INFLUENCE OF LATERAL SIDE-BRANCHES OVER TRANSMISSION LOSS OF ACOUSTIC RESONATORS

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This paper studies the acoustic performance of an acoustic resonator used for noise reduction, using numerical modelling for an open side-branches acoustic resonators.

Plane waves were at the base of the transfer matrix method (TMM), which can provide a fast prototype solution for designers of acoustic resonators. In this paper are presented the principle method for the transmission loss prediction (TL), applied to an acoustic resonator with lateral side-branches. The numerical prediction method is applied in successive stages corresponding to those new sections to a resonator with four lateral side-branches.

Keywords: acoustic resonator, transfer matrix method, transmission loss.

1. Introduction

A current problem that concerns the European Community, as well as the world-wide community, consists in reduction of noise pollution produced by gases passing through pipes (in equipment's and acclimatizers). Acoustic resonators used to reduce the noise level are based on Helmholtz side-branches resonators, which influence the transmission loss.

Side-branches introduced on a principal duct have a high level of transmission loss in a narrow frequency band (resonance frequency).

Internationally, among the first to initiate transmission loss calculations, in incipient design stages were Bell and Beranek [3,4,5], who defined the general characteristic parameters of impedance and resonance frequencies. Later, Rayleigh treated the resonator as a mechanical system, in which the resonator throat acts like mass, while the air in the cavity acts like a spring [11].

Anderson studied the effect of the flow over the resonance frequencies, when a single side-branch Helmholtz resonator is attached to a circular duct [2], using the "cross-correlation" technique analysis. Alfredson (1970) [1], Munjal [9,10], Thawani and Noreen (1988) [15] and Sullivan together with Crocker (1978) [14] presented various approaches regarding the modelling of acoustic resonators, using analytic and finite element methods.

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Further, Craggs (1989) [8] reported a technique that combines the transfer matrix and the finite element approaches, in the study of the acoustics of a duct coated with phono-absorbent materials. Schill, studied the acoustic resonance of cylindrical tubes with side-branches [12] for military applications. Bugaru, Enescu and Vasile studied noise attenuators and developed methods and algorithms for the transmission loss calculation, using the transfer matrix method TMM [6,7,16].

The basic idea used in the present paper for implementing the transfer method matrix consists in considering the sound pressure p and the particle velocity u as acoustic state variables (Fig.1).
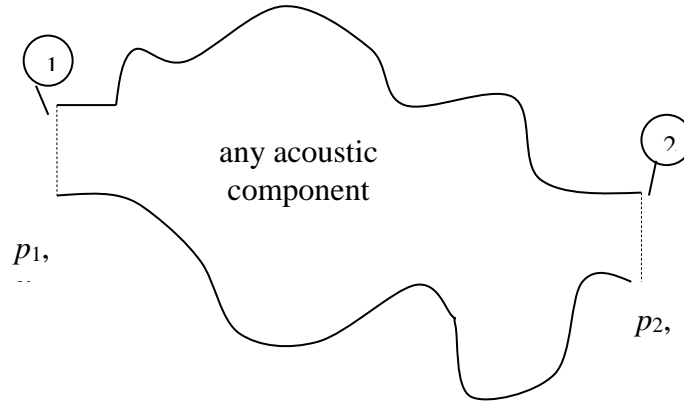

Fig.1. The general case

For any passive linear component:

$$p_1 = Ap_2 + BS_2u_2, \tag{1}$$

$$S_1u_1 = Cp_2 + DS_2p_2, \tag{2}$$

or

$$\begin{Bmatrix} p_1 \\ S_1u_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} p_2 \\ S_2u_2 \end{Bmatrix}, \tag{3}$$

where: p_1, p_2 are the inlet/outlet acoustic pressures, respectively;
                u_1, u_2 are the inlet/outlet particle velocities, respectively;
                S_1, S_2 are the inlet/outlet cross-sectional areas, respectively;
                $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the four poles matrix (dependent on A, B, C and D
                components).

