THE EFFECT OF THE MOLECULAR ATTRACTION FORCES IN HYDRAULICS

Mircea Dimitrie CAZACU[1], Mireille Sorina ANGHEL[2]

Se considera forţele de atracţie moleculara: de adeziune dintre fluid şi corpul solid, cât şi forţele de coeziune dintre lichid si lichid la suprafaţa lui liberă şi din vecinătatea frontierelor solide ale domeniului ocupat de fluidul in curgere.

Aceste consideraţii au o importanţă practică deosebită şi se referă la completarea principiului lui Arhimede din hidrostatica, cât şi la teoria stratului limită a lui Ludwig Prandtl din hidrodinamică.

One considers the molecular attraction forces of: adhesion between fluid and solid body and also the cohesion forces between liquid and liquid on its free surface and at the neighborhood with the solid boundary of the fluid flow domain.

These considerations have a great practical importance and refer at the completion of the Archimedes' principle of Hydrostatics, as well as of the Ludwig Prandtl's boundary layer theory in Hydrodynamics.

Keywords: Molecular attraction forces. Archimedes' principle. Prandtl's boundary layer theory.

1. The importance of the molecular attraction forces in Hydraulics

In Hydrostatics is already very known the Jurin's old law, concerning the meniscus influence at the liquid level measurements in piezometer tubes with the diameters smaller as 8 mm, in which direction we obtained in the last time any meniscus shapes [1][2].

Also, is good known the manufacturing from very thin and of low density table of the Askania manometer needle for very low pressure, to avoid the meniscus formation on its peak.

In Hydrodynamics we introduced for the first time the molecular attraction forces of adhesion and cohesion in the Navier-Stokes equations for the viscous an heavy liquid motion, generated by a very efficient and controllable rotary biphasic contactor [3] ÷ [7], patented already in 1998 year [8].

Other very important technical applications of the molecular attraction forces in Hydrodynamics are: to establish the correct boundary conditions in the

[1] Professor, Dept. of Hydraulics and hydraulic Machines, University POLITEHNICA of Bucharest, Romania

[2] PhD student, Technical College "Iuliu Maniu" , Bucharest, Romania

viscous fluid flows near the solid walls, as well as for the pressure drop determination of the fluid flow through pipe-lines of different materials, in the cavitational bubble dynamics study, for the gas purifying, to raise the efficiency of heat exchangers with low temperature differences, to reduce the temperature of the industrial steam, to the manufacturing of nourishment powders (milk, soups) and also syrups and carbonised waters.

2. The action of molecular attraction forces in Hydrostatics

In this paper we shall present any completions of the Archimedes' old principle of the hydrostatic lift in the case of very small bodies.

Considering the forces acting on a submerged cylindrical very small body, presented in fig. 1, we shall remark first the variation of the hydrostatical pressure

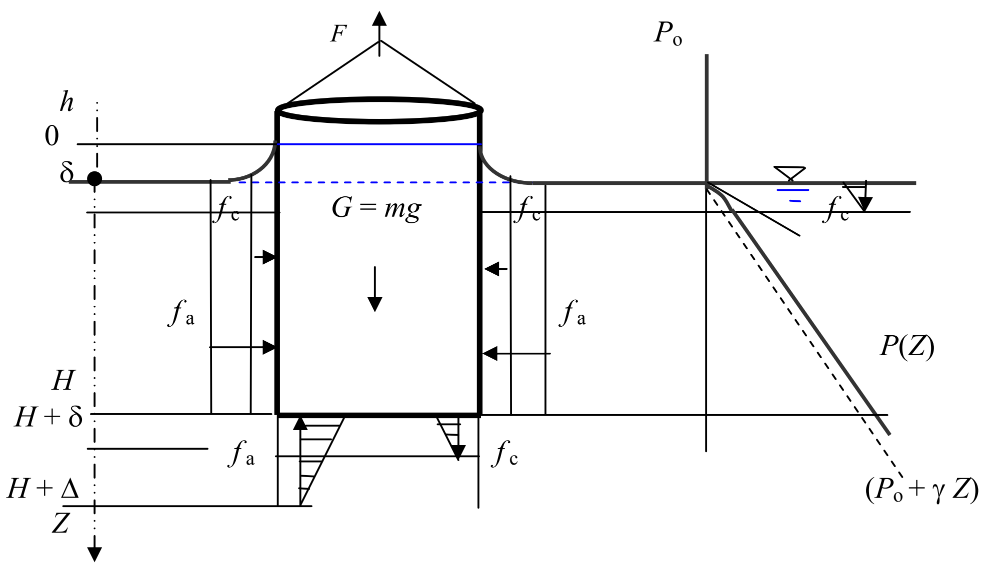

Fig. 1. The pressure variation and the forces acting on a cylindrical small body

variation on the vertical into the liquid, due to the cohesion molecular attraction force acting on its free surface, considering the linear variation of molecular attraction force in the domain D_1 and D_2, according to the relations

$$F_c = \rho f_c \left(1 - \frac{z}{\delta}\right) \quad \text{and} \quad F_a = \rho f_a \left(1 - \frac{z}{\Delta}\right). \tag{1}$$

