SOLUTION COMPUTATION FOR RESISTIVE CIRCUITS CONTAINING COMPANION MODELS

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Se propune rezolvarea în precizie long double a circuitului liniar care conține modele companion. Acest algoritm, care dă rezultate mult mai bune decât iterațiile GMRES, a fost implementat într-un program de analiză tranzitorie care alege pasul de timp pe baza unor erori de bilanț energetic. Sunt prezentate două exemple în care acest program dă rezultate mai bune decât SPICE.

The long double precision solving of the linear circuit containing companion models is proposed. This algorithm, giving much better results than GMRES iterations, is implemented in a transient analysis program which chooses the time step on the basis of energy balance errors. Two examples, for which this program gives better results than SPICE, are presented.

Keywords: transient analysis, time step choice, errors.

1. Introduction

The transient response of a nonlinear circuit is computed using numerical methods as backward Euler, trapezoidal, or Gear methods. These methods have been developed to compute the solution of the state equations in the normal form $\dot{x} = f(x,t)$, where x is the state vector. Nevertheless, no commercial software for circuit analysis uses the state equations due to the tedious computations needed to reach their normal form. All known circuit simulators working in the time domain use the companion models [1] which are resistive circuits as those shown in Fig. 1 and Fig. 2, which correspond to the backward Euler method.

The solution of the dynamic circuit at the time instant t_{n+1} is computed solving the circuit in which each dynamic element is replaced by its companion model. If the backward Euler method is used, the independent sources parameters depend only on the state in the previous time instant t_n The voltage and current subscripts in Fig. 1 and Fig. 2 point out the corresponding time instant. It can be observed that some resistance magnitudes increase as the time step h increases while other resistance magnitudes decrease as h increases. Using other numerical methods

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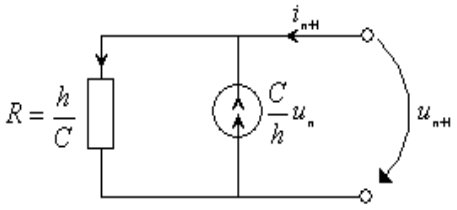
similar dependences of the resistance values on h are obtained [1]. This is because the resistive circuit containing companion models has resistances with order of magnitudes which is extended over some decades. In the case of RF circuits, containing both high frequency carrier signals and low frequency modulator signals, h must take very small values in order to compute high frequency details of the circuit response. For example if $h=10^{-10}\,s$ a broad range of resistance values (between $10^{-6}\Omega$ and $10^{7}\Omega$) can be obtained. In this case a linear algebraic equation system which may have an ill-conditioned matrix must be solved. The main contribution of this paper is related to the solving of this system.



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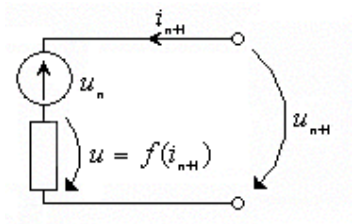
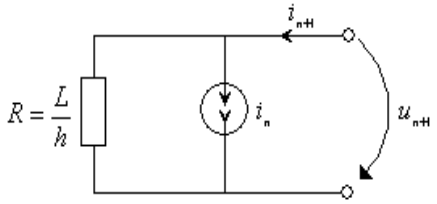


$$f(i_{n+1}) = h\left.\frac{d\hat{u}}{dq}\right|_{n+1} \cdot i_{n+1}$$

b

Fig.1. Companion models of a capacitor a) linear, b) charge controlled $u = \hat{u}(q)$



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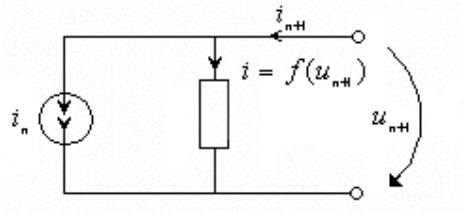

$$f(u_{n+1}) = h\left.\frac{d\hat{i}}{d\varphi}\right|_{n+1} \cdot u_{n+1}$$

b

Fig. 2. Companion models of an inductor a) linear b) flux controlled $i = \hat{i}(\varphi)$