2. Case study for an acoustic resonator

A tube with the length L=249.5 cm and the diameter D=18.5 cm is considered, with four side-branches, with the same diameter $d = 5.35$ cm, but with non-equal lengths: the first one with the length h_1=11 cm, the second with h_2=21 cm, the third with h_3=30.5 cm, and the last one with h_4=31.5 cm.
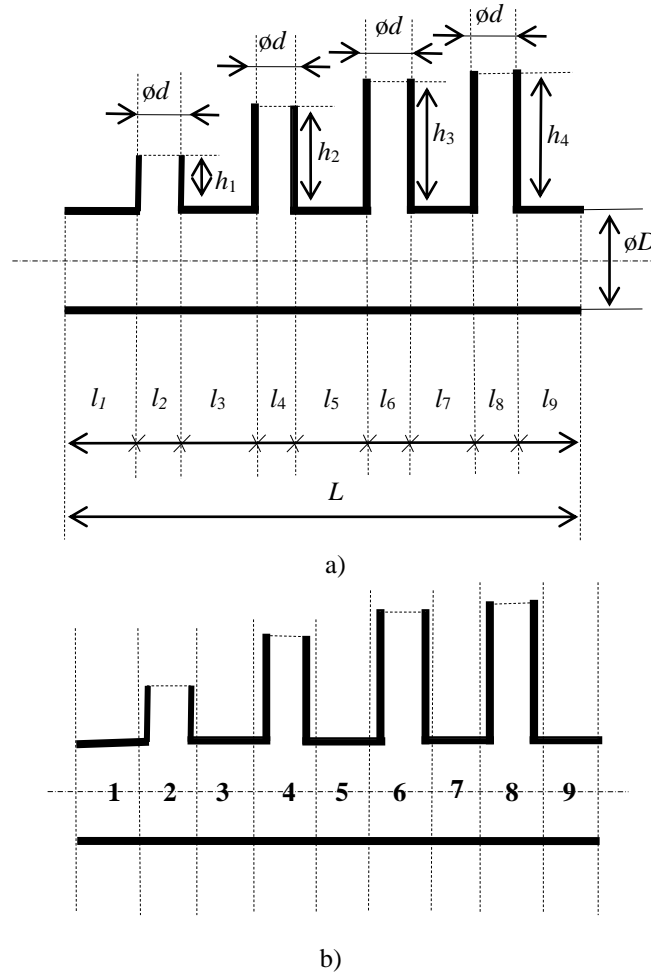

Fig.2. Decomposition of the resonator with side-branches in basic elements:
a) section dimensions; b) the considered sections

The lengths of the uniform cross-section duct segments, represented in Fig. 2, are: $l_1 = 34$ cm, $l_3=l_5=l_7$=45 cm, $l_9 = 59$ cm.

The procedure is illustrated by considering the resonator in Fig. 2. a, which is divided into the basic elements, labeled 1-9, indicated by the dashed lines from Fig. 2. b.

The lengths of the sections l_2, l_4, l_6 and l_8 are equal with the lateral side-branches diameters d.

Elements 1, 3, 5, 7 and 9 are simple pipes of constant cross section.

Elements 2, 4, 6 and 8 are sections with lateral side-branches of non-equal lengths.

2.1. Pipes with uniform cross-section

For a pipe with uniform cross-section (ucs), denoted by index i (Fig. 3), the inlet acoustic pressure and the mass velocity fields are denoted, respectively, by p_i, $\rho_o S_i u_i$, while the corresponding outlet quantities are denoted by p_{i+1}, $\rho_o S_i u_{i+1}$ [9, 13,17].
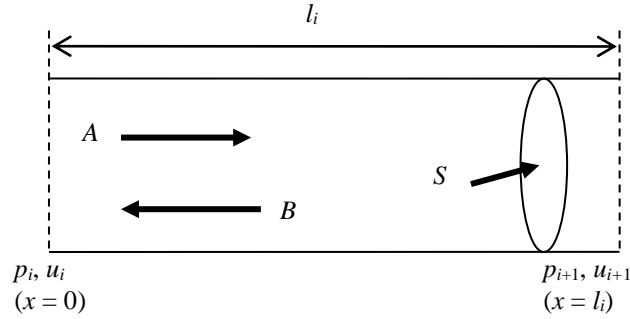

