# CALCULATION OF DETERIORATION DUE TO CRACKS IN TUBULAR SPECIMENS

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*The problem under study is damage caused by cracks. We show the difference between the deterioration/damage produced in a flat plate and in a tubular specimen, respectively. The concept of deterioration is used to calculate the critical normal and tangential stresses of a statically loaded cracked specimen.*

*The general results obtained are applied to the deterioration calculation of tubular specimens with cracks on the inner surface whether axial rectangular or circumferential rectangular and semi-elliptical One highlights the variation of the total deterioration and the total deterioration components depending on crack geometry and the type of loading.*

**Keywords:** deterioration, damage, cracks, critical stress, tubular specimen.

## 1. Introduction

The lifetime of pressure equipment and in general of engineering structures, is influenced by the loading they undergo, namely mechanical (static, fatigue, shock), thermal (under or over creep temperature), mechano-chemical (corrosion, erosion) in a permanent or in transition regime, as well as by deterioration and by residual stresses.

Generally, the requirements concerning the safety of mechanical structures needs to consider cracks/flows in the design stage, as well as in their monitoring during operation.

Khachanov [1] has introduced the deterioration parameter, $D$, a nondimensional variable. $D = 0 -$ for undamaged material, and $D = 1$ – for the fully damaged structure material (failure, excessive deformation etc…).

The deterioration is expressed as a reduction of a certain mechanical or thermo mechanical characteristic, or as a measure of the accumulation of the deformation or duration of mechanical and thermomechanical stress [2; 3]. The state of the art can be found in the papers [3; 4].

Several relations have been proposed for the fatigue life and for the critical stress calculation of a cracked structure [2; 5].

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Mechanical structures are characterized by critical stresses. This is that value of stress that determines the failure or the distruction of mechanical structure.

One considers the general case of the nonlinear, power law, behavior of the structure material, under monotonic loading with the normal stress, σ, or with the shear stress, τ,

$$\sigma = M_{\sigma} \cdot \varepsilon^{k} \quad \text{and} \quad \tau = M_{\tau} \cdot \gamma^{k_1} \tag{1}$$

where ε is strain, γ - shear strain and $M_{\sigma}$, $M_{\tau}$, $k$ and $k_1$ are constants of material.

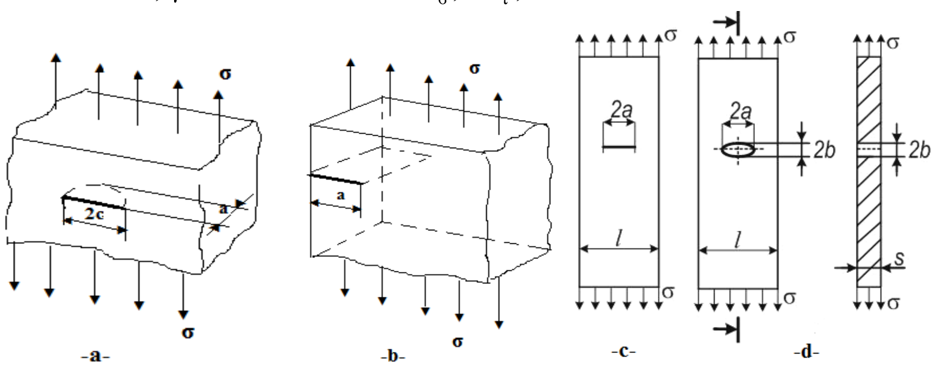


Fig.1. Crack in: *a* - semi-infinite body with elliptical surface crack; *b* - semi-infinite plate with through crack (long shallow surface crack); *c,d* - cracks in plates uniaxial loaded.