Sweeping a time step in transient analysis of a circuit involves solving of a resistive circuit whose parameters are computed depending on a certain value of h followed by an error computation and a decision to accept or reject the assumed value for h. Three kinds of errors are known to estimate the correctness of the value of h; these errors are presented in Section 2. Section 3 deals with solving of linear systems with ill-conditioned matrices and includes our new approach. Two examples are presented in Section 4, while Section 5 contains the conclusions.

2. Errors in transient analysis

The time step magnitude in transient analysis of electrical circuits is chosen depending on certain errors. Three types of errors are used in transient analysis of circuits. The first one is the local truncation error (LTE), which is employed in SPICE-like circuit simulators (SPICE, PSPICE, HSPICE, SPECTRE, SPECTRE RF). Both the LTE of each state variable and the LTE of its time derivative are used. The LTE is estimated in the worst case corresponding to a relative error and to an absolute error. For example, the error of the time derivative of a state variable is:

$$\underset{x}{\varepsilon_{\bullet}} = \varepsilon_r \cdot \max\left(\left|\dot{x}_{n+1}\right|, \left|\dot{x}_n\right|\right) + \varepsilon_a \tag{1}$$

where $\dot{x}_{n+1}$ is the current through a capacitor or the voltage of a inductor. A similar error is defined for x_{n+1}.

For each time step, the maximum allowed LTE is given by:

$$E = \max(\varepsilon_x, \underset{x}{\varepsilon_{\bullet}}) \tag{2}$$

Starting from this value, a maximum time step is computed as:

$$h_{n+1} \le \sqrt{\frac{6E}{\left|\frac{d^3 x_n}{dt^3}\right|}} \tag{3}$$

This algorithm for time step computation includes a "cut and try" procedure based on the previous relationships [1].

The main drawback of this algorithm is the relation (3) which is based on the remainder estimation in Taylor formula [2]. The LTE of the trapezoidal algorithm is estimated as $-\frac{h^3}{12}x'''(\tau)$ where τ is an unknown value in the vicinity of t_{n+1}. Moreover, the third derivative can only be approximated knowing only the sample values given by a numerical method (the form of the solution between the samples is not known).

Another algorithm for time step choice, based on an energy error, is proposed in [3]. The energy accumulated by a nonlinear capacitor in the time step $[t_j, t_{j+1}]$ can be computed exactly as:

$$E_{j+1} - E_j = \int_{v_j}^{v_{j+1}} \frac{dq}{dt} v dv \tag{4}$$

where q is the capacitor charge, v_j is the capacitor voltage at t_j and v_{j+1} is the capacitor voltage at t_{j+1}.

For this capacitor, the energy balance in this time step is the difference between the accumulated energy and the energy fed by circuit into capacitor:

$$\Delta E = E_{j+1} - E_j - \int_{t_j}^{t_{j+1}} i(\tau)v(\tau)d\tau \tag{5}$$

where *i* is the capacitor current.

If $\Delta E \neq 0$, the integration algorithm gives an erroneous estimate of the solution. While the accumulated energy depends only on v_j and v_{j+1}, the energy fed by circuit into capacitor depends on the functions $i(\tau)$ and $v(\tau)$ too.

An algorithm for the computation of the time step based on ΔE control is developed in [3]. The maximum allowed ΔE_{j+1} in the time interval [t_j, t_{j+1}] is computed in a similar manner to (1):

$$\left|\Delta E_{j+1}\right| < \varepsilon_r \left|\Delta E_j\right| + \varepsilon_a \tag{6}$$

The energy balance for a time step may be computed taking into account the energies accumulated by all dynamic elements and the energies absorbed by resistors and sources

$$E_{R,S} = \int_0^h u(t) \cdot i(t)dt \tag{7}$$

The absolute energy balance error is defined as:

$$\Delta E_a = \sum_{k=1}^{n} E_k \tag{8}$$

and the relative energy balance error is defined as:

$$\Delta E_r = \frac{\Delta E_a}{\sqrt{\sum_{k=1}^{n} E_k^2}} \tag{9}$$

where *n* is the number of circuit elements including sources.