Fig. 3. Model of a uniform cross-section pipe

The following is a list of variables and parameters that appear in most transfer matrix relations of reactive elements [17]:

p_i = acoustic pressure at i-th location of element,

u_i = particle velocity at i-th location of element,

ρ_0 = mean density of gas [kg/m^3], c = sound speed [m/s],

S_i = cross section of element at i-th location [m^2], $Y_i = c / S_i$,

A, B = amplitudes of the right- and left-bound fields, respectively,

$k_c = k_0 / (1-M^2)$, assuming negligible frictional energy loss along straight pipe segments,

$k_0 = \omega / c = 2\pi f / c$ [rad/m], $\omega = 2\pi f$, f = frequency [Hz],

$M = V / c$ - mean-flow Mach number in exhaust pipe,

V = mean flow velocity through S, $T_{ij} = (ij)$- element of the transfer matrix,

$j = \sqrt{-1}$ = complex number unites.

The matrix form for the uniform pipe can be written as

$$\begin{bmatrix} p_{i+1} \\ \rho_0 S_i u_{i+1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_i \\ \rho_0 S_i u_i \end{bmatrix}, \tag{4}$$

where the transfer matrix T_{ucs} is given by:

$$T_{ucs} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{ucs} = e^{-jMk_c l_i} \begin{bmatrix} \cos k_c l_i & jY_i \sin k_c l_i \\ \dfrac{j}{Y_i} \sin k_c l_i & \cos k_c l_i \end{bmatrix}. \tag{5}$$

It can be shown (Munjal, 1975) that the four-pole constants for non-viscous medium are:

$$\begin{cases} A = e^{-jMk_c l_i} \cos k_c l_i \\[2mm] B = j\left(\dfrac{\rho_0 c}{S}\right) e^{-jMk_c l_i} \sin k_c l_i \\[2mm] C = j\left(\dfrac{S}{\rho_0 c}\right) e^{-jMk_c l_i} \sin k_c l_i \\[2mm] D = e^{-jMk_c l_i} \cos k_c l_i. \end{cases} \tag{6}$$

2.2. Side-branch section

For a side-branch muffler section (sbs), the transition elements used in modeling the cross-sectional discontinuities are shown in Fig. 4.

The relations between the specific quantities are:

$$p_i = p_{i+1} = p_B, \tag{7}$$

$$Su_i = S_B u_B + Su_{i+1}, \tag{8}$$

$$z_B = \frac{p_B}{S_B u_B} = \frac{p_{i+1}}{S_B u_B}, \tag{9}$$

$$Su_i = \frac{p_{i+1}}{z_B} + Su_{i+1}, \tag{10}$$

$$\begin{Bmatrix} p_i \\ Su_i \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{z_B} & 1 \end{bmatrix} \begin{Bmatrix} p_{i+1} \\ Su_{i+1} \end{Bmatrix}, \tag{11}$$

$$T_{sbs} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{z_B} & 1 \end{bmatrix}, \tag{12}$$

where

$$z_B = -\frac{j\rho_0 c}{S_B}\cot\left(\frac{\omega h_i}{c}\right),$$ (13)

while $S_B = \frac{\pi d^2}{4}$ is the cross-section area of the side-branch muffler [m^2].
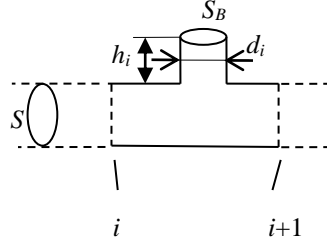

Fig. 4. Model of a side-branch muffler section

2.3. Analysis of the transmission loss

The nine elements are characterized by the transfer matrices $T^{(1)}$ through $T^{(9)}$; therefore, the system matrix $T^{(S)}$ for the whole resonator is obtained by matrix multiplication (matrices corresponding to the nine sections):

$$T^{(S)} = T^{(1)} \cdot T^{(2)} \cdot ... \cdot T^{(9)},$$ (14)

$$T^{(S)} = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$ (15)

where

$T^{(1)}, T^{(3)}, T^{(5)}, T^{(7)}$ and $T^{(9)}$ correspond to constant sections,

$T^{(2)}, T^{(4)}, T^{(6)}$ and $T^{(8)}$ correspond to sections with resonator (side-branches).