In a semi-infinite body with elliptical crack (Fig.1, a) or with through edge crack (Fig.1, b), the depth of the crack is used to calculate the stress intensity factor. We shall use for D(a) the deterioration due to crack in a plate (Fig. 1, c) which has been expressed by the following general relationship [3; 8; 9],

$$D(a) = \left( a / a_{cr} \right)^{\frac{\alpha+1}{2}}, \tag{2}$$

where $a$ is the current or instantaneous crack length; $a_{cr}$ – the critical value of the $a$ and $\alpha = 1/k$ .

The deterioration has been defined as well as the ratio [10],

$$D(a) = a / a_f \tag{3}$$

between the current or instantaneous crack length $a$ and its final length, $a_f$. In the case of linear-elastic behavior k =1 and $\alpha = 1$, such as equation (2) becomes equation (3), with $a_f = a_{cr}$ .

The real crack in a plate is like in figure 1,c of length 2*a*, through the whole thickness, *s*, but of a very small width. In theoretical considerations the crack is regarded as an ellipse whose longer semi-axis is equal to *a* and whose shorter semi-axis is equal to $b \to 0$ (Fig. 1, d).

In the case of shells, components of the pressure equipment, it is obligatory to consider the actual extension of the crack: external circumferential cracks (Fig. 2), external axial cracks (Fig. 3) internal circumferential (Fig. 4, *a*, *b*) and internal axial cracks (Fig. 4, *c*, *d*), as well as through wall cracks (Fig. 5).

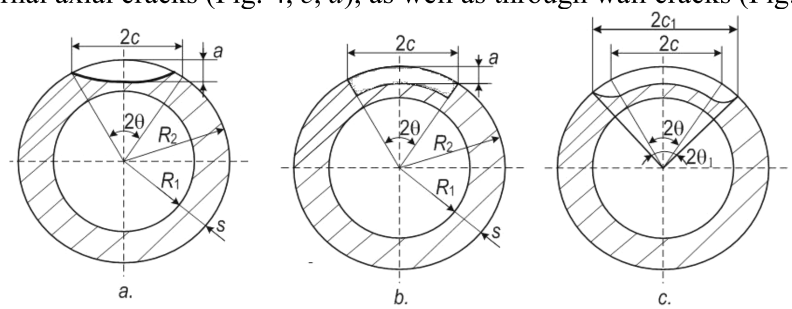


Fig. 2. Circumferential cracks on the outer surface of tubular elements where the cross section is: a - semi-elliptical, b - rectangular, c - rectangular with connected edges

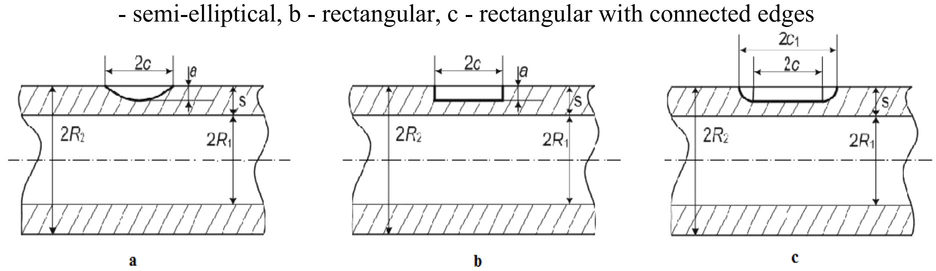


Fig.3. Axial cracks on the outer surface of tubular elements whose cross -section is: a - semi-elliptical; b - rectangular, c - rectangular with connections.

The influence of cracks on mechanical structures has been studied from different points of view, namely: – the influence of axial cracks upon the rupture pressure of a cylindrical pressure vessel [11]; – the influence of cracks caused by corrosion upon the critical circumferential stress and plastic instability in pipes [12-14]; – the influence of the circumferential semi-elliptical crack on the outer surface toughness [15]; – the crack influence upon the bursting pressure of the autoffretaged cylinders with inclined cracks [16] etc.

Kachanov [1], Xue and Wierzbicki [17] have related the effective stress at the crack tip to deterioration and the applied stress. On the other side Gong et al [18] correlate the cohesive energy, cohesive stiffness and cohesive strength with the damage due to cyclic loading. Marin et al. [19] related the plastic Poisson's ratio to its initial value and the deterioration.