The time step is chosen computing ΔE_r and the assumed time step is accepted if $\Delta E_r \leq EER$, where *EER* is the imposed relative energy balance error margin. The algorithm for the time step choice can be outlined as follows [4]:

$t_{n+1} = t_n + h_n$
`solve for` t_{n+1}
`compute` ΔE_r
`if` $\Delta E_r < EER/10$
    `accept` t_{n+1}
    $h_{n+1} = 1.5 \cdot h_n$

$h_{n+1} = \min(h_{n+1}, TMAX)$
continue
else if $EER/10 < \Delta E_r < EER$
accept t_{n+1}
$h_{n+1} = h_n$
continue
else if $\Delta E_r > EER$
reject t_{n+1}
$h_{n+1} = h_n / 1.5$
if $h_{n+1} < H\min$ print TIME STEP TOO SMALL; analysis is aborted

This algorithm allows simple computation of a global estimate of the correctness of the transient analysis for the whole circuit and for the whole time interval (from t_{start} to t_{stop}). The global error $\Delta E_{total} = \sum \Delta E_a$, where ΔE_a are given by (8), and the sum is considered for all accepted steps, is computed to this end.

3. Solving of linear systems with ill-conditioned matrices

Consider the linear system $Ax=b$ corresponding to the resistive circuit containing companion models. This system can be solved, for example, using LU decomposition. If the time step used to integrate the circuit equations is very small, the numerical values of the entries in the A matrix are orders of magnitude apart and the problem becomes ill-conditioned. Due to this fact, the system solution is affected by numerical errors. To improve the system solution, some iterative refinement algorithms are known, the most used being GMRES [8]. This algorithm can be outlined as:

Repeat

Compute the residual $r = b - A \cdot x$

Solve $A \cdot d = r$

Update the solution $x = x + d$

Until {r or d is small enough or stops decreasing, or a maximum iteration count is exceeded}.

This algorithm compensates, up to a point, for bad row-scaling. The residual is never worsened but the solution x, though usually improved, frequently gets worse if the condition number is very big [9].

In order to estimate the correctness of the solution of this linear system, the following power balance errors are used:

- the absolute power balance error

$$\Delta P_{abs} = \sum_{k=1}^{n} P_k \tag{10}$$

- the relative power balance error

$$\Delta P_{rel} = \frac{P_{abs}}{\sqrt{\sum_{k=1}^{n} (P_k)^2}} \tag{11}$$

It seems that using an extra precision, a more accurate solution than that corresponding to the GMRES refinement can be obtained using a shorter CPU time. Our approach is to use *long double* precision computation for solving the resistive circuits containing companion models. The two examples presented below show that our approach gives better results than the SPICE one, which uses diagonal pivoting combined with Markowitz criterion for minimum fill-in number.

3. Examples

The LU decomposition for solving the linear system occurring in the transient analysis has been implemented in a program using the time step choice algorithm in [4].

The first example contains a nonlinear element and a resonant branch (Fig. 3 a). The circuit is driven by a sinusoidal excitation of 1 MHz, the resonance frequency of the *RLC* branch being *100 MHZ*. The nonlinear element is a diode modeled by a *PWL* resistor in series with a voltage source (Fig. 3 b).

