BACK ANALYSIS OF INTERNAL FORCES FOR TUNNEL SUPPORTING STRUCTURE WITH MULTI-CENTER CIRCULAR ARC

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In view of its good mechanical properties, multi-center circular arc is widely adopted in roadway or tunnel section design. Developments of the initial parameter method and large-scale application of field monitoring and measurement provide a new possibility to back analysis of internal forces for such problems. Based on the initial parameter method and field monitoring results of surrounding rock pressure, theory of circular beam on elastic foundation is employed to derive the analytical solution of supporting structure internal forces. Combining the monitored data of rock pressure in a tunnel, solution of lining structure internal force of a section is back analyzed, and safety assessment of the lining weak parts is implemented. Considering the geometric feature of tunnel section and the measurement point arrangement of surrounding rock pressure, the analytical solution for the supporting structure internal forces of multi-center circular arc tunnel proposed in this paper has a strong feasibility in roadway or tunnel construction. The conclusions obtained in this paper, could provide theoretical supports for the roadway or tunnel section design and the optimization of roadway or tunnel construction technology, and thus to provide theoretical references for the construction safety of roadway or tunnel.

Keywords: multi-center circular arc tunnel; supporting structure; internal force; back analysis

1. Introduction

Roadway or tunnel excavation construction is a dynamic changing process. During the roadway or tunnel construction, the mechanics characteristic of concrete supporting structures is a process of continuous development and change. How to analyze the internal forces distribution of tunnel supporting structures under complicated construction surroundings and preprocess the weak parts of supporting structures, is the key procedure to ensure the tunnel construction safety.

Mature applications of site monitoring and measurement methods in tunnel constructions [1-3], provide a new opportunity for back analysis theory and technology. Back analysis of geotechnical parameters by the optimization of a 3D numerical model of the hydroelectric powerhouse cavern was showed in reference

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[4]. A displacement based back-analysis method for the determination of rock mass modulus and the horizontal in situ stress was proposed by Zhang et al. [5]. Sakurai et al. [6] addressed a series of back analysis procedures in which the identification of strain distribution was sought as the primary goal in order to achieve a solid and reliable routine of observations and data interpretations. Displacement based direct back analysis using univariate optimization algorithm were employed by Ghorbani and Sharifzadeh to identify the geotechnical properties of rocks, stress ratio and joints parameters [7]. Based on comprehensive application of evolutionary support vector machines (SVMs), numerical analysis and genetic algorithm, Feng et al. [8] proposed a new back analysis method. Based on measurement data provided by the construction site, Pichler et al. [9] proposed one parameter identification (PI) method for determination of unknown model parameters in geotechnical engineering.

In recent years, some new elements were introduced in the field of tunnel constructions back analysis. Hisatake and Hieda proposed a three-dimensional back-analysis method, which consists of an ordinary 3D finite element method and the secant method [10], and the values for the modulus of elasticity, Poisson's ratio, and the six components of the initial stress levels of the ground are identified through the method. Caia et al. [11] developed a novel method to back-calculate rock mass strength parameters by using acoustic emission (AE) monitoring data in combination with FEM stress analysis. In order to reduce the uncertainties about the parameters evaluated by in situ and laboratory tests, Moreira et al. [12] proposed an Evolution Strategy algorithm for the back analysis of geotechnical parameters in underground structures. According to the theory of circular beam on elastic foundation, based on touch stresses between concrete layer and surrounding rock, Wen et al. [13] obtained the analytical solution of concrete internal force.

Initial parameter method has been applied widely since it was put forward by Haydl [14]. The method of initial parameters was initially extended and applied to the calculation of constant stiffness beam columns and the solution of elastic buckling problems of columns by Haydl [15-16]. Using a method of initial parameters, Orynyak and Radchenko derived an analytical solution for the differential equations for displacements and angles of rotation of the curved shell center line allowing for the action of distributed forces and moments [17]. Based on principle of initial parameter method, Sun et al. [18] established the initial parameter matrix equation of force and displacement of circular beam on elastic foundation, and then obtained the internal force and displacement of the multi-center circular arc tunnel lining. Orynyak et al. [19] proposed a modification of the method of initial parameters as employed to allow for harmonic vibrations of a fluid transported in piping. Multi-center circular arc shaped tunnel is always used in roadway tunnel [20-22], introduction of the initial parameter method could

make up shortage of the analytical solution of internal force and displacement in concentric arcs tunnel lining [13], and make the conclusion more suitable for the practice [18].