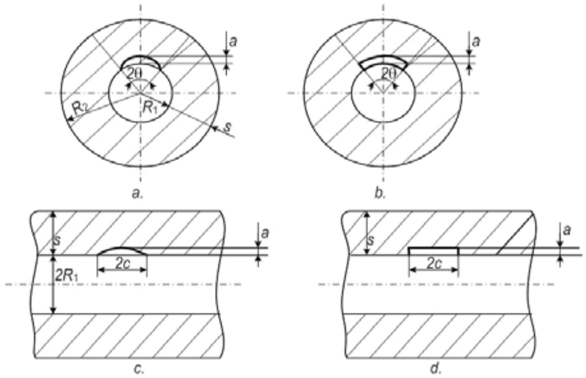


Fig.4. Tubular elements with circumferential (*a*; *b*) and axial (*c*;*d*) cracks in semi-elliptical (*a*; *c*), and rectangular (*b*; *d*) cross-sections.

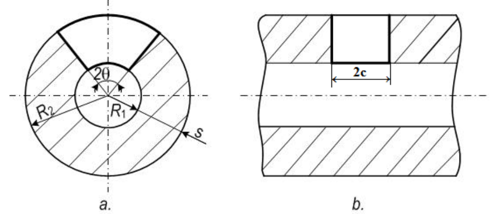


Fig.5. Tubular element with cracks all across the wall thickness:
*a* –- circumferential; *b* –- axial, rectangular.

In this paper one establishes relations for the deterioration produced by cracks in tubular components (Fig. 2 – 5). First the practical use of the deterioration in the strength calculation is explained.

## 2. The deterioration in the critical strength calculation of a statically loaded structure

In a wide plane sample which contains a crack of length 2*a* (Fig. 1), the *critical normal stress* is [20],

$$\sigma_{cr}(a_\sigma) = \sigma_{cr} \cdot \left[P_{cr}(0) - D(a_\sigma)\right]^{\frac{1}{\alpha+1}}, \tag{4}$$

where $\sigma_{cr}$ is the critical normal stress of the crackless structure; $\sigma_{cr}(a_\sigma)$ – the critical normal stress of the cracked structure; $D(a_\sigma)$ – the deterioration due to the crack, $a_\sigma$, dangerous for the normal stress loading; $P_{cr}(0)$ – the critical value of specific energy participation which features a statistical distribution, that is $P_{cr}(0) \in \left[P_{cr,\min}(0); P_{cr,\max}(0)\right]$, a distribution where $P_{cr,\max}(0) \le 1$. If the values of the critical characteristics of the material are deterministic variables, $P_{cr}(0) = 1$.

The participation of the specific energy corresponding to stress σ applied to a cracked structure can be written as [8],

$$P(\sigma)=\left(\frac{\sigma}{\sigma_{cr}(a_\sigma)}\right)^{\alpha+1}, \qquad (5)$$

where the value of the exponent $\alpha = 1/k$ depends on the rate of loading [2; 8].

One attains the material critical state if

$$P(\sigma) = 1. \qquad (6)$$

By a similar procedure, in the case of shear stress it has obtained [20],

$$\tau_{cr}(a_\tau)=\tau_{cr}\cdot[P_{cr}(0)-D(a_\tau)]^{\frac{1}{\alpha_1+1}}, \qquad (7)$$

where $\tau_{cr}$ is the critical shear stress of the crackless structure; $\tau_{cr}(a_\tau)$ – the critical shear stress of the cracked structure; $D(a_\tau)$ – the deterioration due to crack $a_\tau$, dangerous for the shear stress loading. The participation of the specific energy corresponding to the stress τ, is

$$P(\tau)=\left(\frac{\tau}{\tau_{cr}(a_\tau)}\right)^{\frac{1}{\alpha_1+1}}, \qquad (8)$$

where $\alpha_1 = 1/k_1$ depends on the rate of loading [2; 8].