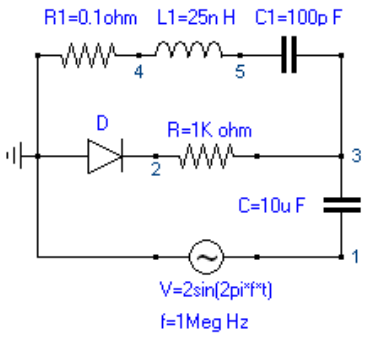

Fig. 3 a. The nonlinear circuit

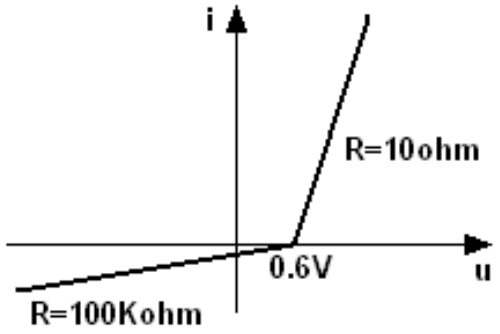

Fig. 3 b. Nonlinear resistor

The inductor voltage detail given in Fig. 4 shows the difference between the *double* precision and the *long double* solutions.

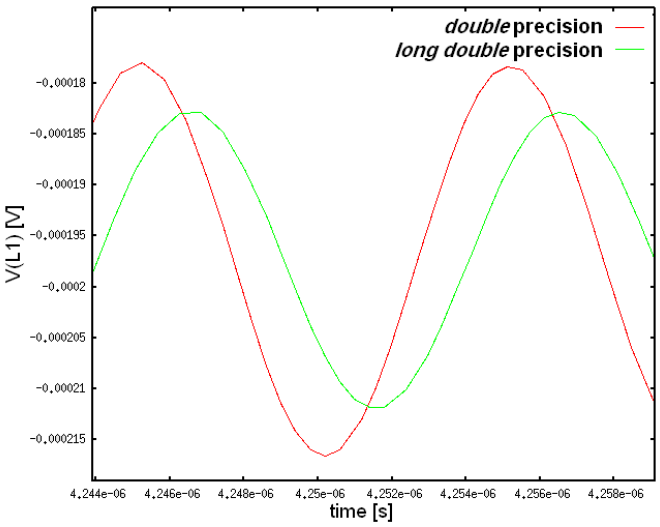

Fig. 4. The inductor voltage

Using double precision, with or without GMRES refinement, the evolution of the relative power balance error is almost the same (Fig. 5).

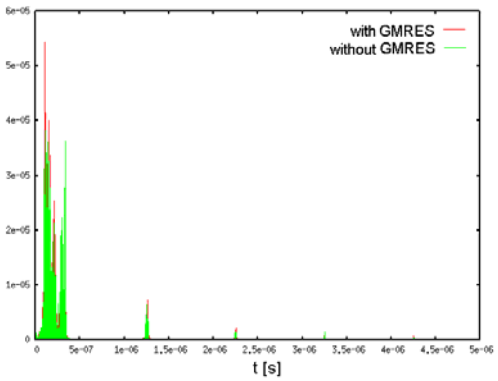

Fig. 5. Relative power balance error – *double* precision

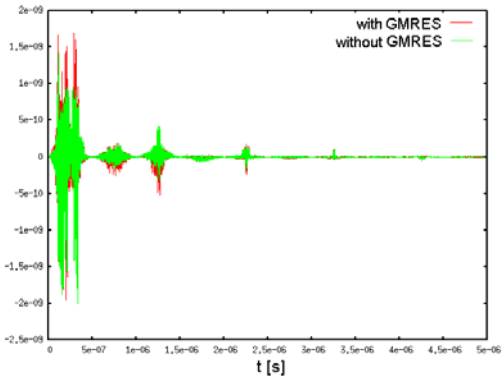

Fig.6. Absolute power balance error – *double* precision

The same evolution can be observed and for the absolute power balance error (Fig. 6).

If instead of *double* precision we use *long double* precision to solve the system, both the relative power balance (Fig. 7) and the absolute power balance (Fig. 8) are significantly improved even without GMRES refinement.