Based on the initial parameter method and field monitoring results of surrounding rock pressure, theory of circular beam on elastic foundation is employed to derive the analytical solution of supporting structure internal forces. Combining the monitored data of rock pressure in a tunnel, solution of lining structure internal force of a section is back analyzed, and safety assessment of the lining weak parts is implemented. Considering the geometric feature of tunnel section and the measurement point arrangement of surrounding rock pressure, the analytical solution for the supporting structure internal forces of multi-center circular arc tunnel proposed in this paper has a strong feasibility in tunnel construction. The conclusions obtained in this paper, could provide theoretical supports for the tunnel section design and the optimization of tunnel construction technology, and thus to provide theoretical references for the construction safety of tunnel.

2. Deformation differential equations and its solution

In tunnel construction, arch wall and horseshoe are usually selected as the tunnel cross section. Tunnel supporting structure especially the primary support structure is flexible and it has a close contact with the surrounding rock mass. Therefore, the arcing structures of tunnel supporting could be considered as circular beam on elastic foundation.

2.1 Governing equations

A segmental arc of the tunnel supporting structure with internal radius r_0 and external radius r_1 is considered in Fig. 1(a), the supporting structure thickness is $h = r_1 - r_0$. Here, we define the section area F, the geometrical moment of inertia I the center radius $r_i = (r_1 + r_0)/2$.
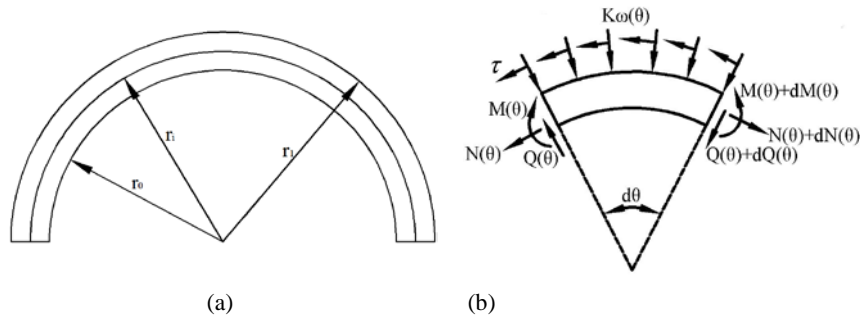

(a)   (b)

Fig. 1. Calculating diagram of ccircular beam on elastic foundation

(a) Tunnel supporting structure, (b) differential element

The differential element $r_i d\theta$ is shown in Fig. 1(b), where $\omega(\theta)$, K, $M(\theta)$, $Q(\theta)$, $N(\theta)$ are the radial displacement, rock resistant coefficient, bending moment, shear force and axial force, respectively, and the shear stress between the supporting structure and surrounding rock is τ.

Omitting second-order term, static equilibrium equations of the differential element can be expressed as

$$\mathrm{d}Q(\theta) + N(\theta)\mathrm{d}\theta + K\omega(\theta)r_1\mathrm{d}\theta = 0 \tag{1}$$

$$\mathrm{d}N(\theta) - Q(\theta)\mathrm{d}\theta - \tau r_1\,\mathrm{d}\theta = 0 \tag{2}$$

$$\mathrm{d}M(\theta) - r_i\mathrm{d}N(\theta) + \tau r_1^2\,\mathrm{d}\theta = 0 \tag{3}$$

The relation between the radial displacement and the section internal force of the circular beam on elastic foundation can be expressed as:

$$\frac{\mathrm{d}^2\omega(\theta)}{\mathrm{d}\theta^2} + \omega(\theta) = \frac{M(\theta)r_i^2}{E_c I} + \frac{N(\theta)r_i}{E_c F}. \tag{4}$$

By the simultaneous four Equations (1)~(4), the governing equation that the radial displacement $\omega(\theta)$ can be obtained and should be satisfied as follow:

$$\frac{\mathrm{d}^5\omega(\theta)}{\mathrm{d}\theta^5} + 2\frac{\mathrm{d}^3\omega(\theta)}{\mathrm{d}\theta^3} + l^2\frac{\mathrm{d}\omega(\theta)}{\mathrm{d}\theta} = \frac{(M_0 - N_0 r_i)r_i^2}{EI} - \frac{\tau r_i^2 r_1^2 \theta}{EI} \tag{5}$$

where, $l^2 = 1 + Kr_1\left(r_i^3/E_c I + r_i/E_c F\right)$, $M_0 = M(0)$, $N_0 = N(0)$.

