INVESTIGATION OF OXYGEN ISOTOPES IN THE D-DIMENSION SCHRÖDINGER EQUATION BY THE PNU METHOD

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In this article, we review few-body systems by analytical methods. As we know, solve equations related to few-body systems for reasons such as the Tensor forces and the coupling between the equations is difficult. Therefore, to describe the motion of nucleons in the nucleus, such as forces modeling techniques must be used. To do this, we examined the Schrödinger equation for a few-body system using, Jacobi coordinates and hyper-spherical functions. We used the improved Hult'en plus Yukawa potential for interactions between nucleons. The D-dimensional Schrödinger equation in the case of $l \neq 0$ had discussed by using Parametric Nikiforov–Uvarov method. And we obtained relations energy values and wave function. The dependence of the few-body binding energies on the potential parameters has been investigated. Also, the energy state of two- and three-body systems have been compared. Finally, the energy of the ground state of some of the isotopes of oxygen was obtained.

Keywords: D-dimensional Schrödinger equation; Jacobi coordinates; Parametric Nikiforov-Uvarov method.

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1. Introduction

As we know, full resolution of the equations for few-body systems is very difficult for reasons such as tensor forces and coupling between the corresponding equations. Therefore, methods such as force modeling should be used to describe the motion of nuclei in the nucleus non relativistic Schrödinger equation, Klein-Gordon (K-G) and relativistic Dirac equation have long been recognized as essential tools for the study of atoms, nuclei, molecules and their spectral behaviors. Different methods have been used to solve these equations with central and non-central potentials. Some of these methods are supersymmetric quantum mechanics [1,2], path integral [3,4], factorization method [5,6]. In recent years, there has been a great desire to solve quantum mechanical systems in the framework of the Parametric Nikiforov–Uvarov (PNU) method. This algebraic technique is used to

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solve second-order linear differential equations. Which has been successfully used to solve the Schrödinger, Dirac, Klein-Gordon, and Duffin-Kemmer-Petiau (DKP) wave equations in the presence of central and non-central potentials [7,8].

The study of nuclei under extreme conditions has always been a necessity to understand the nuclear forces. As early as 1934, Elsasser [9] noticed the existence of special numbers of neutrons and protons which confer a particularly stable configuration to the corresponding nuclei. In analogy with atomic electrons, he correlated these numbers with closed shells in a model of non-interacting nucleons occupying energy levels generated by a potential well. Using the macroscopic-microscopic (M-M) model with isospin-dependent spin-orbit potential Qijun Zhi et al. showed that systematic calculation of the ground state properties of nuclei with proton number Z = 8–20. The calculated binding energies agree well with the experimental data. [10-11].

Understanding the evolution of the shell structure from the valley of stability to neutron-rich extremes represent a key challenge in nuclear structure. With a closed proton shell, the ^{17}O, ^{18}O, ^{19}O and ^{20}O isotopes provide an ideal region to investigate the shell formation and evolution in medium mass nuclei from nuclear forces [12,13]. These isotopes have a double magic number with 1, 2, 3 and 4 neutrons on top of the closed core. For example, the nuclei ^{17}O and ^{19}O can be modeled as a doubly magic ^{17}O=n+ (N=Z=8) and ^{19}O=3n+ (N=Z=8), with additional (valence) nucleons in the $ld_{5/2}$ level. The ground state spin and parity of ^{17}O and ^{19}O are $J^{\pi} = 5/2^{+}$, which corresponds to the spin and parity of the level where the valence nucleon resides [14].

We use non-relativistic shell model for calculation of the energy levels for $^{17\text{-}20}$O isotopes. Since these isotopes have some nucleons out of the core, Schrödinger equations in D-dimensional is utilized to investigate them in non-relativistic shell model. We apply the improved Hult'en plus Yukawa potential between the core and additional (valence) nucleons because these potentials are important nuclear potentials for a description of the interaction between single nucleon and whole nuclei.