Writing the Euler's hydrostatics equation, we obtain by integration in D_1

$$\frac{1}{\rho} P'_Z = g + f_c \left(1 - \frac{Z}{\delta}\right) \quad \rightarrow \quad P(Z)\Big|_{D_1} = g\,Z + \rho f_c \left(Z - \frac{Z^2}{2\delta}\right), \tag{2}$$

a parabolic supplement regarding the classical linear variation $P_o + \gamma\, Z$.

Concerning the pressure variation around the solid body, we ascertain that the pressure distribution on its vertical wall, being horizontal, that have not any influence under the vertical force F, remaining only to consider the pressure distribution on the body bottom, which is the following

$$P_{\text{Bot}} = P(H) + F_{\text{a}} - F_{\text{c}} = P_0 + \rho\, f_{\text{c}} \frac{\delta}{2} + \gamma\, H + \rho\, f_{\text{a}} - \rho\, f_{\text{c}}\,, \tag{3}$$

the Archimedean hydrostatic lift being grater in this case as in classical case

$$L_{\text{A}} = \left(P_{\text{Bottom}} - P_0\right) S = \gamma\, H\, S + \rho\, S \left(f_{\text{a}} - f_{\text{c}} \frac{\delta}{2} \right) \succ \gamma\, H\, S\,. \tag{4}$$

3. The action of molecular attraction forces in Hydrodynamics

For instance we shall present the steady two-dimensional viscous fluid flow in the boundary layer, adjacent to a flat plate, completing the Navier-Stokes' equation system with the terms of the attraction molecular forces of adhesion and cohesion, represented as in figure number 2

$$U'_{\text{X}} U + U'_{\text{Y}} V + \frac{1}{\rho} P'_{\text{X}} = \upsilon \left(U''_{\text{X}^2} + U''_{\text{Y}^2} \right), \tag{5}$$

$$V'_{\text{X}} U + V'_{\text{Y}} V + \frac{1}{\rho} P'_{\text{Y}} = \upsilon \left(V''_{\text{X}^2} + V''_{\text{Y}^2} \right) - g + f_{\text{c}} \left(1 - \frac{Y}{\delta} \right) \Bigg|_{\text{D}_2} - f_{\text{a}} \left(1 - \frac{Y}{\Delta} \right) \Bigg|_{\text{D}_3}\,, \tag{6}$$

and mass conservation equation for an incompressible liquid flow with ρ = const.

$$U'_{\text{X}} + V'_{\text{Y}} = 0\,. \tag{7}$$

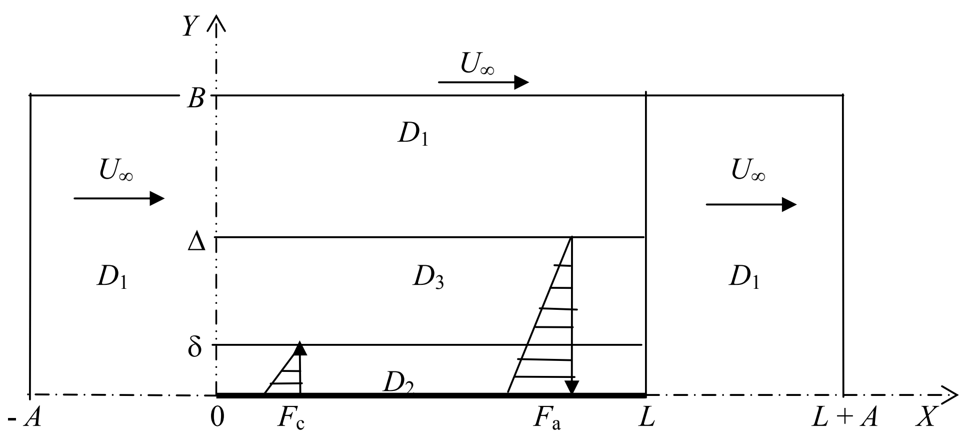

Fig. 2. The fluid flow domain in the neighbourhood of a flat plate

3.1. Elimination of the pressure function

Because the pressure values are not known on all the domain boundaries, we can eliminate this continuous, uniform and bounded function, by virtue of the Schwarz's commutative relation of mixed partial differential of second degree $P''_{XY} = P''_{YX}$ by subtraction of the equation (6) differentiated with respect to Y variable from the equation (5) differentiated with respect to X variable, obtaining

$$U\left(U''_{XY} - V''_{X^2}\right) + V\left(U''_{Y^2} - V''_{XY}\right) = \upsilon\left(U'''_{X^2Y} + U'''_{Y^3} - V'''_{X^3} - V'''_{XY^2}\right), \qquad (8)$$

which prove that the molecular attraction forces not influence the fluid flow.