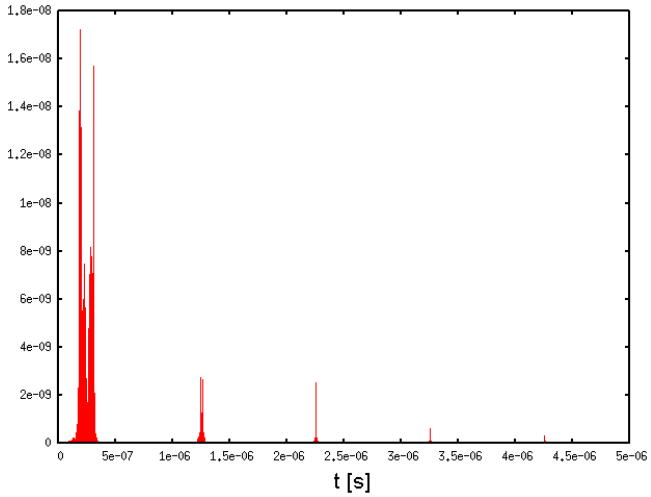

Fig. 7. Relative power balance error - *long double* precision

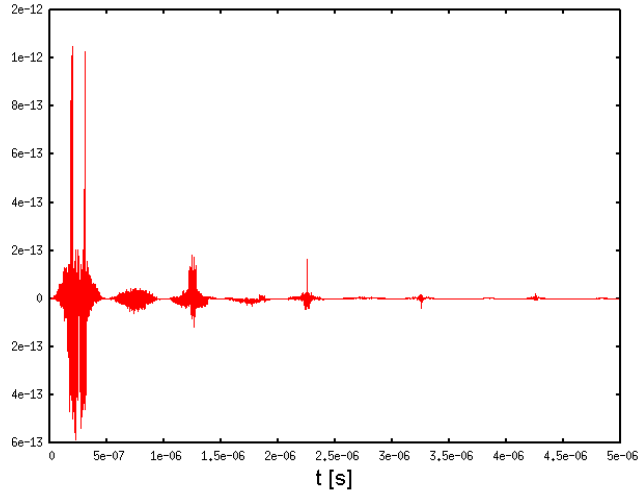

Fig. 8. Absolute power balance error – *long double* precision

The results are summarized in Table 1:

Table 1

precision	GMRES	ΔP_{rel} max	ΔP_{abs} max	CPU time
double	yes	5.5E-5	2E-9	0.54s
long double	no	1.7E-8	1.05E-12	0.43s

It can be observed that with *double* precision we cannot obtain the error limit obtained with *long double* precision, no matter how many GMRES iteration

are made (even 100). Using *double* precision with GMRES refinement is more time consuming than *long double* precision without GMRES refinement.

The second example is a band-pass filter built with two bulk acoustic wave resonators and was analyzed with SPICE and the proposed algorithm. A nonlinear circuit model having elements with polynomial nonlinearities in the motional branch [7] is used for each resonator (Fig. 9). This circuit is driven by a sinusoidal excitation of 2.025 GHz (the series resonance frequency of the first resonator).

The first resonator has the following implementation:

$$u_{Rm1}(i_{Rm1}) = 4.56 \cdot (i_{Rm1} + 0.5e-5 \cdot i_{Rm1}^2 + 0.5e-6 \cdot i_{Rm1}^3)$$
$$\varphi_{Lm1}(i_{Lm1}) = 69.91e-9 \cdot (i_{Lm1} + 5e-3 \cdot i_{Lm1}^2 + 1e-5 \cdot i_{Lm1}^3)$$
$$q_{Cm1}(u_{Cm1}) = 88.29e-15 \cdot (u_{Cm1} + 6e-8 \cdot u_{Cm1}^2 + 6e-10 \cdot u_{Cm1}^3)$$

while that of the second one is:

$$u_{Rm2}(i_{Rm2}) = 4.56 \cdot (i_{Rm2} + 0.5e-5 \cdot i_{Rm2}^2 + 0.5e-6 \cdot i_{Rm2}^3)$$
$$\varphi_{Lm2}(i_{Lm2}) = 70e-9 \cdot (i_{Lm2} + 5e-3 \cdot i_{Lm2}^2 + 1e-5 \cdot i_{Lm2}^3)$$
$$q_{Cm2}(u_{Cm2}) = 93.166e-15 \cdot (u_{Cm2} + 6e-8 \cdot u_{Cm2}^2 + 6e-10 \cdot u_{Cm2}^3)$$