For structure with cracks *statically loaded*: $\sigma_{cr} = \sigma_y$ (yield stress) if it is imperative that the material should not exceed the yield stress, or $\sigma_{cr} = \sigma_u$ (ultimate stress) if one allows to be exceed the yield stress. A similar observation is valid for the shear stress, namely $\tau_{cr} = \tau_y$ or $\tau_{cr} = \tau_u$.

As to use the above strength criteria, the deterioration must be calculated. For wide plane sample the relation (2) or (3) may be used.

### 3. The deterioration of tubular sample

For *tubular specimens*, taking into account relations (4) and (7), as well as the experimental results reported in [21, 22] there have been proposed the following relations for damage with:

- circumferential cracks (Fig. 2, *a – c*, Fig. 4, *a*, *b* şi Fig. 5, *a*),

$$D(a;\theta)=D(\theta)\cdot(1+D(a)); \qquad (9)$$

- axial cracks (Fig. 3, Fig. 4. *c*, *d* şi Fig. 5, *b*),

$$D(a;c)=D(c)\cdot(1+D(a)), \qquad (10)$$

where D (a) is given by the general relationship (2); D (θ) depends on the ratio $\theta/\pi$ and D (c) depends on the ratio $c/\sqrt{(R_m\cdot s)}$ where $R_m = 0.5(R_1+R_2)$ is the mean radius of the section.

Consequently, for *statically loaded tubular specimens* with cracks, the critical stress is calculated with relations (4) and (7) written as it follows, for:

- normal critical stress

$$\left.\begin{aligned}\sigma_{cr}(a_\sigma;c)&=\sigma_{cr}\cdot[P_{cr}(0)-D(a_\sigma;c)]^{\frac{1}{\alpha+1}},\\ \sigma_{cr}(a_\sigma;\theta)&=\sigma_{cr}\cdot[P_{cr}(0)-D(a_\sigma;\theta)]^{\frac{1}{\alpha+1}};\end{aligned}\right\}; \tag{11}$$

- critical shear stress

$$\left.\begin{aligned}\tau_{cr}(a_\tau;c)&=\tau_{cr}\cdot[P_{cr}(0)-D(a_\tau;c)]^{\frac{1}{\alpha_1+1}},\\ \tau_{cr}(a_\tau;\theta)&=\tau_{cr}\cdot[P_{cr}(0)-D(a_\tau;\theta)]^{\frac{1}{\alpha_1+1}}.\end{aligned}\right\}. \tag{12}$$

We further analyze the loads: under - internal pressure, $p$; - axial force, $N$ and bending moment $M_b$. In all cases, the loading occurred down to the yield point [21, 22], the latter being considered the limit stress ($\sigma_{cr} = \sigma_y$).

The experimental researches used to calculate deterioration refer to tubular specimens from the category of cylindrical shells, as the dimensionless ratio, $\beta = R_2/R_1 = 1 + s/R_1 \le 1.2$, which yields $s/R_1 \le 0.2$ or $s/R_1 \ge 5$. By replacing $R_1 = R_m + 0.5s$ one gets $R_m/s \ge 4.5$ in the case of shells. In our considerations we have used $R_m/s = 5$ and 20, values that comply with the aforestated condition.

### 4. Statically loaded tubular specimens with rectangular axial cracks on the inside surface (Fig. 4, d)

In cylindrical shells under internal pressure, the maximum stress is the hoop or circumferential stress,

$$\sigma_\theta = p\cdot R_m/s\ . \tag{13}$$

Consequently, with $\sigma = \sigma_\theta$ and $\sigma_{cr} = \sigma_{\theta,cr}$, one gets

$$p_{cr}(a;c)/p_{cr} = \sigma_{cr}(a;c)/\sigma_{cr}.$$

Fig. 6 shows (processed according to [21]) the dimensionless ratio variation $\sigma_{cr}(a)/\sigma_{cr}$ (where $\sigma_{cr} = \sigma_y$) depending on the dimensionless variable $c/\sqrt{R_m\cdot s}$, for four values of the reported crack depth a/s = 0.25; 0.5; 0.75 and 1.0.