2.2 Analysis of internal forces

The general solution of equation (5) can be expressed as [23]:

$$\omega(\theta) = e^{-\alpha\theta}(C_1\cos\beta\theta + C_2\sin\beta\theta) + e^{\alpha\theta}(C_3\cos\beta\theta + C_4\sin\beta\theta)$$
$$+ \frac{(M_0 - N_0 r_i)r_i^2}{EI m^2} - \frac{\tau r_i^2 r_1^2 \theta}{EI m^2} \tag{6}$$

where, $\alpha = \sqrt{\dfrac{l-1}{2}}$, $\beta = \sqrt{\dfrac{l+1}{2}}$, and C_1, C_2, C_3, C_4 are integral constant.

By the simultaneous four equations (1)~(4) and equation (6), the solutions of internal force of supporting structure can be obtained as follow [24]

$$Q(\theta) = \frac{a_M}{r_i}e^{-\alpha\theta}[(\beta C_2 - \alpha C_1)\sin\beta\theta + (\beta C_1 + \alpha C_2)\cos\beta\theta]$$
$$- \frac{a_M}{r_i}e^{-\alpha\theta}[(\alpha C_3 + \beta C_4)\sin\beta\theta + (\beta C_3 - \alpha C_4)\cos\beta\theta] + \tau r_i b_M - \tau r_1 \tag{7}$$

$$M(\theta) = a_M e^{-\alpha\theta}(C_1 \sin\beta\theta - C_2 \cos\beta\theta) - a_M e^{\alpha\theta}(C_3 \sin\beta\theta - C_4 \cos\beta\theta)$$
$$+ \tau r_i^2 \theta b_M - \tau r_1^2 \theta + (c_M + 1)(M_0 - N_0 r_i) \tag{8}$$

$$N(\theta) = \frac{a_M}{r_i} e^{-\alpha\theta}(C_1 \sin\beta\theta - C_2 \cos\beta\theta) - \frac{a_M}{r_i} e^{\alpha\theta}(C_3 \sin\beta\theta - C_4 \cos\beta\theta)$$
$$+ \tau r_i \theta b_M + \frac{c_M}{r_i}(M_0 - N_0 r_i) \tag{9}$$

The angular displacement $\phi(\theta)$ of circular beam can be expressed as

$$\phi(\theta) = \frac{1}{r} e^{-\alpha\theta}(-\alpha C_1 + \beta C_2)\cos\beta\theta - (\beta C_1 + \alpha C_2)\sin\beta\theta$$
$$+ \frac{1}{r} e^{\alpha\theta}(\alpha C_3 + \beta C_4)\cos\beta\theta + (-\beta C_3 + \alpha C_4)\sin\beta\theta - \tau d_M \tag{10}$$

where, $a_M = \sqrt{m^2 - 1}\Big/(1/EF + r_i^2/EI)$, $b_M = \dfrac{(m^2 - 1)r_1^2}{EIm^2}\Big/(1/EF + r_i^2/EI)$,

$c_M = \dfrac{(1 - m^2)r_i^2}{EIm^2}\Big/(1/EF + r_i^2/EI)$, $d_M = r_i r_1^2/EIm^2$.

3. Supporting structure internal force expressed by initial parameters

For Equations (6)~(10), setting θ equal to zero, the expressions of initial parameters can be obtained as follow:

$$\omega_0 = \omega(0),\ Q_0 = Q(0),\ M_0 = M(0),\ N_0 = N(0),\ \phi_0 = \phi(0) \tag{11}$$

By solving the system of equations (11), one can obtain the integration constants C1-C4, as functions of the initial parameters (ω_0, Q_0, M_0, N_0 and ϕ_0).

Substituting expressions of $C_1 \sim C_4$ into Equations(6)~(10), the initial parameters expressions of $\omega(\theta)$, $Q(\theta)$, $M(\theta)$, $N(\theta)$ and $\phi(\theta)$ can be obtained.

The matrix equation of radial displacement and internal forces of the tunnel supporting structure can be expressed by initial parameters as follow:

$$\boldsymbol{W}_\theta = \boldsymbol{T}_\theta \boldsymbol{W}_0 + \boldsymbol{P}_\theta \tag{12}$$

where, $\boldsymbol{W}_0$ is the initial parameters point of column matrix of radial displacement and internal forces, $W_0 = [\omega_0, Q_0, M_0, N_0, \phi_0]^{\mathrm{T}}$, $\boldsymbol{W}_\theta$ is the column matrix of radial displacement and internal forces, $W_\theta = [\omega_\theta, Q_\theta, M_\theta, N_\theta, \phi_\theta]^{\mathrm{T}}$; T_θ and P_θ are the homologous coefficient matrix and constant matrix, respectively.