2. Review of Parametric Nikiforov–Uvarov Method

Nikiforov–Uvarov (NU) method is based on reducing the second-order differential equation to a generalized equation of hyper-geometric type. This powerful mathematical tool solves second order differential equations. Let us consider the following differential equation

$$\Psi''_{n,\ell}(s)+\frac{\tilde{\tau}(s)}{\sigma(s)}\Psi'_{n,\ell}(s)+\frac{\tilde{\sigma}(s)}{\sigma^{2}(s)}\psi_{n,\ell}(s)=0\,. \tag{1}$$

Where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials, at most of the second degree, and $\tilde{\tau}(s)$ is a first-degree polynomial. Also, *n* and *l* are the radial quantum number and the orbital angular momentum quantum numbers, respectively.

The application of the NU method can be made simpler and direct without the need to check the validity of the solution. We present a shortcut for the method. So, at first, we write the general form of the Schrödinger -like Eq. (1) in a more general form as [15,16]:

$$\left[\frac{d^2}{ds^2}+\frac{\varepsilon_1-\varepsilon_2 s}{s(1-\varepsilon_3 s)}\frac{d}{ds}+\frac{(-\chi_2 s^2+\chi_1 s-\chi_0)}{s^2(1-\varepsilon_3 s)^2}\right]\Psi_{n,\ell}(s)=0\,. \tag{2}$$

For the Schrödinger equation, in the presence of potentials that can be written as in Eq. (2), the relation of energy Eigen-values and wave function is given by the following relationships, respectively:

$$n\varepsilon_2-(2n+1)\varepsilon_5+(2n+1)(\sqrt{\varepsilon_9}+\varepsilon_3\sqrt{\varepsilon_8})+n(n-1)\varepsilon_3+\varepsilon_7+2\varepsilon_3\varepsilon_8+2\sqrt{\varepsilon_8\varepsilon_9}=0\,, \tag{3}$$

$$\Psi_{n,\ell}(s)=N_{n,\ell}s^{\varepsilon_{12}}(1-\varepsilon_3 s)^{\varepsilon_{13}}P_n^{(\varepsilon_{10},\varepsilon_{11})}(1-2\varepsilon_3 s)\,. \tag{4}$$

In these relations $\psi(s)$ the wave function and ε_i are constant coefficients which are obtained with respect to the initial parameters χ_i (i = 0,1,2) in Eq. (5). Also, *n* is the quantum number of the system, $N_{n,l}$ is the normalization coefficient and the $P_n^{(\mu,\nu)}(x)$ Jacobi polynomials.

$$\begin{aligned}
&\varepsilon_4=\frac{1}{2}(1-\varepsilon_1)\,, && \varepsilon_5=\frac{1}{2}(\varepsilon_2-2\varepsilon_3)\\
&\varepsilon_6=\varepsilon_5^2+\chi_2\,, && \varepsilon_7=2\varepsilon_4\varepsilon_5-\chi_1\,,\\
&\varepsilon_8=\varepsilon_4^2+\chi_0\,, && \varepsilon_9=\varepsilon_3(\varepsilon_7+\varepsilon_3\varepsilon_8)+\varepsilon_6\,,\\
&\varepsilon_{10}=\varepsilon_1+2\varepsilon_4+2\sqrt{\varepsilon_8}\rangle-1\,,\\
&\varepsilon_{11}=1-\varepsilon_1-2\varepsilon_4+\frac{2}{\varepsilon_3}\sqrt{\varepsilon_9}\rangle-1,\varepsilon_3\neq 0\,,\\
&\varepsilon_{12}=\varepsilon_4+\sqrt{\varepsilon_8}\rangle 0\,,\ \varepsilon_{13}=-\varepsilon_4+\frac{1}{\varepsilon_3}\left(\sqrt{\varepsilon_9}-\varepsilon_5\right)\rangle 0,\varepsilon_3\neq 0\,.
\end{aligned} \tag{5}$$

The parametric method NU can solve the second-order differential equations with the conditions mentioned. We can study the Dirac, Klein Gordon and Schrödinger equations and other similar equations with the help of this method for some specific potentials. However, this method can only be solved for some of the potentials summarized in Eq. (1) [17,18].

