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ON THE OPTIMIZATION OF AN ACKERMANN STEERING LINKAGE

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In the paper the optimization problem of the Ackermann linkage is examined taking into account in turn two criteria: the maximum of the absolute value of the steering error and the root-mean-square of the steering error. For this purpose the transmission function of the mechanism is directly used without resorting to the method of increasing the degree of the freedom.

Keywords: Ackermann linkage, optimization, steering, steering angle, vehicle

1. Introduction

The steering system of an automobile with a rigid steering axle (beam and steering knuckles hinge-connected to the beam with king pins) comprises the steering mechanism which is represented by the steering trapezium otherwise known as Ackermann steering linkage. The analysis and synthesis of the Ackermann steering linkage has been tackled in numerous papers [1]. Although in these works the problem of solution finding which should lead to enough good approximation of the correct steering condition has been set, a rigorous approach to optimization has been undertaken only relatively recently. So, in [2] the optimization problem of a planar Ackermann linkage has been investigated by the method of increasing the degree of freedom of mechanism. It is supposed that the function which is generated by the mechanism should ensure positions which are as near as possible to certain chosen positions (design points). For the optimization of a planar Ackermann linkage in [3] a method is applied which minimizes the strength energy of the bars by considering this elasticity. There are some difficulties when performing the numerical optimization.

In the present paper the synthesis of the Ackermann linkage is tackled using systematically normalized lengths and paying more attention to the constraints imposed by the peculiarities of the mechanism [4]. The optimization problem of the Ackermann linkage is examined taking into account two criteria: the criterion of the absolute value of the steering error and the criterion of the root-mean-square of the steering error. For this purpose the transmission function of the mechanism is directly used, without resorting to the method of increasing the degree of freedom.

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In the paper we consider the planar Ackermann linkage and the classical condition of correct steering of a vehicle given by the Ackermann's relation [1, 5]. Also, in the following we use the notations and the relations derived in [4].

2. The steering error

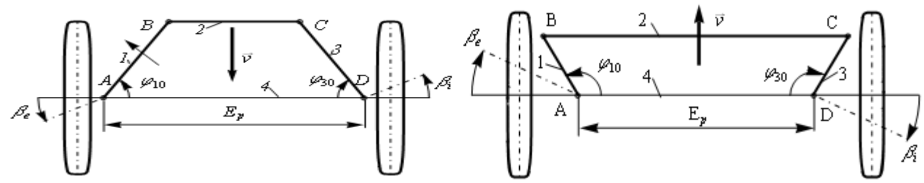


Fig.1. Geometrical elements of an Ackermann steering linkage: a) trailing; b) leading

Schematically, the disposal of an Ackerman steering linkage is shown in Fig.1. The shown position corresponds to the straight-line motion of automobile. The following notations are used: l_j-length of the element $j(j=1,2,3,4)$; φ_1=angle(BAD); φ_2=angle between BC and AD (the positive direction is counter clockwise rotation of BC); φ_3=angle(CDA); $E_p=l_4$-king pin track; β_e and β_i-turning angle of outer wheel and inner wheel, respectively; $\varphi_{10}=\varphi_{30}$-the values of φ_1 and φ_3 when the wheel are not turned. Also we introduce the normalized lengths: $\lambda_1=\lambda_3=l_1/l_4=l_3/l_4$, $\lambda_2=l_2/l_4$ $(l_1=l_3)$.

In [4] it has been proved that

$$\varphi_2(\varphi_1)=\arccos\frac{1+\lambda_2^2-2\lambda_1\cos\varphi_1}{2\lambda_1\sqrt{1+\lambda_1^2-2\lambda_1\cos\varphi_1}}-arctg\left(\frac{\lambda_1-1}{\lambda_1+1}\cdot\cot g\frac{\varphi_1}{2}\right)-\frac{\pi-\varphi_1}{2}, \quad (1)$$

$$\varphi_3(\varphi_1)=\arccos\frac{1+2\lambda_1^2-\lambda_2^2-2\lambda_1\cos\varphi_1}{2\lambda_1\sqrt{1+\lambda_1^2-2\lambda_1\cos\varphi_1}}+arctg\left(\frac{\lambda_1-1}{\lambda_1+1}\cdot\cot g\frac{\varphi_1}{2}\right)+\frac{\pi-\varphi_1}{2}, \quad (2)$$

$$i_{31}=\frac{d\varphi_3}{d\varphi_1}=\frac{\sin(\varphi_2-\varphi_1)}{\sin(\varphi_2+\varphi_3)}. \quad (3)$$