The transmission loss (*TL*) can be evaluated using the relation [9,13,17]:

$$TL = 10\log\left(\frac{1}{4}\left|A + \frac{B}{Y_i} + CY_i + D\right|^2\right).$$ (16)

3. Modelling results

For the analysis of this acoustic resonator, the influence of the lateral side-branches is studied. First, sections 1÷3 are analyzed, where a side-branch ø 5.35cm and length 11 cm is added. In the Fig. 5, resonance frequencies of 773 Hz and 2318 Hz, with transmission loss amplitudes of 37.55 dB and 41.07 dB were found (calculated with equation 16).
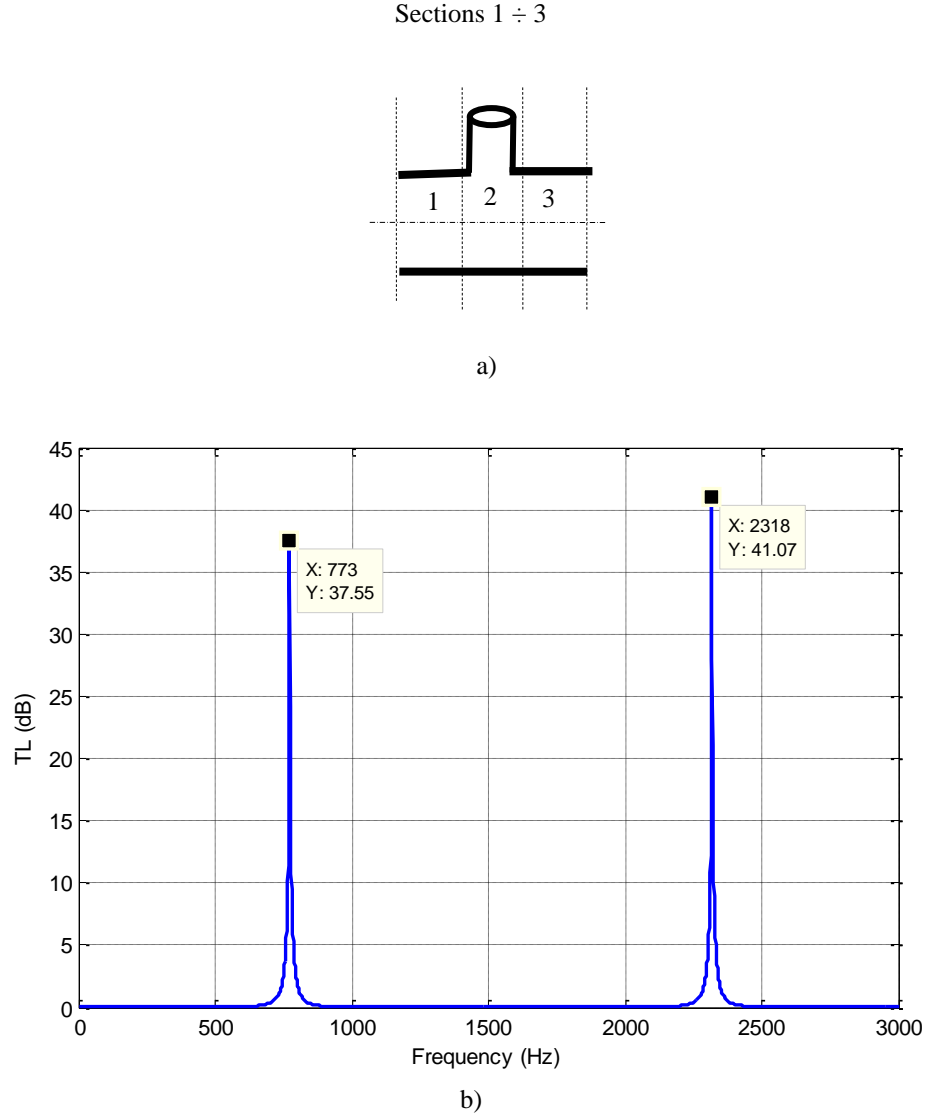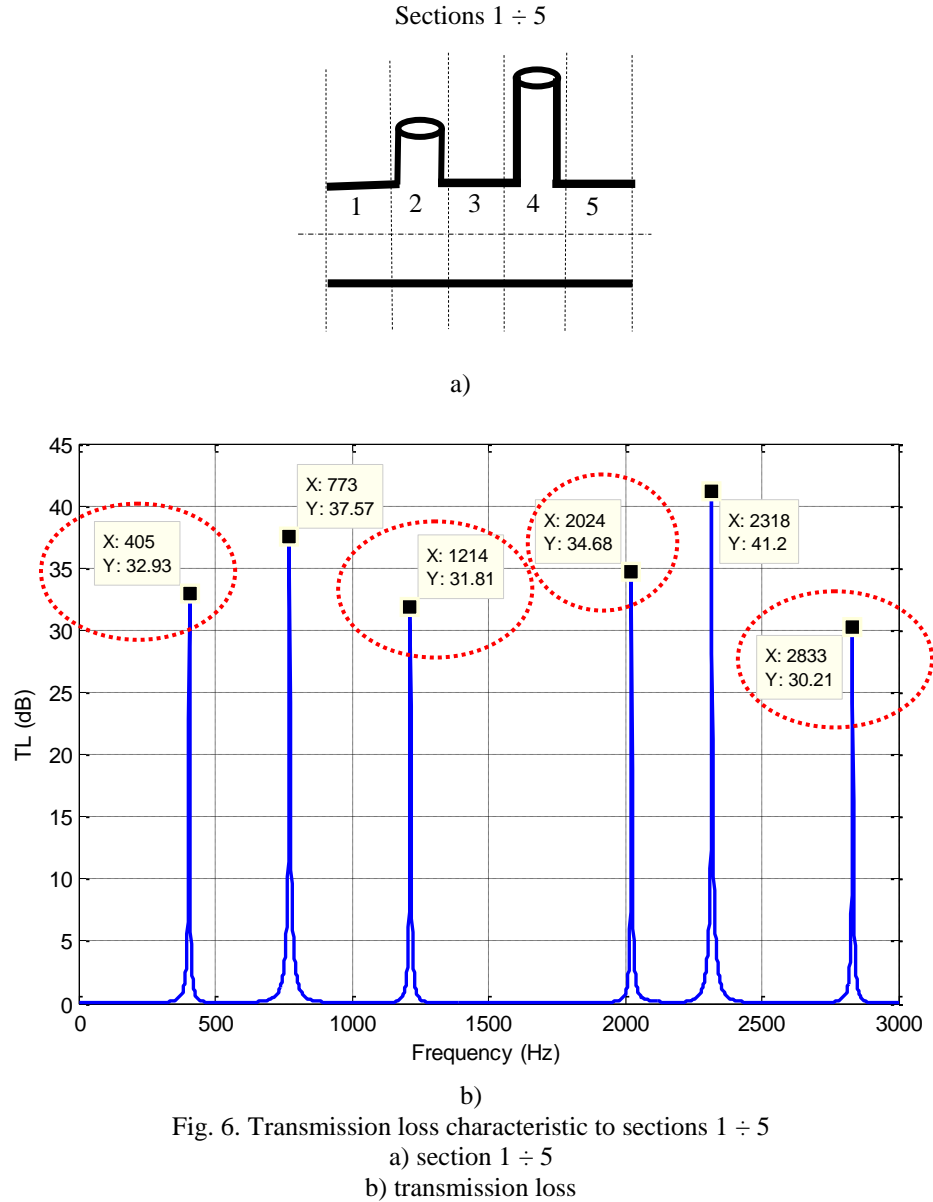
Sections 1 ÷ 3

a)

b)