From the first relation (11) one gets the equation of deterioration,

$$D(a_\sigma;c) = P_{cr}(0) - \left[\frac{\sigma_{cr}(a_\sigma;c)}{\sigma_{cr}}\right]^{\alpha+1}. \tag{14}$$

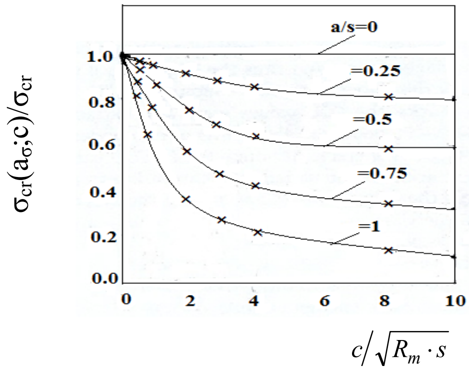


Fig. 6. Tubular specimens with axial rectangular cracks on the inner surface (Fig. 4, d). Dependency of $\sigma_{cr}(a;c)/\sigma_{cr}$ ratio on reported length of axial crack $c/\sqrt{R_m \cdot s}$ , for different values of the *a/s* ratio (processed according to [21]).

From equation (14) with $P_{cr}(0) = 1$, based on the experimental data in Figure 6 there have been obtained the values of D $(a_\sigma;c)$ shown in Fig. 7, *a*. From equation (11) one determines the influence of crack length via the damage component,

$$D(c) = \frac{D(a;c)}{1 + D(a)} \quad , \tag{15}$$

wherein D (a) was calculated with equation (2) where the material behavior under load ($\sigma < \sigma_y$) being linear elastic, k = 1 and α = 1.The variation of D (c) is shown in Figure 7.*b,* depending on the ratio $c/\sqrt{R_m \cdot s}$ for four values of the reported depth a/s.

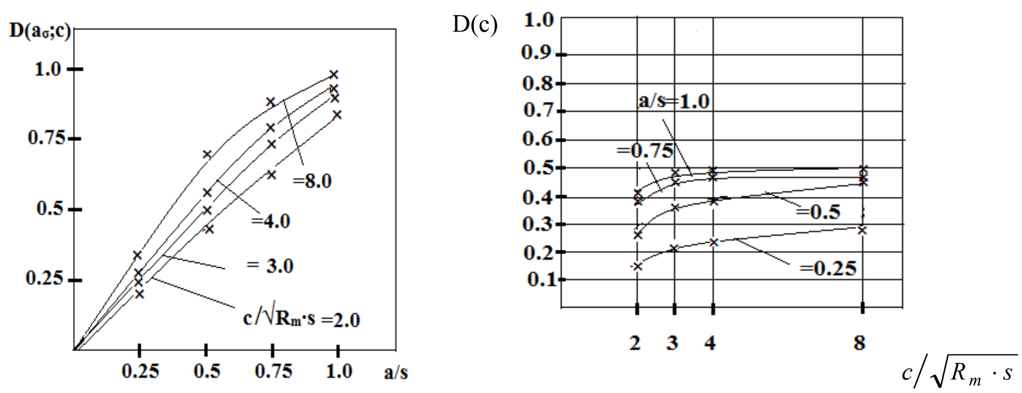


Fig.7. Dependency of deterioration D $(a_\sigma;c)$ on the *a/s* ratio for four values of the crack reported axial length (*a*) and deterioration D(c) depending on $c/\sqrt{R_m \cdot s}$ for four values of ratio a/s (*b*), in the case of internal pressure loading.