Inspecting Fig 1 one can write the relationship

$$\beta_e=\pm(\varphi_1-\varphi_{10}),\ \beta_i=\pm(\varphi_{10}-\varphi_3), \quad (4)$$

where the sign + is adopted for the trailing Ackerman steering linkage and the sign – corresponds to the leading Ackermann steering linkage. Also the angles φ_{10} and φ_{30} are given by the relation

$$\varphi_{10} = \varphi_{30} = \frac{1-\lambda_2}{2\lambda_1}. \tag{5}$$

The transmission angle between element 2 and element 3 for the trailing Ackermann linkage and the leading Ackermann linkage, respectively, is given by the relations [4]

$$\gamma_{23} = \varphi_2 + \varphi_3, \qquad \gamma_{23} = \pi - (\varphi_2 + \varphi_3). \tag{6}$$

The condition of correct steering is written as [1, 5]:

$$\cot g\beta_e - \cot g\beta_i = E_p / L, \tag{7}$$

where L is the wheel base of the automobile. If it is supposed that β_e is the input magnitude and β_i is output magnitude then the relation (7) leads to the relation

$$\beta_i = arctg\left(\frac{tg\beta_e}{1-(E_p / L)\cdot tg\beta_e}\right). \tag{8}$$

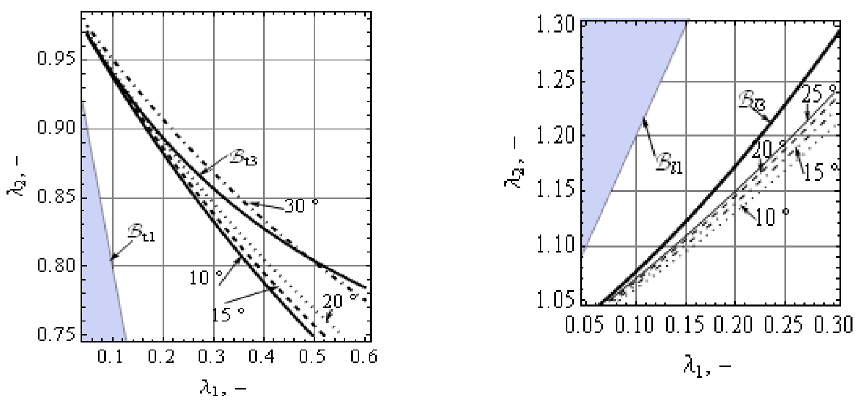
The steering error is defined by the difference between the turning angle of inner wheel which is carried out by the Ackermann steering linkage and the turning angle of the same wheel corresponding to the condition of correct steering for a given turning angle of outer wheel. Therefore, the expression of this error can be written as

$$e_s(\beta_e, \lambda_1, \lambda_2) = \pm[\varphi_{10} - \varphi_3(\pm\beta_e + \varphi_{10})] - arctg[\frac{tg\beta_e}{1-(E_p / L)tg\beta_e}] \tag{9}$$

where sign + is for the trailing Ackerman steering linkage and the sign – is for the leading Ackermann steering linkage(the function φ_3 has been previously defined by the relation (1)).

The function (9) corresponding to the trailing Ackermann linkage is defined on the domain $\mathcal{D}_{t2}$ [4]. The function (9) is defined in the point $(\lambda_{1\varepsilon},1)$ with $\lambda_{1\varepsilon}>0$, but it is not defined in the point $O_t(0,1)$. It is readily verified that point O_t is an accumulation point of the set defined by $\mathcal{D}_{t2}$ with $\lambda_{1\varepsilon}>0$. Also, it can be readily verified that there is no limit at the point O_t (on variable ensemble). Indeed, putting $\lambda_2=q.\lambda_1+1$ it is found that the limit of this function is dependent on q. This finding is very important, having outcomes which will be discussed later. The function (9) corresponding to the leading Ackermann linkage is defined on the domain $\mathcal{D}_{l2}$ (see [4]) but it is not defined in the point O_t (the domain $\mathcal{D}_{l2}$ is similar to the domain $\mathcal{D}_{t2}$). Also, this function doesn't have a limit at the point O_t.