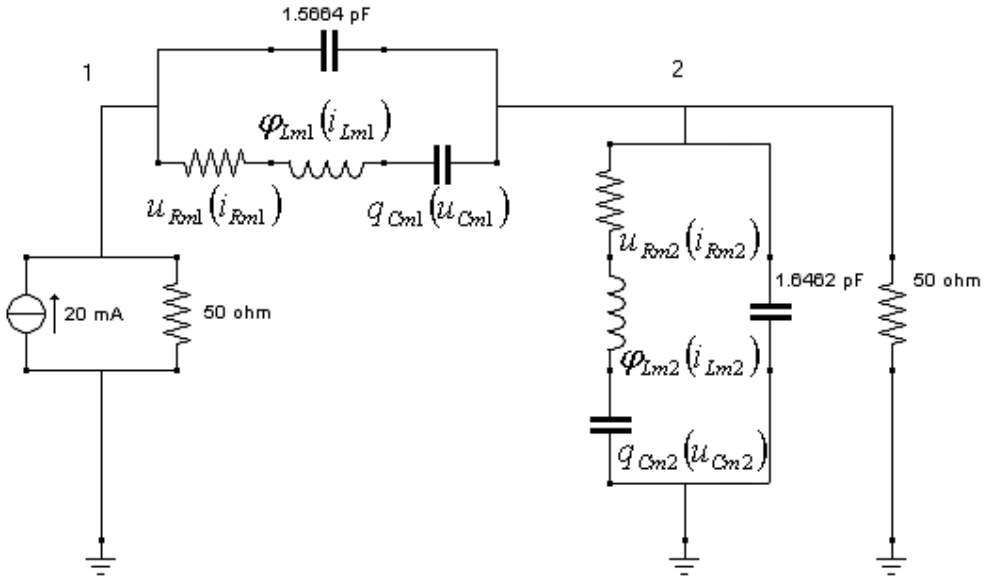

Fig. 9. Bandpass filter

After sweeping 100 excitation periods the output voltage *V(2)* computed with the proposed algorithm is practically the same with that computed with SPICE (Fig. 10).

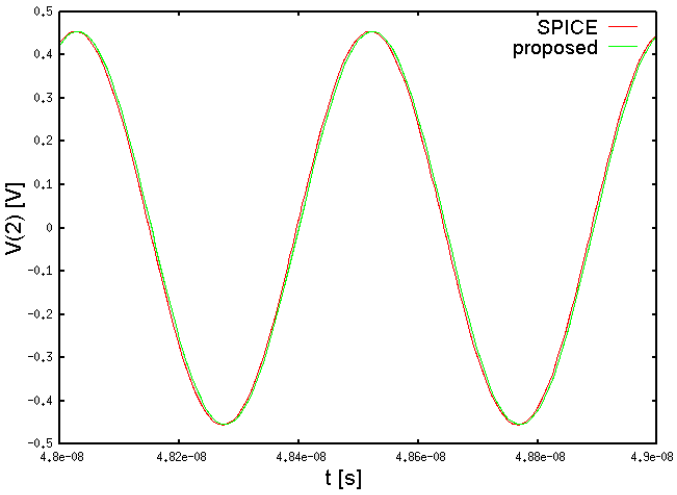

Fig. 10. Output voltage of the bandpass filter - detail

The error limits in these two algorithms have been set so that the number of accepted steps to be similar. Using the same numerical method (trapezoidal rule) for the integration of circuit equations, similar waveforms have been obtained as expected. The waveforms *V(2)* obtained using these two algorithms have the properties in Table 2.

