\ddot{y}}{\dot{y}}$ and $\dot{y}'' = \frac{d\dot{y}'}{dy} = \frac{1}{\dot{y}}\frac{d}{dt}\left(\frac{\ddot{y}}{\dot{y}}\right)$ of the function $\dot{y}$ in the representation space $\dot{y}, y$.

5. Characteristic and transition Growth stages. Fine structure of the Growth Plot

The examination of Table 1 points out that - for the identified growth stages – the parameters of the used theoretical models: *β, γ* (West), *C, n* (power laws) have distinct (hence discontinuous) values, while – due to the huge number (at least of the magnitude order of 10^8 people), the high-volume statistics smooth and eliminate so the eventual discontinuities initially present in the growth data.

It results so that there are 2 types of growth stages:

a) *the "characteristic" growth stages* [presenting a certain stability of the values of the West and power laws parameters, given by the expressions (22) and (23)],

b) *the transition growth stages*, presenting a continuous change of the West and power laws parameters between some successive growth stages.

Taking into account that both the newly proposed version of West's model (1) and the power laws (2) are defined in terms of the first derivatives $\dot{y}', \dot{y}''$ of the time derivative $\dot{y}$ of the non-dimensional growth advancement parameter *y*, it results that the transition growth stages will be located around the extreme values points: $\dot{y}' = 0$ and of the inflexion points: $\dot{y}'' = 0$ of the $\dot{y} = f(y)$ plot, where this plot slope and curvature, respectively, change their signs (see e.g. Fig. 1).

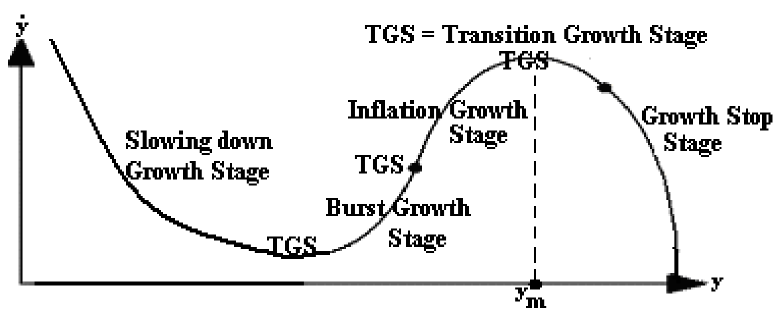

Fig. 1. Typical characteristic and transition growth stages

In order to check this assumption, we have calculated – by means of relations (22a) and (23a) - the parameters *β, γ* of the used West model for each point ("differential": $y \cong y_o$) growth process in the interval 128…159 months (after birth).

The partial derivatives intervening in the relations (22a) and (23a) were calculated by means of the Finite Differences Method, using the logical scheme indicated by Figure 2. The obtained results were synthesized in Table 2.

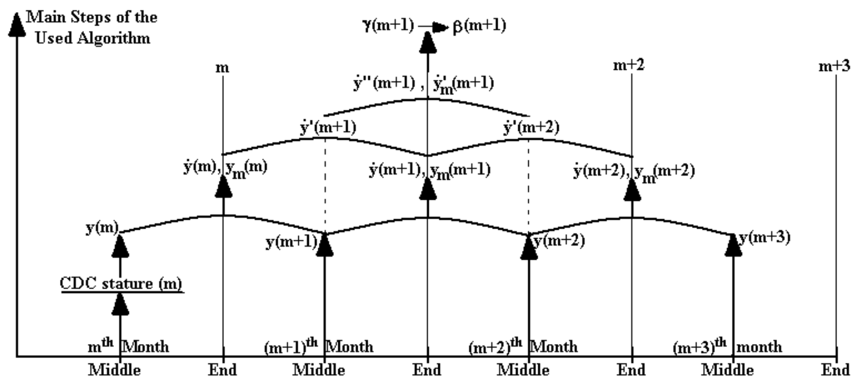

Fig. 2. Logical Scheme of the Finite Differences evaluation of the generation/ annihilation γ and amplification/damping β West's coefficients (the symbol $y_m(m)$ stands for the medium (average) value of parameter *y* during the m-th month)

The analysis of the results synthesized by Table 2 points out that the parameters *β, γ* of the West differential segments between two successive measurements present intervals of: a) relative stability corresponding to the Burst characteristic growth stage and to the Inflation stage, and: b) transition between the main stages of a macroscopic structure (Burst and Inflation, ones), establishing so a certain continuity of the successive stages of the growth process.

While the intervals between 2 successive measurements, whose West parameters are given in the 2nd and 3rd columns of Table 2, represent *the hyperfine structure of the growth process*, the broader intervals corresponding to the main stages of the macroscopic structure of the growth process form together with the transition intervals *the fine structure* of this growth process.

3. Conclusions

As it is well known, the dominant evolution trends of the whole human body growth process are described by the equations corresponding to the Universality classes: U1 (Gompertz), U2 (West), etc.

The human body growth is not however a uniform process, the growing parameters [length/height, weight, head circumference, etc] *x(t)* presenting both decelerating stages ($\frac{1}{x}\frac{dx}{dt} < 0$) and (considerably shorter) accelerating stages ($\frac{1}{x}\frac{dx}{dt} > 0$).