In Fig.2 the curves $e_s(\beta_{e0},\lambda_1,\lambda_2)=0$ for different given values of β_{e0} are shown. Also, in the same figure are shown the curves $\mathcal{B}_{t3}$ and $\mathcal{B}_{l3}$ corresponding to the admissible transmission angle that settle the limits of the admissible domains $\mathcal{D}_{t3}$ and $\mathcal{D}_{l3}$ [4]($\mathcal{B}_{t1}$ and $\mathcal{B}_{l1}$ are the boundaries of the existence domains of the Ackermann steering linkage).



a) b)

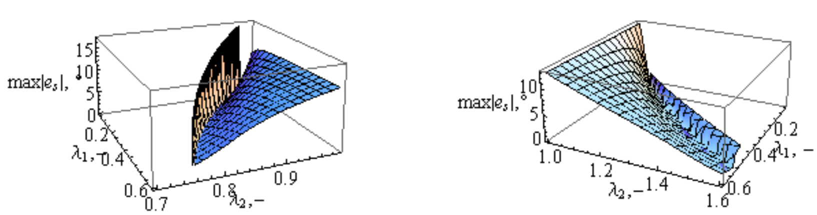
Fig. 2. Relating to the annulment of the steering error: a) trailing Ackermann linkage; b) leading Ackermann linkage (E_p/L=0.36, γ_a=25°)

It is found that in the case of trailing Ackermann steering linkage some curves are situated out of the admissible domain, at least on certain length. Also, it is noticed that for $\lambda_1 \in [0.05, 0.20]$ the curves for β_{e0}>20° are very near one with another and they are included in the admissible domain. In exchange, in the case of the leading Ackermann linkage the curves are included in the admissible domain.

3. Criterion of the maximum of absolute steering error value

In comparison with the above discussion it is necessary to use a more adequate criterion for the determination of the parameters λ_1 and λ_2. This criterion has been used in [2]. More precisely, one uses an objective function as a maximum of the absolute value of the steering error on the interval [0, β_{emax}]: :

$$f_0(\lambda_1, \lambda_2) = \underbrace{\max}_{0 \le \beta_e \le \beta_{e\max}} \left| e_s(\beta_e, \lambda_1, \lambda_2) \right| \cdot \tag{10}$$

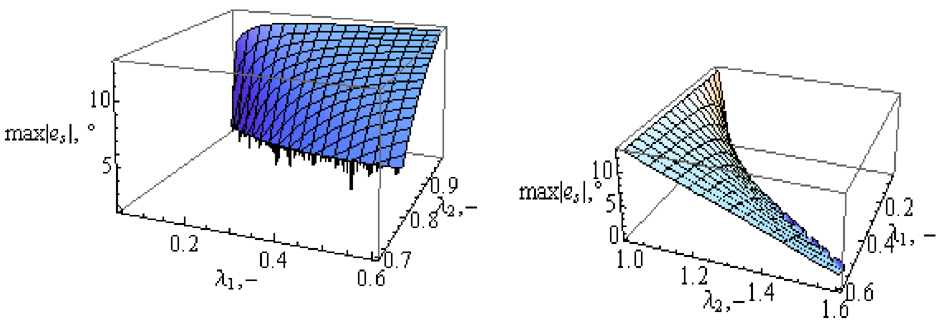


a) b)

Fig.3. Maximum of absolute value of the steering error as a function of λ_1 and λ_2 (β_{emax}=35°, E_p/L=1/1.9): a) trailing Ackermann linkage; b) leading Ackermann linkage