3. The energy Eigen-values and wave functions

The many-body forces are more easily introduced and treated within the hyper-spherical harmonics formalism. For N-particle system after eliminating the center-

of-mass motion becomes a D-dimensional one where *D=3N−3*. When the particles are nucleons, it is possible to ignore the difference in mass between protons and neutrons. For such as system, we can define the N Jacobi vector as follows [19].

$$\xi_i = \sqrt{\frac{i}{i+1}} \left(r_{i+1} - \frac{1}{i}\sum_{j=1}^{i} r_i \right), \; i=1,2,...,N-1. \tag{6}$$

Where The ξ_i is the location of each point relative to the center of mass of the previous points, and the r_i is the coordinates of the particles in the laboratory system. The volume element in this coordinate is as follows:

$$\prod_{i=1}^{N} dr_i = N^{\frac{3}{2}} \, dR \prod_{j=1}^{N-1} d\xi_i = dx \quad . \tag{7}$$

In the hyper-spherical method, a point in the (D=3N-3)-dimensional configuration space is represented as lying on a (D-1)-dimensional hypersphere of radius *x*. The variable x is called the hyper-radius [19]. The potential *V(x)* is assumed to depend on the hyper-radius x only. The potential *V(x)* is called hyper-central in the sense that it is invariant for any rotation in the D-dimensional space. The Schrödinger equation in D-dimension [20] is given as follows.

$$\frac{d^2R}{dx^2} + \frac{(D-1)}{x}\frac{dR}{dx} + \frac{2\mu}{\hbar^2}\left[E_{n,\ell} - V(x) - \frac{\hbar^2}{2\mu}\left(\frac{\ell(\ell+D-2)}{x^2} \right) \right] R = 0, \tag{8}$$

where *D=3N-3* and μ is the mass reduced of the N-particle system. In this study, we consider the improved Hult'en plus Yukawa potential [21-22] follows:

$$V(x) = -\frac{v_0 \alpha e^{-\alpha x}}{(1-e^{-\alpha x})} + v_1 \frac{e^{-\alpha x}}{x^2}, \tag{9}$$

where the parameters v_0 and v_1 are real parameters, these are strength parameters, and the parameter α is related to the range of the potential.

Using the change of variables, $U(x) = x^{\frac{D-1}{2}} R(x)$, $\lambda = \ell + \frac{D-3}{2}$ and putting the potential in the Schrödinger equation, Eq. (10) is given as:

$$\frac{d^2U(x)}{dx^2} + \frac{2\mu}{\hbar^2}\left[E_{n,\ell} + \frac{v_0 \alpha e^{-\alpha x}}{(1-e^{-\alpha x})} - v_1 \frac{e^{-\alpha x}}{x^2} - \frac{\hbar^2 \lambda(\lambda+1)}{2\mu x^2} \right] U(x) = 0. \tag{10}$$

Eq. (10) can be precisely solved only for $\lambda = 0, -1$. We consider the approximation proposed by Greene and Aldrich to solve the analytical Eq. (10) [23]. This approximation is valid for $\alpha x << 1$. The main characteristic of these solutions lies in

the substitution of the centrifugal term by an approximation so that one can obtain an equation, normally hyper-geometric, which is solvable [24].

$$\frac{1}{x^2} \approx \frac{\alpha^2 e^{-\alpha x}}{(1-e^{-\alpha x})^2}. \tag{11}$$

Using the change of variable $s = \exp(-\alpha x)$, the Eq. (10) is written as follows:

$$U''_{n,\ell}(s) + \frac{(1-s)}{s(1-s)} U'_{n,\ell}(s) + \frac{1}{s^2(1-s)^2}\left[-\chi_2 s^2 + \chi_1 s - \chi_0\right] U_{n,\ell}(s) = 0, \tag{12}$$

where the parameters χ_2, χ_1 and χ_0 are considered as follows:

$$\begin{aligned} \chi_2 &= \frac{2\mu}{\hbar^2\alpha^2}\left[v_1\alpha^2 + v_0\alpha - E_{n,\ell}\right] \\ \chi_1 &= \frac{2\mu}{\hbar^2\alpha^2}\left(v_0\alpha - 2E_{n,\ell}\right) - \lambda(\lambda+1). \\ \chi_0 &= -\frac{2\mu E_{n,\ell}}{\hbar^2\alpha^2} \end{aligned} \tag{13}$$

Now, by comparing Eq. (12) with Eq. (2), the coefficients ε_i (i = 1, 2, 3) are easily obtained.