Table 2

Waveform properties

	PROPOSED	SPICE
IMPOSED ERROR LIMIT	EER=9E-5	RELTOL=5.5E-6
ACCEPTED STEPS	7863	7816
REJECTED STEPS	448	3031
ΔE_{total}	3.906E-15	1.04E-14
CPU TIME	0.25S	0.32S

It follows that the proposed algorithm is better from the viewpoint of the rejected steps number, the global error ΔE_{total}, the relative energy balance error and CPU time. It is interesting to observe the evolution of the relative energy balance error for the SPICE solution and to compare it with the relative energy balance error of the proposed algorithm (Fig. 11).

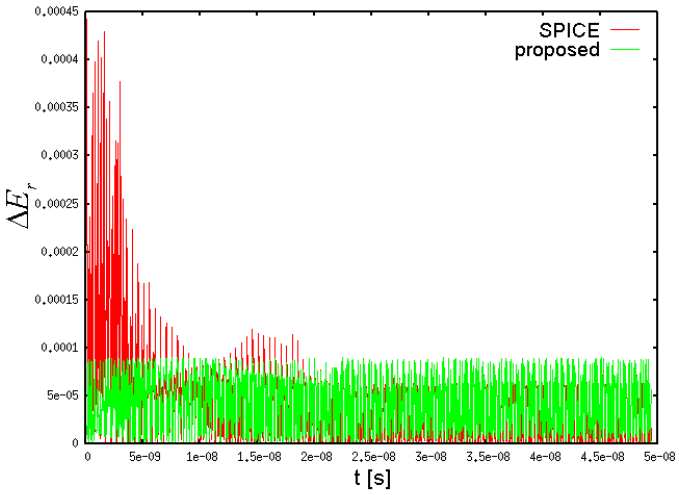

Fig. 11. Relative energy balance error for SPICE solution and for the proposed algorithm

The evolution of time step for the SPICE solution and for the proposed algorithm is given in Fig. 12.

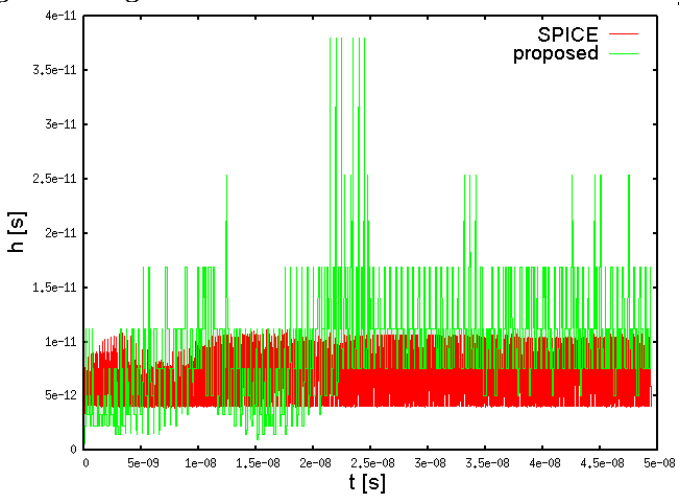

Fig. 12. Time step evolution for the SPICE solution and for the proposed algorithm

5. Conclusions

It was shown that transient analysis of RF circuits may lead to ill-conditioned systems of linear equations. Their solutions are computed with errors even though GMRES iterations are used. The computation of these solutions without GMRES refinement using *long double* precision leads to more accurate

solutions than that corresponding to the GMRES refinement using *double* precision. Moreover, a shorter CPU time is needed in the first case.

This algorithm was implemented in a transient analysis program which chooses the time step on the basis of energy balance errors. Two examples, for which this program gives better results than SPICE, have been presented.

6. Acknowledgment

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