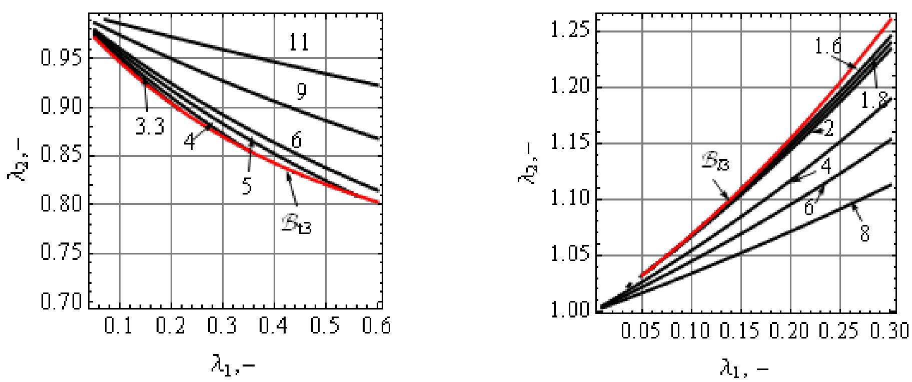
The optimal parameters are those which achieve minimization of f_0 with respect to λ_1 and λ_2, being satisfied the above discussed constraints. Although the function $e_s(\beta_e,\lambda_1,\lambda_2)$ is differentiable with respect to β_e, generally, the function $|e_s(\beta_e,\lambda_1,\lambda_2)|$ is not differentiable. This fact brings forth difficulties to solve the optimization problem. Also, an analytic approach of the optimization problem is in practice impossible.

Starting from (2) and (3) we have defined the function (10) by the function *NMaximize* of the programming environment *Mathematica*®.



a) b)

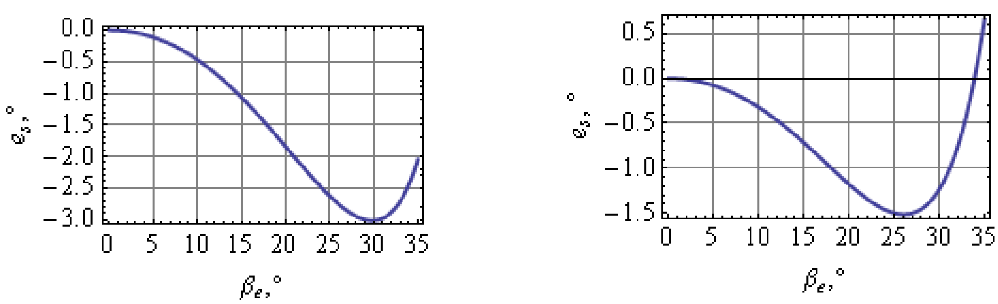
Fig.4. Maximum of absolute value of the steering error as a function of λ_1 and λ_2 on the the admissible domain ($\gamma_{23} \geq \gamma_a$=25°) :a) trailing Ackermann linkage; b) leading Ackermann linkage



a) b)

Fig.5. Contour lines of the function f_0 : a) trailing Ackermann steering linkage ; b) leading Ackermann steering linkage

For the above reasons one used the optimization methods without derivatives: Simulated Annealing and Differential Method. For a given value of the ratio E_p/L the graph of the function (10) is shown in Fig. 3. It is found that f_0 has very large values (but finite) on the boundary of the definition domain of function e_s. This fact can be explained because the critical position of the mechanism the transmission ratio i_{13}=0 [4]. Also, it has been ascertained that the function f_0 decreases fast in proportion as the point (λ_1, λ_2) moves away from the boundary. After that the function f_0 increases. But, what really matters is the domain corresponding to the motion transmitting. In this case the graph of the function f_0 is shown in Fig.4. It is found that the minimum values of f_0 are obtained on the boundary of the admissible domain. In a direct way this fact is highlighted by contours lines of the function f_0 as shown in Fig.5. In this figure are also shown the boundaries of the admissible domains defined by the admissible value γ_a of transmission angle. In the case of the trailing Ackermann linkage it is found that as $\lambda_1$decreases on the domain boundary the value of the criterion decreases also in proportion. The ensemble of the contour lines is focused on the point (0, 1) without it being attained. Of course, for small values of λ_1, large variations of the criterion are produced when the variations of λ_2 are small. For optimization it is sufficiently to choose a value of λ_1 and after that to solve the equation obtained by equating to zero the function of the transmission angle with respect to λ_2. For example, for λ_1=0.16, E_p/L=1/1.9, β_{emax}=35° and γ_a=25° the results are: λ_2=0.91911 and f_0=3.026°. In the case of the leading Ackermann linkage as $\lambda_1$increases on the domain boundary the optimization criterion decreases in proportion. For example, for λ_1=0.16 the obtained results are: λ_2=1.1184 and f_0=1.55°. For λ_1=0.30 we have: λ_2=1.260 and f_0=1.1528°. It is clear that the diminution of min f_0 is not very marked when λ_1 increases.