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1. \tag{14}$$

The coefficients ε_i (i = 4, 5 ... 13) are also obtained according to Eq. (5), which is given in Eq. (15) as following:

$$\begin{aligned} &\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1, \quad \varepsilon_4 = 0, \quad \varepsilon_5 = -\frac{1}{2}, \quad \varepsilon_6 = \chi_2 + \frac{1}{4} \\ &\varepsilon_7 = -\chi_1, \quad \varepsilon_8 = \chi_0, \quad \varepsilon_9 = \chi_2 - \chi_1 + \chi_0 + \frac{1}{4}, \quad \varepsilon_{10} = 2\sqrt{\chi_0} \\ &\varepsilon_{11} = 2\sqrt{\chi_2 - \chi_1 + \chi_0 + \frac{1}{4}}, \quad \varepsilon_{12} = \sqrt{\chi_0}, \quad \varepsilon_{13} = \frac{1}{2} + \sqrt{\chi_2 - \chi_1 + \chi_0 + \frac{1}{4}} \end{aligned} \tag{15}$$

Applying PNU method, we obtain the energy equation (with referring to Eq. (3)) as:

$$(2n+1)\left(\sqrt{\chi_2 - \chi_1 + \chi_0 + \frac{1}{4}} + \sqrt{\chi_0} + \frac{1}{4}(2n+1)\right) + 2\sqrt{\chi_0(\chi_2 - \chi_1 + \chi_0 + \frac{1}{4})} + 2\chi_0 - \chi_1 + \frac{1}{4} = 0 \tag{16}$$

The energy equation with referring to Eq. (13) summarized as:

$$E_{n,\ell} = -\frac{\hbar^2\alpha^2\left\{\left[\frac{2\mu}{\hbar^2\alpha^2}v_0\alpha-\lambda(\lambda+1)-\frac{1}{4}\right]-(2n+1)\left[\sqrt{\left[\frac{2\mu}{\hbar^2\alpha^2}v_1\alpha^2+\lambda(\lambda+1)+\frac{1}{4}\right]}+\frac{1}{4}(2n+1)\right]\right\}^2}{2\mu\left\{(2n+1)+2\sqrt{\left[\frac{2\mu}{\hbar^2\alpha^2}v_1\alpha^2+\lambda(\lambda+1)+\frac{1}{4}\right]}\right\}^2}. \quad (17)$$

In the following, we can obtain the wave function according to Eq. (4) and Eq. (15):

$$R_{n,\ell}(x) = N_{n,\ell}\,x^{-\left(\frac{D-1}{2}\right)}\left(e^{-\alpha x}\right)^{\left(\sqrt{\chi_0}\right)}\left(1-e^{-\alpha x}\right)^{\left(\frac{1}{2}+\sqrt{\frac{1}{4}+\chi_2-\chi_1+\chi_0}\right)}P_n^{\left(2\sqrt{\chi_0},2\sqrt{\frac{1}{4}+\chi_2-\chi_1+\chi_0}\right)}\left(1-e^{-\alpha x}\right), \quad (18)$$

where $N_{n,l}$ is the normalization coefficient.

4. Results:

According to Eq. (17), the energy eigenvalues depend on the hyper-central potential parameters. We carried out calculations for the ground state of two- and three-body bound systems. Having fixed parameter values of $\alpha=0.08fm^{-1}$, $m=8fm^{-1}$ and $\hbar=c=1$ in the natural units we have investigated the dependence of the two- and three-body binding energies on the parameter v_0, by performing calculations for several v_1, in Fig. 1a–b, respectively. These parameters are taken for the good behavior of energy values. For a particular v_1, few-body binding energies are found to decrease with increasing v_0, as it should be. Similarly, the dependence of the two- and three-nucleon binding energies on the parameter v_1 has been investigated in Fig. 2a–b. It can be seen that for a particular v_0, few-body binding energies increase with increasing v_1.