**5. Statically loaded tubular specimens with rectangular circumferential cracks on the inside surface (Fig. 4, b).**

***a.*** With cylindrical shells under the *axial force* load *N* the axial stress is

$$\sigma = N/A, \tag{16}$$

where $A = \pi R_m^2$ is the cross-sectional area of the non-cracked tubular element. In tubular elements with circumferential cracks (Fig.2),

$$N_{cr}(a_\sigma;\theta)/N_{cr} = \sigma_{cr}(a_\sigma;\theta)/\sigma_{cr}.$$

From the second equation (11) one gets the deterioration,

$$D(a_\sigma;\theta) = P_{cr}(0) - \left[\frac{\sigma_{cr}(a_\sigma;\theta)}{\sigma_{cr}}\right]^{\alpha+1}, \tag{17}$$

while from equation (9) one obtains,

$$D(\theta) = \frac{D(a_\sigma;\theta)}{1 + D(a)}, \tag{18}$$

where $D(a_\sigma)$ with $a = a_\sigma$ and $a_{cr} = a_{\sigma,cr}$ is calculated with relation (2).

For the tubular elements with a rectangular circumferential crack on the inner surface [21], the ratio $\sigma_{cr}(a_\sigma;\theta)/\sigma_{cr}$ variation dependent on the *a/s* ratio for four values of the reported angle θ/π is shown in Figure 8. Based on these data, with equation (17) one calculated the total deterioration $D(a_\sigma;\theta)$ shown in Figure 9, *a*, while with relation (18) one obtained the influence of the crack circumferential length, D(θ), shown in Figure 9, *b*.

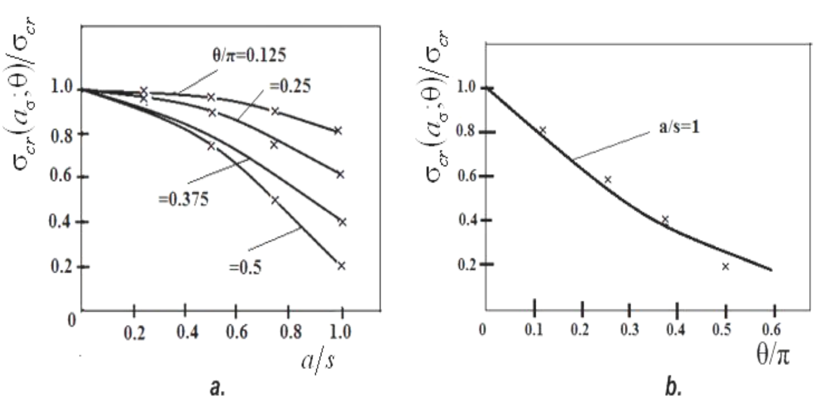


Fig. 8. Dependency ratio $\sigma_{cr}(a_\sigma;\theta)/\sigma_{cr}$ for steel pipes with circumferential semi-elliptical crack under axial force (processed according to [21]) featuring: - crack depth $a/s$, for four values of half angle θ/π (*a*) – half angle θ/π for through-wall cracks (Figure 5, a) *(b).*

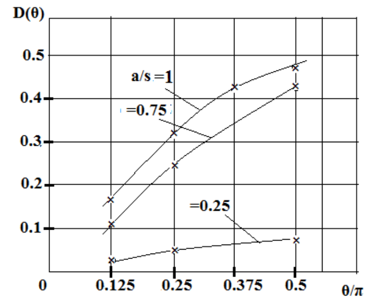


Fig.9. Specimen of circumferentially cracked pipe axial loaded.
Deterioration dependency $D(a_\sigma;\theta)$ calculated with relation (17) on ratio $a/s$, for different values of reported half angle θ/π (*a*) and deterioration dependency $D(\theta)$, calculated with equation (18) on θ/π for three values of the ratio $a/s$ (b).