a) b)

Fig.6.The steering error as a function of the outer steering angle in the case of optimization according to the absolute steering error criterion (see text): a) trailing Ackermann linkage; b) leading Ackermann linkage

The variation of the steering error with respect to the outer turning angle is shown in Fig. 6. It is found that in the case of the leading Ackermann steering linkage the absolute value of the steering error is smaller than that for the trailing Ackermann steering linkage.

4. Criterion of root-mean-square steering error

Since the steering error should be reduced on the whole, we can use an integral criterion under the form of

$$I(\lambda_1,\lambda_2,E_p/L)=\int_0^{\beta_{e\max}} e_s^2(\beta_e,\lambda_1,\lambda_2,E_p/L).f_w(\beta_e)d\beta_e, \tag{11}$$

where $f_w(\beta_e)>0$ is a weight function with the property

$$\int_0^{\beta_{e\max}} f_w(\beta_e)d\beta_e=1. \tag{12}$$

Obviously, the parameters λ_1 and λ_2 are chosen so that $I(\lambda_1, \lambda_2)$ to have the minimum value. A similar criterion has been used in [6] for the case of a six-bar steering linkage. The weight function is chosen taking into account the concrete operation conditions of the vehicle. In [6] it is assumed that f_w=1, in the way and in the present paper we shall act. To make comparisons more easily we use the root mean square steering error defined as

$$e_{rmss}(\lambda_1,\lambda_2,E_p/L)=\frac{180}{\pi}\cdot\sqrt{\left(\int_0^{\beta_{e\max}} e_s^2(\beta_e,\lambda_1,\lambda_2,E_p/L)d\beta_e\right)/\beta_{e\max}}\quad [°]. \tag{13}$$

The optimization consists in the minimization of the root-mean-square steering error with respect to λ_1 and λ_2. So we have a constrained optimization problem. The optimum point may be or the critical point of the function e_{rmss} either a point situated on the boundary of the admissible domain defined by the constraints. The critical point is defined by the relations

$$\partial e_{rmss}/\partial\lambda_1=0,\quad \partial e_{rmss}/\partial\lambda_2=0, \tag{14}$$

which lead to

$$\frac{\partial}{\partial\lambda_j}\int_0^{\beta_{e\max}} e_s^2(\beta_e,\lambda_1,\lambda_2,E_p/L)d\beta_e=0,\ (j=1,2). \tag{15}$$

The condition (15) becomes

$$\int_0^{\beta_{e\max}} e_s(\beta_e,\lambda_1,\lambda_2,E_p/L)\frac{\partial e_s}{\partial\lambda_j}d\beta_e=0,(j=1,2). \tag{16}$$

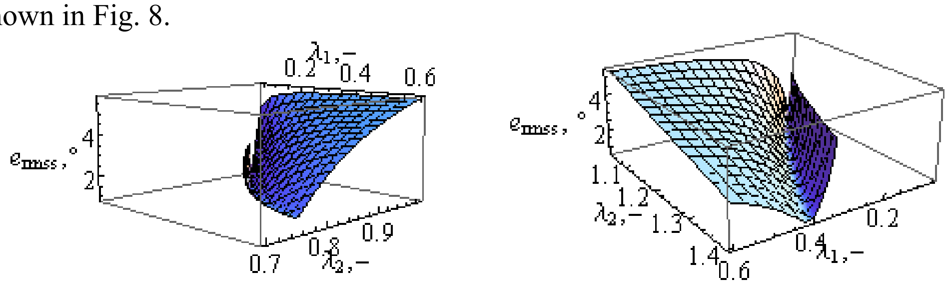
For the trailing Ackermann linkage the preceding condition is written as

$$\frac{\partial\varphi_{10}}{\partial\lambda_j}\cdot\int_0^{\beta_{e\max}}(1-\left.\frac{d\varphi_3}{d\varphi_1}\right|_{(\beta_e+\varphi_{10})})\cdot e_s(\beta_e,\lambda_1,\lambda_2,E_p/L)d\beta_e=0,\ (j=1,2) \tag{17}$$

with $\partial\varphi_{10}/\partial\lambda_j\neq 0$. Further we arrive at

$$I_t(\lambda_1,\lambda_2,E_p/L)=\int_0^{\beta_{e\max}}\left(1-\left.\frac{d\varphi_3}{d\varphi_1}\right|_{(\beta_e+\varphi_{10})}\right).e_s(\beta_e,\lambda_1,\lambda_2,E_p/L)d\beta_e=0. \tag{18}$$