Fig. 5. Transmission loss characteristic to sections 1 ÷ 3
a) section 1 ÷ 3
b) transmission loss

By adding another side-branch ø 5.35cm and length 11 cm, considering sections 1÷5 (Fig. 2), with lengths $l_1...l_5$, four other resonance frequencies can be remarked (marked on the Fig. 6): 405 Hz, 1214 Hz, 2024 Hz and 2833 Hz.

Sections 1 ÷ 5

a)

b)

Fig. 6. Transmission loss characteristic to sections 1 ÷ 5
a) section 1 ÷ 5
b) transmission loss

In a similar way, by adding a new side-branch, ø 5.35cm and length 30.5 cm, a resonator with three lateral side-branches is obtained, corresponding to sections 1÷7. The lengths of the considered sections, $l_1 \dots l_7$ are mark in Fig. 2.

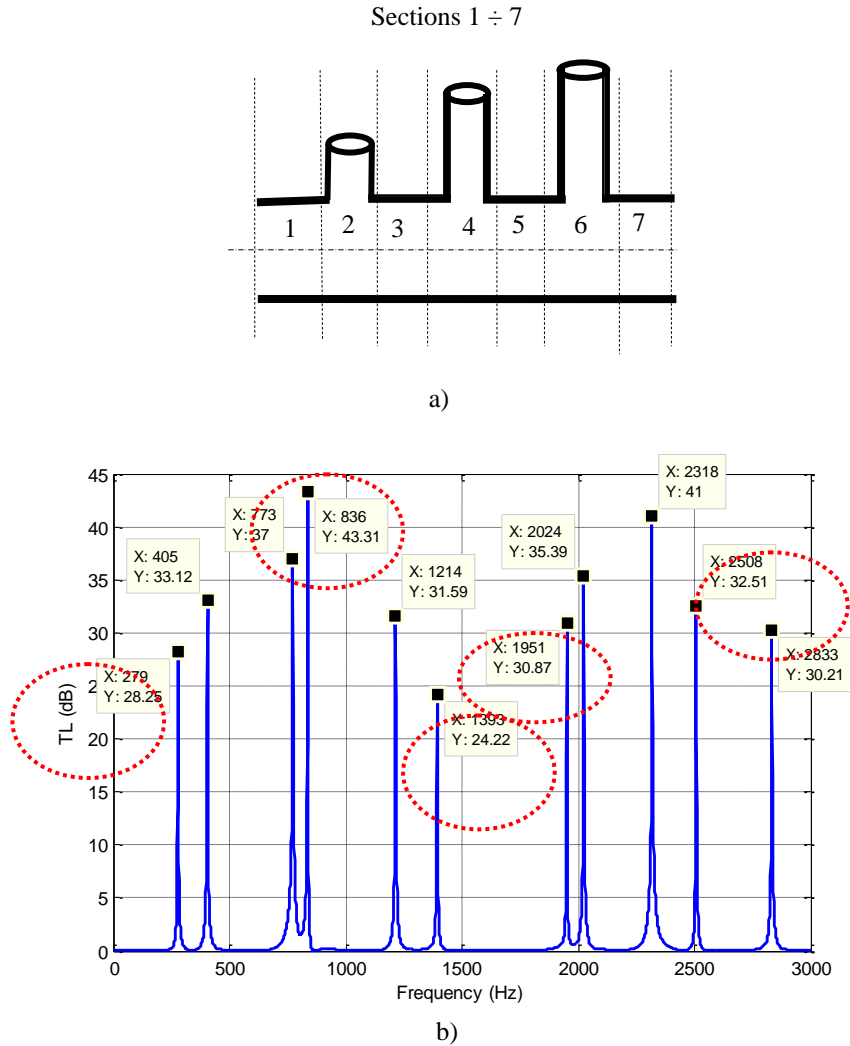In Fig. 7b, five supplementary resonant frequencies are remarked: 279 Hz, 836 Hz,1393 Hz, 1951 Hz and 2508 Hz.