***b.*** In a tubular specimen under the bending moment $M_b$, stress induced is,

$$\sigma_b = \pm M_b / W\,, \qquad (19)$$

where $W = \pi \cdot R_m^2 \cdot s$ is the pipe section strength modulus. Consequently, $M_{b,cr}(a_\sigma;\theta)/M_{b,cr} = \sigma_{b,cr}(a_\sigma;\theta)/\sigma_{b,cr}$

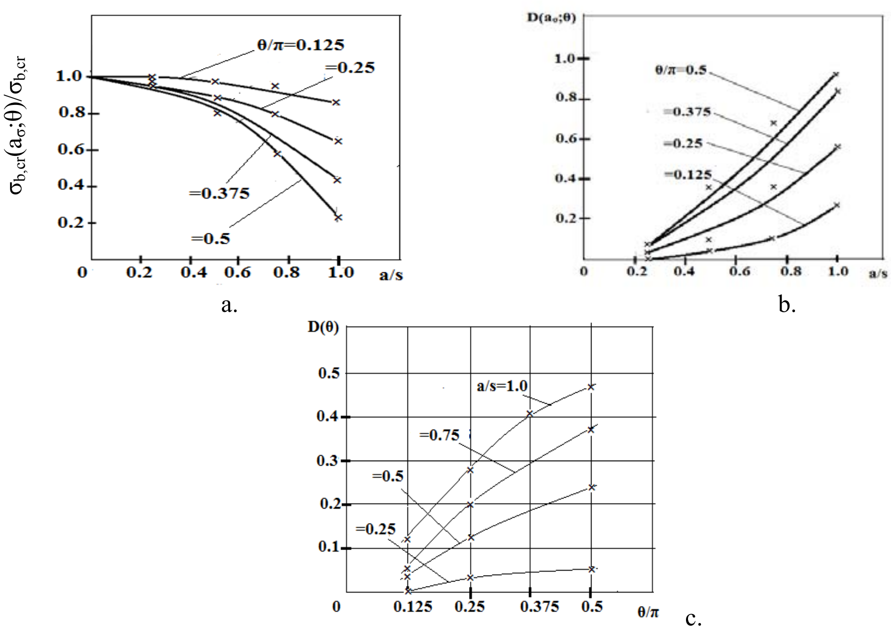


Fig.10. Pipe specimen with rectangular circumferential cracks on the inner surface loaded by a bending moment: a-dependency of ratio of critical bending stresses on the ratio $a/s$ for four values of the reported half angle θ/π (processed according to [21]); b- total deterioration $D(a_\sigma;\theta)$ calculated with (17), depending on the ratio $a/s$ for the four values of reported half angle θ/π; c- total deterioration $D(\theta)$ calculated with (18) depending on θ/π for four values of ratio $a/s$.

For tubular specimens with rectangular circumferential cracks on the inner surface whose ends are loaded with bending moments , the results are shown in Figure 10,a. Taking into account that in equation (17) $\sigma_{cr}(a_\sigma;\theta) \equiv \sigma_{b,cr}(a_\sigma;\theta)$ and $\sigma_{cr} \equiv \sigma_{b,cr}$ one draw the curves in Figure 10, b for total deterioration $D(a_\sigma;\theta)$, depending on the reported crack depth *a/s*. With these values and with (18), where $D(a)$ was calculated with equation (2), one acquired the resulting curves shown in Figure 10, c.

## 6. Tubular specimens with circumferential semi-elliptical cracks on the inner surface, statically loaded (Fig. 4, a and c)

For a tubular specimen with semi-elliptical circumferential cracks on the inner surface (Fig. 4 a), under *axial force* N, by processing the experimental data in [22], we have obtained the curves in Figure 11, a, for the dependence of $\sigma_{cr}(a_\sigma;\theta)/\sigma_{cr}$ on the crack reported depth $a/s$, with three values of the reported angle θ/π.