Therefore, the conditions (14) are satisfied if a single condition (18) takes place. As such, the critical points of the mentioned function are not isolated points. They are situated on a curve of which projection on the plane (λ_1, λ_2) is defined by the equation (18). In this case the critical points are of type trough as we can see on the graphical representation in Fig. 7. To establish the expression of the integrand function in (18) one uses the relations (1),(2),(3),(4) and (5). The integral of (18) cannot be expressed by elementary functions and its evaluation is numerically carried out. Using an adequate program in *Mathematica®* the curve $I_t(\lambda_1,\lambda_2,E_p/L)=0$ has been drawn for given values of E_p/L. In the case of the leading Ackermann linkage a similar method has been used. The results are shown in Fig. 8.



a) b)

Fig.7. Root mean square steering error as a function of λ_1 and λ_2 (E_p/L=1/1.9, β_{emax}=35°) : a) trailing Ackermann linkage ; b) leading Ackermann linkage

In the case of the trailing trapezium the curves are situated higher in proportion as E_p/L decreases (one can verify that for E_p/L=0.26 the curve is completely situated within the admissible domain, which is not case for E_p/L=0.46). In the case of the leading trapezium in proportion as the mentioned ratio decreases the curves are situated more down.

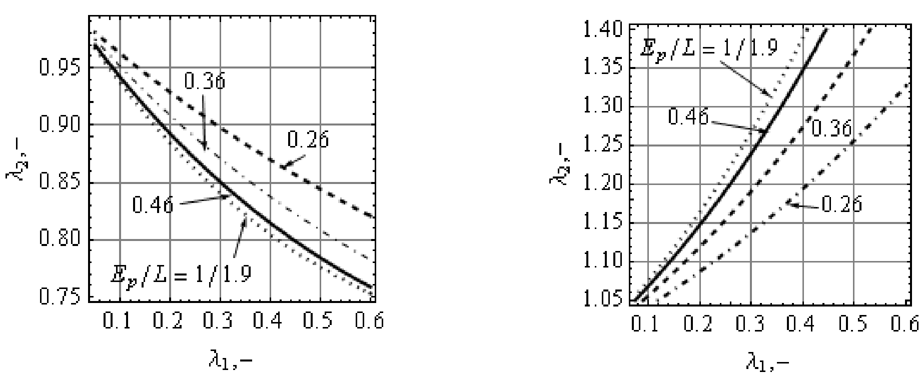


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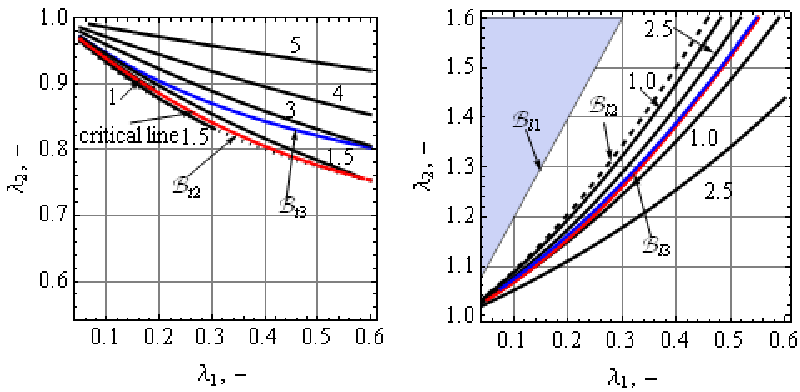
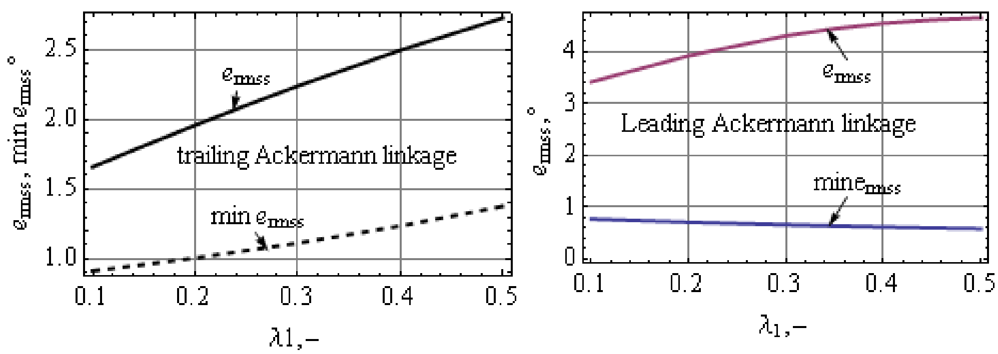
Fig. 8. Projections of critical lines on the plane (λ_1, λ_2) : a) trailing Ackermann linkage ; b) leading Ackermann linkage