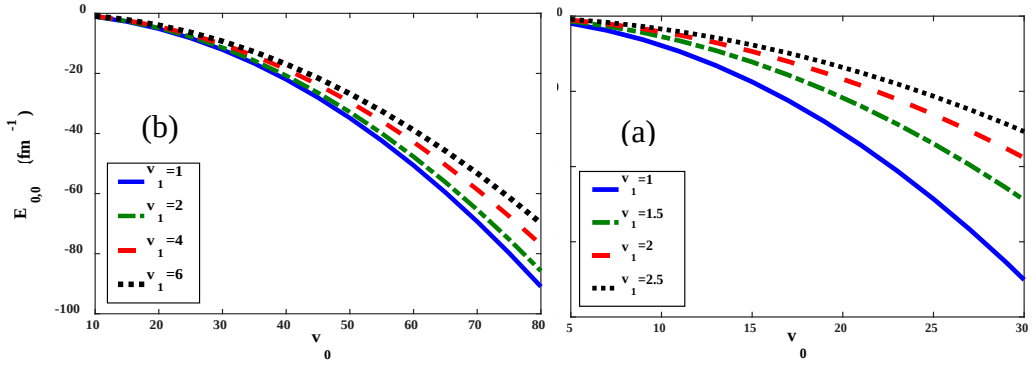

Fig. 1. The variation of the ground state binding energy of the Schrödinger equation on the parameter v_0 with different values of v_1 for the fixed value of $\alpha = 0.08fm^{-1}$, $m=8fm^{-1}$ and $\hbar=c=1$ in the natural units, for two-body (**a**), three-body (**b**) systems

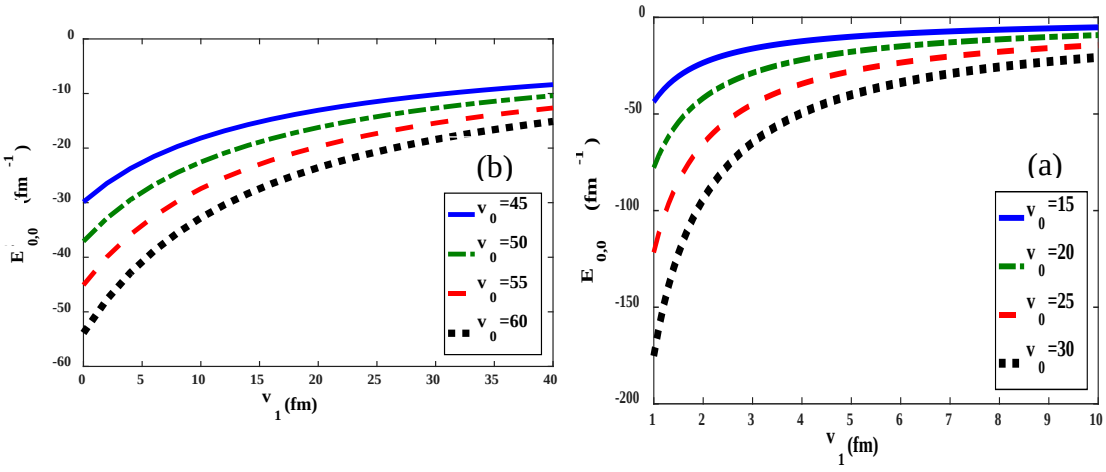

Fig.2. The variation of the ground state binding energy of the Schrödinger equation on the parameter v_1 with different values of v_0 for the fixed value of $\alpha = 0.08fm^{-1}$, $m=8fm^{-1}$ *and* $\hbar=c=1$ in the natural units, for two-body (**a**), three-body (**b**) systems.

With referring to Eq. (17), the binding energy for the 2, 3, and 4 body systems are also compared. This comparison is shown in Fig. 3 for different values of potential parameters in v_0 and v_1.