3.2. Elimination of the fluid mass conservation equation

To eliminate the mass conservation equation (7), unstable in the iterative numerical calculus, we shall introduce the streamlines function by the relations:

$$U = \Psi'_Y \quad \text{and} \quad V = -\Psi'_X . \qquad (9)$$

Calculating all the partial differentials of the velocity components (9) till to the three order:

$$U'_X = \Psi''_{XY}, \quad U'_Y = \Psi''_{Y^2}, \quad V'_X = -\Psi''_{X^2}, \quad V'_Y = -\Psi''_{XY}, \qquad (10)$$

$$U''_{X^2} = \Psi'''_{X^2Y}, \quad U''_{XY} = \Psi'''_{XY^2}, \quad U''_{Y^2} = \Psi'''_{Y^3}, \quad V''_{X^2} = -\Psi'''_{X^3}, \quad V''_{XY} = -\Psi'''_{X^2Y}, \qquad (11)$$

$$U'''_{Y^3} = \Psi^{IV}_{Y^4}, \quad U'''_{X^2Y} = \Psi^{IV}_{X^2Y^2}, \quad V'''_{X^3} = -\Psi^{IV}_{X^4}, \quad V'''_{XY^2} = -\Psi^{IV}_{X^2Y^2}, \qquad (12)$$

and introducing these expressions in the equation (8) one obtains the partial differential equation verified by the streamlines function

$$\Psi'_Y\left(\Psi'''_{XY^2} + \Psi'''_{X^3}\right) - \Psi'_X\left(\Psi'''_{Y^3} + \Psi'''_{X^2Y}\right) = \upsilon\left(\Psi^{IV}_{X^4} + 2\Psi^{IV}_{X^2Y^2} + \Psi^{IV}_{Y^4}\right). \qquad (13)$$

3.3. Dimensionless form of the equation system

For more generality of the numerical solution we take as characteristic magnitudes the values: the domain wide B, the fluid velocity at the upstream of the domain U_∞ and the pressure in the exterior of the flow domain P_0.

With the dimensionless new variables and functions:

$$x = \frac{X}{B}, \quad y = \frac{Y}{B}, \quad u = \frac{U}{U_\infty}, \quad v = \frac{V}{U_\infty}, \quad \psi = \frac{\Psi}{U_\infty B}, \qquad (14)$$

the equation system becomes:

$$u'_{\mathrm{x}}u + u'_{\mathrm{y}}v + \mathrm{Eu}\, p'_{\mathrm{x}} = \frac{1}{\mathrm{Re}}\left(u''_{\mathrm{x}^2} + u''_{\mathrm{y}^2}\right),\ \ \psi''_{\mathrm{xy}}\psi'_{\mathrm{y}} - \psi''_{\mathrm{y}^2}\psi'_{\mathrm{x}} + \mathrm{Eu}\, p'_{\mathrm{x}} = \frac{1}{\mathrm{Re}}\left(\psi'''_{\mathrm{x}^2\mathrm{y}} + \psi'''_{\mathrm{y}^3}\right), \quad (5')$$

$$v'_{\mathrm{x}}u + v'_{\mathrm{y}}v + \mathrm{Eu}\, p'_{\mathrm{y}} = -\frac{1}{\mathrm{Fr}} + \frac{1}{\mathrm{Re}}\left(v''_{\mathrm{x}^2} + v''_{\mathrm{y}^2}\right) + \mathrm{Fc}\left(1 - \frac{y}{\delta/B}\right)\Bigg|_{\mathrm{D}_2} - \mathrm{Fa}\left(1 - \frac{y}{\Delta/B}\right)\Bigg|_{\mathrm{D}_3},$$

and also (6')