a) b)

Fig.9. Contour lines of the function e_{rmss} and the curves I_t=0 and curves for β_{emax} and γ_a (E_p/L=1/1.9, β_{emax}=35°, γ_a=25°, critical line-red line): a) trailing Ackermann linkage; b) leading Ackermann linkage

The contour lines of the function e_{rmss}, the curve I_t=0, the curve corresponding to the critical position of the mechanism for β_{emax} and the curve corresponding to the admissible transmission angle γ_a =25° are plotted in Fig. 9(for E_p/L=1/1.9). The last two curves are depicted by $\mathcal{B}_{l2}$, $\mathcal{B}_{l3}$, $\mathcal{B}_{t2}$ and $\mathcal{B}_{t3}$ (see [4]). For a given value of λ_1 there is a certain value of λ_2 corresponding to the curve I_t=0 for which the root mean square steering error has the minimum value. The minimum value is dependent on the position of the figurative point on the mentioned curve. This fact is made evident on Fig. 9. At the same time it is necessary to observe that there are situations when curve I_t=0 is not in part or totally included in the admissible domain. In the case of Fig.9a curve I_t=0 is situated out of admissible domain defined by γ_a. Therefore, the constrained optimum is given by the curve corresponding to γ_a. For a given λ_1 one can determine λ_2 from equation derived from (6) putting γ_{23}=γ_a (obviously, in turn, for each type of Ackermann linkage). After that one can determine the value of e_{rmss}, which is dependent on the position of the figurative point on the mentioned curve. This fact is highlighted in Fig. 10a. It is found that the constraint related to the transmission angle leads to a large value of e_{rmss} comparatively with that of mine_{rmss}. In the case of Fig.9b the curve I_t=0 is near the curve corresponding to γ_a. It is situated on the admissible domain. as $\lambda_1$increases the optimum value of e_{rmss} decreases in proportion (see Fig. 10b). Although the curve corresponding to γ_a is very close to the curve corresponding to I_t=0 the values of e_{rmss} are large enough. This fact is in concordance with the variation of e_{rmss} shown in Fig.10b.

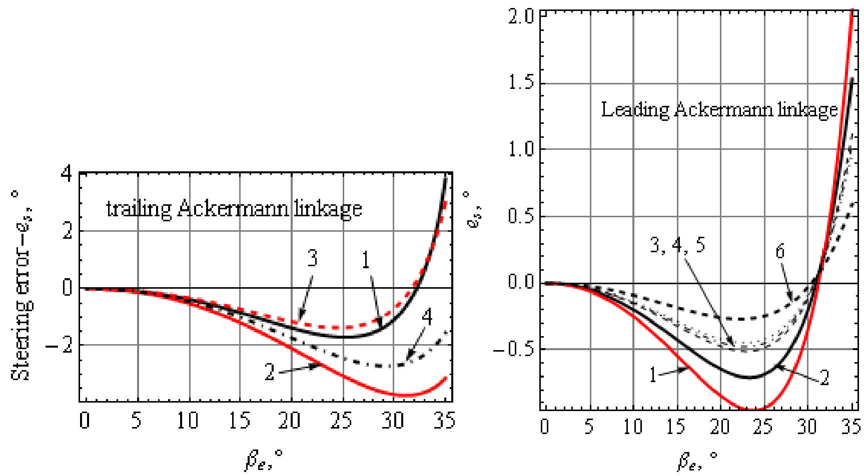


a) b)