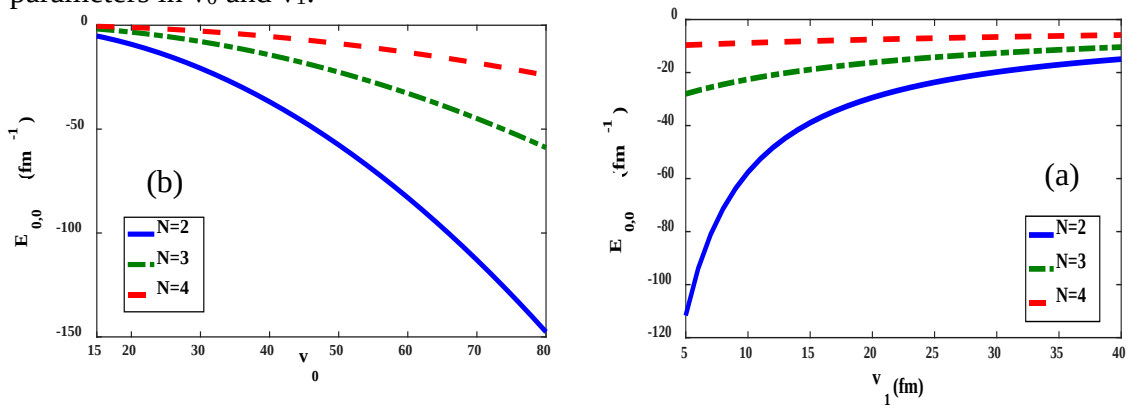

Fig. 3. Comparison between the Schrödinger ground state binding energy for the two-body, three-body and four-body systems versus different values of (a) v_1 and (b) v_0 for the fixed value of $\alpha=0.08fm^{-1}$, $m=8fm^{-1}$ *and* $\hbar=c=1$ in the natural units.

Finally, as an application of this argument, we have obtained the ground state energy by using the Eq. (17) for some of the oxygen isotopes. We have investigated energy levels these isotopes in Non-Relativistic Shell model. These isotopes can be considered as a doubly-magic close shell ^{16}O with additional nucleons (valence) at the $1d_{5/2}$ level. The results are compared with experimental results and other tasks

as shown in Table (1). We have considered the parameters $\alpha=0.014fm^{-1}$, $n=1$, $l=2$ *(in* $1d_{5/2}$*)* and $N= 2, 3,...$ in Eq. (17) for nuclei ^{17}O, ^{18}O, ^{19}O, ^{20}O in the proposed mathematical model. The α parameter was considered by fitting the potential parameters for the specific nucleus.

Table 1:

The ground state energy values of some oxygen isotopes ($\alpha=0.014fm^{-1}$). Column (Our) contains our calculation, Column (Other) contains other calculation and column (Exp) contains the experimental data.

Oxygen Isotopes	Potential parameters		$E_{n,l}$ (MeV)		
	v_0(MeV.fm)	v_1(MeV.fm^2)	Our	Other	Exp[26]
^{17}O	82.4235	3.2182	-132.1423	-132.880[11]	-131.7624
^{18}O	94.4301	0.5387	-140.1993	-139.909[25]	-139.8087
^{19}O	206.4566	4.1022	-145.0045	-146.870[11]	-143.7600
^{20}O	153.5082	2.1093	-152.2033	-152.300[25]	-151.3714

According to Figures 1 to 3, the parameter v_1 is smaller than v_0, and given that the binding energies values are different for each isotope, it is logical that the values of the potential parameters for each isotope are different. These values are calculated according to the experimental values and the solving process for oxygen isotopes.

5. Conclusion

In this paper, we investigated a Non-Relativistic few-body bound system problem by presenting the analytical solution of D-dimensional Schrödinger equation by using Jacobi coordinates and improved Hult'en plus Yukawa potential. Applying Parametric Nikiforov–Uvarov method, the hyper-radial wave functions, expressed in terms of the hypergeometric functions, and the energy equation are obtained. We investigated the dependence of the binding energies for the systems of two and three Non-Relativistic nucleons interacting by the improved Hult'en plus Yukawa potential, on the potential parameters. Finally, the ground state energy of the some of the oxygen isotopes was obtained. We can say that our proposed approach can be useful in investigating the Non-Relativistic corrections relevant to the observable characterizing the properties of few-body nuclear systems, within a simple treatment.

R E F E R E N C E S

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