$$\psi''_{\mathrm{x}^2}\psi'_{\mathrm{y}} - \psi''_{\mathrm{xy}}\psi'_{\mathrm{x}} - \mathrm{Eu}\, p'_{\mathrm{y}} = \frac{1}{\mathrm{Fr}} + \frac{1}{\mathrm{Re}}\left(\psi'''_{\mathrm{x}^3} + \psi'''_{\mathrm{x}^2\mathrm{y}}\right) - \mathrm{Fc}\left(1 - \frac{y}{\delta/B}\right)\Bigg|_{\mathrm{D}_2} + \mathrm{Fa}\left(1 - \frac{y}{\Delta/B}\right)\Bigg|_{\mathrm{D}_3},$$

$$\psi'_{\mathrm{y}}\left(\psi'''_{\mathrm{xy}^2} + \psi'''_{\mathrm{x}^3}\right) - \psi'_{\mathrm{x}}\left(\psi'''_{\mathrm{y}^3} + \psi_{\mathrm{x}^2\mathrm{y}}\right) = \frac{1}{\mathrm{Re}}\left(\psi^{\mathrm{iv}}_{\mathrm{x}^4} + 2\psi^{\mathrm{iv}}_{\mathrm{x}^2\mathrm{y}^2} + \psi^{\mathrm{iv}}_{\mathrm{y}^4}\right). \quad (13')$$

in which we have denoted by: $\mathrm{Eu} = P_0/\rho U_\infty$ the Euler's number, $\mathrm{Re} = U_\infty B/\upsilon$ the Reynolds' number, $\mathrm{Fr} = U_\infty^2/g\,B$ the Froude's number, $\mathrm{Fc} = f_{\mathrm{c}}\,B/U_\infty^2$ the cohesion's number and by $\mathrm{Fa} = f_{\mathrm{a}}\,B/U_\infty^2$ the adhesion's number.

3.4. Numerical integration of the streamlines function

Generally, it is very known the fact that the approximation of a continuous, uniform and bounded analytical function in the neighbourhood of a regular point of a domain, by development in infinite series, consist in addition of the partial derivatives in the regular considered point, multiplied by the convenient exponent of the step and some coefficients, whose size diminish in a sufficient measure with the partial differential order, to assure the process of convergence of the approximated function value.

To solve numerically the equation (13') we used the development of the streamlines function ψ (*x,y*) in our proper series [9] as follows (fig.3)

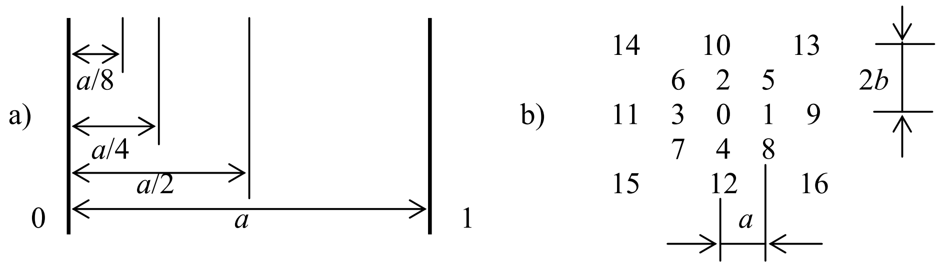

Fig.3 a) grid step division and b) knots numbering in two-dimensional grid

Considering the idea of centered finite differences, we shall write the development in series as

$$f_1 = f_0 + a f'_x + \frac{a^2}{2} f''_{x^2} + \frac{a^3}{8} f'''_{x^3} + \frac{a^4}{64} f^{iv}_{x^4} + \cdots = f_0 + \sum_{n=0}^{\infty} \left(\prod_{i=0}^{n} \frac{a}{2^i} \right) f^{(n+1)}_{x^{n+1}}(0), \qquad (15)$$

justified by the expressions of the centred finite differences:

$$\frac{f_1 - f_0}{a} = f'_x(a/2) = f'_x(0) + \frac{a}{2} f''_{x^2}(0) + \frac{a^2}{8} f'''_{x^3}(0) + \frac{a^3}{64} f^{iv}_{x^4}(0) + \cdots, \qquad (16)$$

$$\frac{\frac{f_1 - f_0}{a} - f'_x(0)}{a/2} = f''_{x^2}(a/4) = f''_{x^2}(0) + \frac{a}{4} f'''_{x^3}(0) + \frac{a^2}{32} f^{iv}_{x^4}(0) + \cdots, \qquad (17)$$

$$\frac{\frac{\frac{f_1 - f_0}{a} - f'_x(0)}{a/2} - f''_{x^2}(0)}{a/4} = f'''_{x^3}(a/8) = f'''_{x^3}(0) + \frac{a}{8} f^{iv}_{x^4}(0) + \cdots \qquad (18)$$

$$\frac{\frac{\frac{\frac{f_1 - f_0}{a} - f'_x(0)}{a/2} - f''_{x^2}(0)}{a/4} - f'''_{x^3}(0)}{a/8} = f^{iv}_{x^4}(a/16) = f^{iv}_{x^4}(0) + \cdots, \qquad (19)$$