Fig.10. Root-mean-square steering error as a function of λ_1 : minimum values and values corresponding to curve for γ_a

Generally, it is found that the errors are reduced by the leading Ackermann linkage. In the case of the trailing Ackermann linkage the errors decrease when λ_1 decreases, which is also contrary to the case of leading trapezium. From analysis of the surface shown in Fig. 7 it is noticed that the variations of the *rms* steering error are very important in the side near to the boundary of the admissible domain. Therefore, for a given λ_1, the small variations of λ_2 around the optimum value leads to the important increases of the *rms* steering error. This result is in accordance with the results of Fig .9, finding that for the small values of λ_1 the curves are much nearer together. In consequence, the small variations of λ_2 lead to the significant variations of the angles involved by these curves. The general recommendations relating to choosing of $\lambda_1 \in [0.12, 0.20]$ [1] are in accordance with this above mentioned especially for the trailing trapezium (it is necessary to have a reduced size). But, at the same time, it is necessary to notice that the manufacture accuracy of the bars for the lengths should be high, otherwise the steering errors will become large. For the leading Ackermann linkage it is desired that λ_1 to have large values, but there is a size limitation because the knuckle arm nears to the wheel.

Obviously, it is important how the steering error changes as a function of the outer steering angle. To illustrate this dependence also the consequences of choosing of the parameters in concordance with the mentioned criterion one considered certain optimal values of λ_1 and λ_2 and plotted the curves shown in Fig. 11 for different values of E_p/L.



a) b)

Fig.11. The steering error as a function of the outer steering angle in the case of optimization according to the root-mean-square steering error criterion (see text): a) trailing Ackermann linkage: E_p/L=1/1.9, β_{emax}=35°,γ_a=25°; 1-λ_1=0.30, λ_2=critical value;2- λ_1=0.30, λ_2=the value corresponding to γ_a; 3-λ_1=0.10, λ_2=critical value; 4-λ_1=0.10, λ_2= the value corresponding to γ_a; b) leading Ackermann linkage: β_{emax}=35°, γ_a=25°; 1-E_p/L=0.46, λ_1=0.30; 2-E_p/L=1/1.9, λ_1=0.30; 3-E_p/L=0.36, λ_1=0.10; 4-E_p/L=0.36, λ_1=0.20; 5- E_p/L=0.36, λ_1=0.30; 6-E_p/L=0.26, λ_1=0.30

It is found that the largest errors are obtained by large turning angle. Generally, for "long "vehicles the steering error is smaller. In any way, the steering error is enough reduced in comparison with that indicated in literature [1]. If, for instance, for E_p/L=0.46 one chooses λ_1=0.30 and λ_2=0.80, as in [7], then the steering error becomes very large, especially for β_e=30°, namely 15°. In [2], for E_p/L=1/1.9, β_{imax}=40°and λ_1=0.16 the optimum design of a trailing Ackermann steering linkage is obtained by φ_{10}=180°-109.15921° (see table1 [2]). The mentioned values lead to β_{emax}=30.2015° and λ_2=0.894978. For E_p/L=1/1.9, E_p/L=1/1.9 and λ_1=0.16 the method used in the present paper results in λ_2=0.895693. Therefore, the results by the two methods are very close together (percentage difference is 0.08%).

5. Conclusions

The maximum absolute value of the steering error and the root-square-mean steering error as functions of λ_1 and λ_2 (normalized lengths of the knuckle arm and of the tie road, respectively) has not a proper minimum point.

In the paper it is shown how to determine the normalized lengths of the bar mechanism so that the mentioned functions get values as reduced as possible taking into account different constraints. Generally, these quantities correspond to the minimum admissible value of transmission angle.

The obtained results are in accordance with those which have been obtained in other works, but the method used in the paper is more simple and direct.

In the paper it was demonstrated that the Ackermann steering linkage has intrinsic limitations in term of achieving the correct steering condition.

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