Table 2

Values of West's parameters *β* and *γ* for the point ("differential") growth processes around the end of a certain month, during the adolescence growth between 128 and 159 months (after birth). In bold are indicated the values which belong to the intervals centered around the *β, γ* values obtained by means of gradient method [11] (see Table 1) of amplitude 10 dB.

The month (after birth)	$\beta\ (yr^{-1})$	γ	Type of the growth stage
128	- 2.786	81.836	**Connection with the previous slowing down Growth Stage**
129	- 4.677	131.373	
130	- 0.3936	13.656	
131	- 3.113	91.137	
132	**- 1.670**	**50.42**	Characteristic Burst stage, **centered around** $\beta = -1.035\,yr^{-1}$ and $\gamma = 31.86$
133	**- 0.751**	**24.69**	
134	**- 1.225**	**37.85**	
135	**- 1.028**	**32.48**	
136	**- 0.864**	**28.04**	
137	0.1549	0.9058	Partial characteristic burst stage prolongation
138	**- 1.2936**	**38.916**	
139	+ 0.6218	- 10.522	
140	**- 0.90047**	**28.105**	
141	0.4558	- 5.698	**Gompertzian transition stage (the signs and values of *β* and *γ* are gradually changed)**
142	0.09103	3.2297	
143	0.1124	2.716	
144	0.620	- 9.252	
145	0.268	- 1.112	
146	**1.129**	**- 20.66**	Partial characteristic Inflation stage prolongation
147	0.405	- 4.4968	
148	**1.3194**	**- 24.54**	**Characteristic Inflation stage centered around** $\beta = 1.326\ yr^{-1}$ $\gamma = -24.713$
149	**1.157**	**- 21.016**	
150	**1.635**	**- 31.147**	
151	**2.016**	**- 39.11**	
152	**1.962**	**- 37.94**	
153	4.139	- 82.46	**Connection (- γ >> 1) with the following growth stop stage**
154	3.571	- 70.846	
155	9.695	- 194.165	
156	19.2297	- 385.07	
157	- 28.253	567.39	**Connection (γ>> 1) with the previous inflation stage**
158	- 7.132	142.016	
159	- 5.277	64.195	

For the low-volume statistics data (e.g. [2], [3]), the number of existing data between two extremes of the function $\frac{1}{x}\frac{dx}{dt}$ is rather reduced, and they can be put in correspondence with one or few West and/or Gompertz growth stages [rarely of the auto-catalytic (U0) type]. These main stages of the human body growth form *the macroscopic structure* of the growth process.

The new version of the West expressions [expressed by the relations (1) and (20) of this work] allows also the description of very short ("differential") growth stages. Or, the high-volume statistics data provide both monthly data up to 20 years (all [1]-[9] sources) and even weekly and daily data [9b, c] up to the age of 5 years. Using these data, the parameters of the West very short (between 2 successive measurements) growth stages can be evaluated, they corresponding to *the hyperfine structure of the growth process*. On finds that during some growth stages, the values of the West parameters are somewhat stable (inside intervals of ±5 dB). These West type intervals (of the types of slowing down, burst, inflation, etc), as well as the intervals joining them (as the Gompertzian , auto-catalytic, etc) form *the fine structure of the growth process.*

The most important findings of the accomplished study are:

a) the qualitative conclusions resulted by means of the analysis of the low-volume statistics growth data [1], [12], [13], are confirmed by the study of the high-volume statistics data,

b) due to the considerably better accuracy of the high-volume statistics growth data, the discrimination power of these data is sensibly higher; in consequence, the number of post-natal growth stages distinguished by the high-volume statistics is somewhat larger: 5 post-natal length/height growth stages (instead of only 4 given by the low-volume statistics), the "overall" burst phase observed for the low-volume statistics data being split into a burst phase and an inflation one (see Table 1),

c) both the West's type expressions and the power law ones are compatible with the studied experimental growth data,

d) as it concerns the slopes s and the coordinates c of the crossing with the $\dot{y}$ axis of the regression (best fit) straight-line $\dot{y} = f(\dot{y}_{calc})$, for a very good $\dot{y} = F(y)$ correlation, these parameters have to fulfil the conditions $\frac{1}{|1-s|}, \frac{1}{c} >> 1$, requirements also very well respected (see Table 1),

e) due to the considerably better stability of the power laws (relative to the West's type expressions, which are very sensitive to any discontinuity, leading quickly to instabilities of the iterative evaluation procedure), these parameters of these relations can be evaluated for considerably higher numbers N (almost 100) of representative pairs of data $\{y_i, \dot{y}_i | i = 1, N\}$; the parameters of the West's type relations can be fitted only for around 10 representative pairs of representative data, but the accuracy of their descriptions (in these cases) is sensibly better,

e) as it was to be expected, there are some significant differences between the basic parameters of the high-volume (World) statistics and the ones for the low-volume (regional) statistics; as a unique example, the maximum value of the non-dimensional length growth rate $\dot{y}$ is reached after 157 months (approximately 13 years) for the CDC statistics, and after 15 years according to the low-volume (regional) statistics given by the treatise [2],

f) the obtained results seem to be valid also (at least, qualitatively) for the growth of all living organisms [14], of tumors [15], [16], of some economic activities [15c], of the population dynamics [17], and even for the qualitative description of the Universe evolution [18], [19], [13].

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