Using the formula (15) we shall fused written the following function developments in finite series for a two-dimensional grid with different steps $a \neq b$ [10], for the neighbouring points of an ordinary point 0, corresponding to the notations from fig.1,b: for the points 1÷ 4, disposed at the first steps in the small cross position

$$f_{1,3} = f_0 \pm a f'_x + \frac{a^2}{2} f''_{x^2} \pm \frac{a^3}{8} f'''_{x^3} + \frac{a^4}{64} f^{iv}_{x^4} \pm \cdots, \qquad (20)$$

$$f_{2,4} = f_0 \pm b f'_y + \frac{b^2}{2} f''_{y^2} \pm \frac{b^3}{8} f'''_{y^3} + \frac{b^4}{64} f^{iv}_{y^4} \pm \cdots, \qquad (21)$$

for the points numbered with 5 ÷ 8, placed in the small square position, we have:

$$f_{5,7} = f_0 \pm \left(a f'_x + b f'_y \right) + \frac{1}{2}\left(a^2 f''_{x^2} + 2ab f''_{xy} + b^2 f''_{y^2} \right) \pm \frac{1}{8}\left(a^3 f'''_{x^3} + 3a^2 b f'''_{x^2 y} + 3ab^2 f'''_{xy^2} + b^3 f'''_{y^3} \right) +$$
$$+ \frac{1}{64}\left(a^4 f^{iv}_{x^4} + 4a^3 b f^{iv}_{x^3 y} + 6a^2 b^2 f^{iv}_{x^2 y^2} + 4ab^3 f^{iv}_{xy^3} + b^4 f^{iv}_{y^4} \right) \pm \cdots, \qquad (22)$$

$$f_{6,8} = f_0 \pm \left(-a f'_x + b f'_y \right) + \frac{1}{2}\left(a^2 f''_{x^2} - 2ab f''_{xy} + b^2 f''_{y^2} \right) \pm \frac{1}{8}\left(-a^3 f'''_{x^3} + 3a^2 b f'''_{x^2 y} - 3ab^2 f'''_{xy^2} + b^3 f'''_{y^3} \right) +$$
$$+ \frac{1}{64}\left(a^4 f^{iv}_{x^4} - 4a^3 b f^{iv}_{x^3 y} + 6a^2 b^2 f^{iv}_{x^2 y^2} - 4ab^3 f^{iv}_{xy^3} + b^4 f^{iv}_{y^4} \right) \pm \cdots, \qquad (23)$$

for the points numbered with 9 ÷ 12, placed at the double step in the big cross position:

$$f_{9,11}=f_0\pm 2af'_{\mathrm{x}}+2a^2f''_{\mathrm{x}^2}\pm a^3f'''_{\mathrm{x}^3}+\frac{a^4}{4}f^{\mathrm{iv}}_{\mathrm{x}^4}\pm\cdots, \tag{24}$$

$$f_{10,12}=f_0\pm 2bf'_{\mathrm{y}}+2b^2f''_{\mathrm{y}^2}\pm b^3f'''_{\mathrm{y}^3}+\frac{b^4}{4}f^{\mathrm{iv}}_{\mathrm{y}^4}\pm\cdots \tag{25}$$

and for the points numbered with 13 ÷ 16, placed at the double steps in the big square position

$$f_{13,15}=f_0\pm 2\left(af'_{\mathrm{x}}+bf'_{\mathrm{y}}\right)+2\left(a^2f''_{\mathrm{x}^2}+2abf''_{\mathrm{xy}}+b^2f''_{\mathrm{y}^2}\right)\pm\begin{pmatrix}a^3f'''_{\mathrm{x}^3}+3a^2bf'''_{\mathrm{x}^2\mathrm{y}}+\\+3ab^2f'''_{\mathrm{xy}^2}+b^3f'''_{\mathrm{y}^3}\end{pmatrix}+$$

$$+\frac{1}{4}\left(a^4f^{\mathrm{iv}}_{\mathrm{x}^4}+4a^3bf^{\mathrm{iv}}_{\mathrm{x}^3\mathrm{y}}+6a^2b^2f^{\mathrm{iv}}_{\mathrm{x}^2\mathrm{y}^2}+4ab^3f^{\mathrm{iv}}_{\mathrm{xy}^3}+b^4f^{\mathrm{iv}}_{\mathrm{y}^4}\right)\pm\cdots, \tag{26}$$