Sections 1 ÷ 7

a)

b)

Fig. 7. Transmission loss characteristic to sections 1 ÷ 7
a) section 1 ÷ 7
b) transmission loss

Finally, by introducing the fourth side-branch, ø 5.35cm and length 31.5 cm, the transmission loss for the whole acoustic resonator in Fig. 2 is obtained, with all nine sections (Fig. 8).

The analysis leads to the conclusion that the frequencies marked in the previous situations (Fig. 5÷7) remain present, while the amplitudes of the transmission loss are modified.
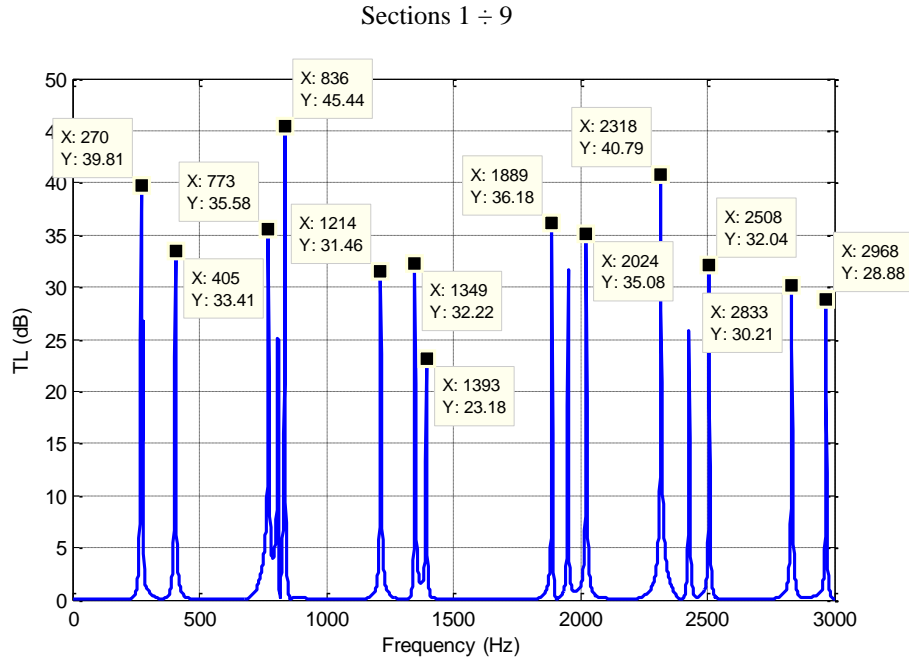
Sections 1 ÷ 9

Fig. 8. Transmission loss characteristic to sections 1 ÷ 9

4. Conclusions

By using several side-branches, the transmission loss increases.

It can be remarked that the frequency bands where the transmission loss is higher values are narrow, on domains between 30÷200 Hz. This signifies that the use of acoustic resonators is efficient in narrow and different bands, according to the chosen configuration.

In practice, if noise reduction in pipes is intended in certain frequency bands, where the spectral analysis shows dominant picks, these picks can be significantly reduced by introducing a specific acoustic resonator, with resonance frequencies coinciding with the picks of the amplitude of the measured noise.

The method of the transfer matrix, applied in this paper that is used commonly in the analysis of the noise mufflers can be successfully used also for acoustic resonators, already in the design stage.

This innovative approach of the transfer matrix method, for an acoustic resonator with several side-branches, taking into account the characteristic matrix of system $T^{(s)}$ in equation (14), can be applied for any type of acoustic resonator with open side-branches.

In further studies, the authors will develop this analytical method also for acoustic resonators with closed side-branches, based on the general principle of Helmholtz resonators, taking into account the equivalent acoustic impedance.

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