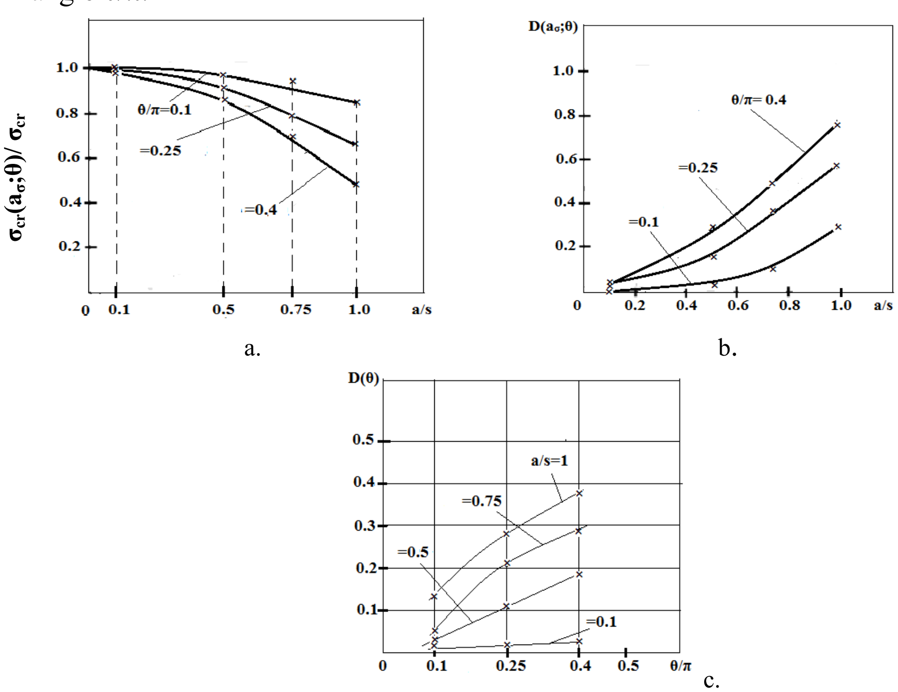


Fig. 11. Pipe specimens with circumferential semi-elliptical cracks on the inner surface under axial force: *a*-dependence of ratio $\sigma_{cr}(a_\sigma;\theta)/\sigma_{cr}$ on ratio *a/s*, for three values of θ/π (processed according to [22]); b-variation of total deterioration $D(a_\sigma;\theta)$ dependent on *a/s* for three values of the ratio θ/π; c- variation of the deterioration component $D(\theta)$ dependent on ratio θ/π for four values of ratio *a/s*.

The behavior of the pipe material has been considered to be ideally-plastic, i.e. the maximum stress is the yield limit. Consequently, in this case $k = 1$ and $\alpha = 1$. From Figure 11, a, by using relations (17) and (18) we obtained the curves in Figure 11, b, and c for $D(a_\sigma;\theta)$ and $D(\theta)$.

**7. Conclusions**

The objective of our study has been to establish relationships for calculating deterioration or damage in tubular specimens. We have shown the specificities of deterioration/damage induced by cracks (axial, circumferential ...) in tube specimens as compared to crack in flat specimens.

There have been presented the general expressions of normal and shear critical stresses for flat plates under static uniaxial loads (4 and 7).

After examining the features of deteriorated tubular specimens there have been proposed: expressions for calculating deterioration caused by circumferential (9) and axial (10) cracks; expressions for the critical normal and shear stresses under static loads in tube specimens with axial and circumferential cracks (11;12).

The relationships were applied to the calculation of deteriorations $D(a_\sigma;c)$, $D(a_\sigma;\theta)$, $D(c)$ and $D(\theta)$ produced by the single loading with rectangular and semi-elliptical cross-section cracks on the inner surface of the tubular specimen.

It was found that the crack shape (semi-elliptical or rectangular) influences the values of the deterioration and critical stresses, an issue that should be taken into consideration in evaluating mechanical structures with cracks.

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