$$f_{14,16}=f_0\pm 2\left(-af'_{\mathrm{x}}+bf'_{\mathrm{y}}\right)+2\left(a^2f''_{\mathrm{x}^2}-2abf''_{\mathrm{xy}}+b^2f''_{\mathrm{y}^2}\right)\pm\begin{pmatrix}-a^3f'''_{\mathrm{x}^3}+3a^2bf'''_{\mathrm{x}^2\mathrm{y}}-\\-3ab^2f'''_{\mathrm{xy}^2}+b^3f'''_{\mathrm{y}^3}\end{pmatrix}+$$

$$+\frac{1}{4}\left(a^4f^{\mathrm{iv}}_{\mathrm{x}^4}-4a^3bf^{\mathrm{iv}}_{\mathrm{x}^3\mathrm{y}}+6a^2b^2f^{\mathrm{iv}}_{\mathrm{x}^2\mathrm{y}^2}-4ab^3f^{\mathrm{iv}}_{\mathrm{xy}^3}+b^4f^{\mathrm{iv}}_{\mathrm{y}^4}\right)\pm\cdots. \tag{27}$$

By simple algebraic calculus we can deduce the new partial differential expressions, fused written, which are the same with these obtained from Taylor's series developments till the 2$^{\mathrm{nd}}$ order

$$f'_{\mathrm{x,y}}=\frac{f_{1,2}-f_{3,4}}{2(a,b)},\quad f''_{(\mathrm{x,y})^2}=\frac{f_{1,2}-2f_0+f_{3,4}}{(a,b)^2},\quad f''_{\mathrm{xy}}=\frac{f_5+f_7-f_6-f_8}{4ab} \tag{28}$$

and also for the 1$^{\mathrm{st}}$ and 2$^{\mathrm{nd}}$ order partial differentials with respect to the finite Taylor's series developments to the 4$^{\mathrm{th}}$ order partial differentials

$$-v_0,u_0=f'_{\mathrm{x,y}}=\frac{1}{a,b}\left[\frac{2}{3}\left(f_{1,2}-f_{3,4}\right)-\frac{1}{12}\left(f_{9,10}-f_{11,12}\right)\right], \tag{29}$$

$$f''_{(\mathrm{x,y})^2}=\frac{1}{(a,b)^2}\left[\frac{4}{3}\left(f_{1,2}+f_{3,4}\right)-\frac{5}{2}f_0-\frac{1}{12}\left(f_{9,10}+f_{11,12}\right)\right], \tag{30}$$

instead what for the partial differential of superior orders we have the expressions:

$$f'''_{(\mathrm{x,y})^3}=\frac{1}{(a,b)^3}\left[\frac{2}{3}\left(f_{9,10}-f_{11,12}\right)-\frac{4}{3}\left(f_{1,2}-f_{3,4}\right)\right], \tag{31}$$

$$f'''_{\mathrm{x}^2\mathrm{y}}=\frac{1}{a^2b}\left[\frac{2}{3}\left(f_5+f_6-f_7-f_8\right)-\frac{4}{3}\left(f_2-f_4\right)\right], \tag{32}$$

$$f'''_{xy^2} = \frac{1}{ab^2}\left[\frac{2}{3}\left(f_5 - f_6 - f_7 + f_8\right) - \frac{4}{3}\left(f_1 - f_3\right)\right], \qquad (33)$$

the partial differentials of 3rd order being 4/3 time greater as these obtained from Taylor's series developments and 16/3 time greater as these obtained from finite difference method, instead that the differentials of 4th order being 8/3 time greater as these obtained from Taylor's series developments and 42.66…time greater as these obtained by finite difference method

$$f^{iv}_{(x,y)^4} = \frac{1}{\left(a,b\right)^4}\left[16 f_0 - \frac{32}{3}\left(f_{1,2} + f_{3,4}\right) + \frac{8}{3}\left(f_{9,10} + f_{11,12}\right)\right], \qquad (34)$$

$$f^{iv}_{x^2y^2} = \frac{1}{a^2b^2}\left[\frac{32}{3} f_0 - \frac{16}{3}\sum_{i=1}^{4} f_i + \frac{8}{3}\sum_{i=5}^{8} f_i\right]. \qquad (35)$$

3.5. Algebraic relation associated to partial differential equation for *a* = *b* = χ

Introducing the partial differential expressions (29) ÷ (35) in the stream line equation, deduced from the Navier and Stokes' equation system of a viscous fluid flow in the case of square grid

$$\psi^{iv}_{x^4} + 2\psi^{iv}_{x^2y^2} + \psi^{iv}_{y^4} = \mathrm{Re}\left[\psi'_y\left(\psi'''_{x^3} + \psi'''_{xy^2}\right) - \psi'_x\left(\psi'''_{x^2y} + \psi'''_{y^3}\right)\right], \qquad (13')$$

one obtain the associate algebraic relation as general numerical solution for the stream line current

$$\psi_0 = \frac{2}{5}\sum_{i=1}^{4}\psi_i - \frac{1}{10}\sum_{i=5}^{8}\psi_i - \frac{1}{20}\sum_{i=9}^{12}\psi_i + \qquad (36)$$

$$+\frac{\mathrm{Re}}{960}\left\{\begin{matrix}\left[8\left(\psi_2 - \psi_4\right) + \psi_{12} - \psi_{10}\right]\left[\psi_5 + \psi_8 + \psi_9 - \psi_6 - \psi_7 - \psi_{11} + 4\left(\psi_3 - \psi_1\right)\right] + \\ + \left[8\left(\psi_3 - \psi_1\right) + \psi_9 - \psi_{11}\right]\left[\psi_5 + \psi_6 + \psi_{10} - \psi_7 - \psi_8 - \psi_{12} + 4\left(\psi_4 - \psi_2\right)\right]\end{matrix}\right\}$$

3.6. The solution stability and the relaxation diagram

For the case of a parallel current with 0*x* axle, corresponding to a stable relaxed grid, the error propagation relations in the current direction and perpendicular to it [9] are for local Reynolds number denoted by R = Re χ u_o:

$$\delta\psi_{n+1}^{\pm x}=\left(\frac{2}{5}\pm\frac{\mathrm{Re}\,\chi\,u_o}{20}\right)\delta\psi_n-\left(\frac{1}{20}\pm\frac{R}{80}\right)\delta\psi_{n-1}\ \text{and}\ \delta\psi_{n+1}^{\pm y}=\frac{2}{5}\delta\psi_n-\frac{1}{20}\delta\psi_{n-1},\quad(37)$$

with that we can trace the error propagation diagram as in figure 4, where we represented the extreme curves, corresponding to the limit values of so called Reynolds local number R = Re χ (u or v), which assure the numerical solution stability by diminishing of the errors.

In the case, when we lead the calculus on the diagonal directions, the stability is better [10]

$$\delta\psi_{n+1}^{\square}=\left(\frac{2}{5}\pm\frac{\mathrm{Re}\,\chi\,u}{20}\right)\delta\psi_n-\left(\frac{1}{20}\pm\frac{R}{80}\right)\delta\psi_{n-1}\qquad(38)$$

and

$$\delta\psi_{n+1}^{\square}=\left(\frac{2}{5}\pm\frac{\mathrm{Re}\,\chi\,u_0}{20}\right)\delta\psi_n-\left(\frac{1}{20}\pm\frac{R}{80}\right)\delta\psi_{n-1}\,.\qquad(39)$$

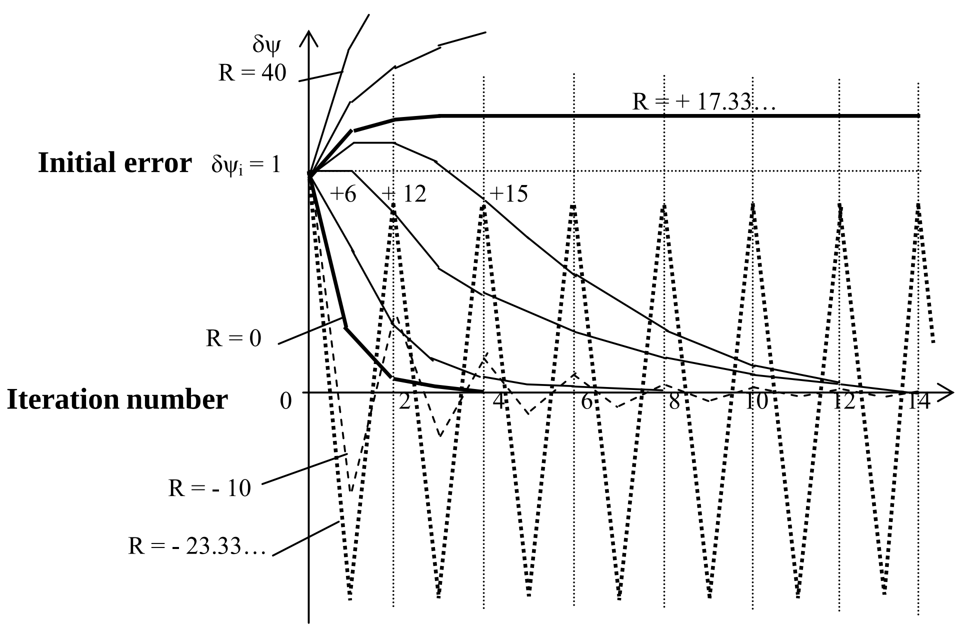

Fig. 4. Error relaxation diagram on the ± x current direction and perpendicularly on it ± y

4. Conclusions

The introduction of the molecular attraction forces of adhesion and cohesion represents a valuable method, such in Hydrostatics, as well as in Hydrodynamics due to the multitude of technical applications in diverse industrial domains, presented in the first point.

The study of the numerical solution stability constitute a guarantee of our future research in the field of the profile with Coandă effect, without separation flow around these special profiles [11], as well as in the field of cavitational bubble dynamics.

REFERENCES

[1] *M.D.Cazacu, R.M.R.Neacsu, A.Ciocanea, Mihaela Preda-Testiban, Camelia Elvira Steliana Tanasie,* Contract Tehno-GAR 191/3896/1995 (Contract UPB 17318/1995). Cercetari teoretice si experimentale privind fenomenul de aderenta moleculara in hidrodinamica. Beneficiar: Academia Romana.

[2] *M.D.Cazacu, R.M.R.Neacsu, Camelia Elvira Steliana Tanasie,* Modelarea fizico-matematica a fortelor de atractie moleculara si calculul formei meniscului in apropierea unui perete solid. Al V-lea Simpozion “Tehnologii, instalatii si echipamente pentru imbunatatirea calitatii mediului”, Univ. Politehnica, Bucuresti, 13-15 Noembrie 1996, **Vol. I**, p. 77-84.

[3] *M.D.Cazacu, R.M.R.Neacsu*, *Camelia Elvira Steliana Tanasie*, Cercetari experimentale asupra contactorului bifazic rotativ. Al V-lea Simpozion “Tehnologii, instalatii si echipamente pentru imbunatatirea calitatii mediului”, Univ. Politehnica, Bucuresti, 13-15 Noembrie 1996, **Vol. I**, p. 91- 98.

[4] *M.D.Cazacu, R.M.R.Neacsu, Camelia Elvira Steliana Tanasie,* Miscarea picaturilor de lichid generate de contactorul bifazic lichid-gaz. Al V-lea Simpozion “Tehnologii, instalatii si echipamente pentru imbunatatirea calitatii mediului”, Univ. Politehnica, Bucuresti, 13-15 Noembrie 1996, **Vol. I**, p. 67-76.

[5] *M.D.Cazacu,* Mathematical model of viscous and heavy fluid in steady flow, under the action of molecular attraction and centrifugal inertia forces.Bul.Inst.Politehnic din Iasi, Tom XLVI(L), Supliment 2000 Mecanica Fluidelor I, Sect.Constr.de Masini, 27 – 31.

[6] *M.D.Cazacu, Iuliana Monica Boiangiu*, Experimental study of heavy and viscous liquid motion, generated by a rotational cone. A III-a Conf.Dorin Pavel a Hidroenergeticienilor din Romania, 28-29 mai 2004, Univ. Politehnica, Bucuresti, **Vol. I**, 57-62.

[7] *M.D.Cazacu, Iuliana Monica Boiangiu*. Efficient surface aerator for waste water treatment. The 3rd International Conference on Energy and Environment – CIEM, June 28 – July 4, 2007, Section 5 – Wastes Management. Univ.Politehnica of Bucharest. CD, 6 p.

[8] M.*D.Cazacu, R.M.R.Neacşu, Iuliana Monica Boiangiu,* Metodă şi dispozitiv de pulverizare a unui lichid. Brevet nr. 118266 B/09.12.1998. Existent în Laboratorul de Curgeri bifazice şi Cavitaţie de la Universitatea POLITEHNICA din Bucureşti.

[9] *M.D.Cazacu,* A better numerical solution stability by using the function developments in proper series. International Conference on Theory and Applications of Mathematics and Informatics – ICTAMI 2005, September 15 – 18th , Albac – Alba Iulia, Romania, CD.

[10] *M.D.Cazacu, Monica Felicia Popovici,* Asupra stabilităţii soluţiei numerice a sistemului de ecuaţii al lui Navier şi Stokes cu aplicaţii la curgerea lichidului vîscos prin labirinţii maşinilor hidraulice, Conf. Maş. Hidr. şi Hidrodinamicã, Timişoara, 18 - 19 oct. 1985, I, 233 - 240 .

[11] *M.D.Cazacu, Mireille Sorina Anghel,* Profil hidrodinamic şi aerodinamic antişoc. Brevet inregistrat la OSIM cu nr. A/00265/10.04.2008.