4. Back analysis of internal forces

It is difficult to accurately measure the radial displacement of surrounding rock and supporting structure but the contact pressure is easy to obtain in tunnel construction process. The analytical solutions for the supporting structure internal forces of multi-center circular arc tunnel were proposed based on the initial parameter method and Winkler's assumption which is expressed by the contact pressure.

The tunnel section with multi-center circular arc is considered in Fig. 2. As shown in Fig. 2, left and right sides of the tunnel are symmetrical to the center line, points A and E are located at the axis of symmetry; B, C and D are the point of intersection of corresponding two circular arches. Benching tunneling method was adopted in the tunnel excavation process and $G'G$ line was defined as the joint line of the two steps.

We define that the load of tunnel surrounding rock was symmetrical about the center line and point A is the initial parameter point of supporting structure, $Q_A = Q(0) = 0$. And so its matrix equation of radial displacement and internal forces could be written as follow:

$$W_A = [\omega_A, 0, M_A, N_A, 0]^T \qquad (13)$$

The three initial parameters ω_1, M_A and N_A can be obtained based on the equation (13), and then the internal forces of the segmental arc AB and BG can be solved.
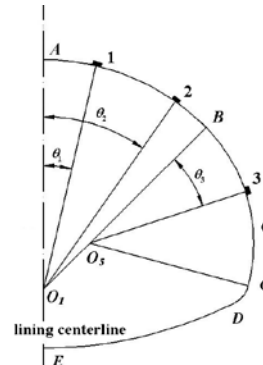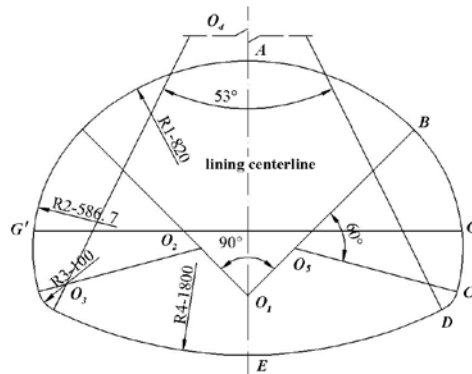

Fig. 2. Schematic diagram of lining section (unit: cm)   Fig. 3. Measured point sketch of surrounding rock pressure

Winkler's assumption mean that the point pressure of the circular beam on elastic foundation is proportional to the displacement of the foundation settlement, expressed as:

$$P(\theta) = K\omega(\theta) \qquad (14)$$

where, $P(\theta)$ is the radial pressure of the tunnel surrounding rock.

The solution of the three initial parameters of point A, ω_A, M_A and N_A in equation (14) is depended on the information of surrounding rock pressure. According to the construction environment, measuring points could be arranged arbitrarily in the arches of AB and BG. In Fig. 3, θ_1 and θ_2 are the central angle between the point 1 and point 2 to the initial parameter point A, respectively. θ_3 is the central angle between the point 1 and point 3 to the initial parameters constant of point B, respectively.

by coupling the equations (6) and (10):

$$P(\theta_1) = K_{AB}\omega_1 \qquad (15)$$

$$P(\theta_2) = K_{AB}\omega_2 \qquad (16)$$

$$P(\theta_3) = K_{BG}\omega_3 \qquad (17)$$

where, ω_1, ω_2 and ω_3 are the radial displacement of measuring points, K_{AB} and K_{BG} are the coefficient of elastic resistance of the surrounding rock.

Based on the Equation (12), ω_1 and ω_2 can be expressed by the initial parameters constant of point A because that they are located in the same arc, ω_3 could be also expressed by the initial parameters constant of point A through the second iteration of point B.

By coupling the equations (15)~(17), the initial parameters constant ω_A, M_A and N_A of point A were obtained. And then the radial displacement and the internal forces of surrounding rock each arc could be solved based on the solution of Equation (12) repeatedly while the point A was thought as the initial point.

5. Example and discussion

One tunnel section with multi-center circular arc is shown in Fig. 2 and the geometric parameters of lining and section parameters of tunnel are listed in Tables 1 and 2.

Table 1

Geometric parameters of lining		
segmental arc	r_0 (cm)	θ (rad)
AB	820	π/4
BG	586.7	π/5
BC	586.7	π/3

Table 2

Tunnel section parameters

parameters	value
h (cm)	50
τ (MPa)	0.2
F (cm^2)	5000
E_c (GPa)	30
K (MPa/m)	510

The radial direction contact pressure between the surrounding rock and primary support was measured using the TXR-2020 pressure meters. And the pressure values are shown in Table 3.

The constants ω_A, M_A and N_A were determined when the data of Table 3 were substituted into the formulas (15)~(17). By coupling the equations (6)~(9), the solutions of internal forces and radial displacements of supporting structure can be obtained.

Table 3

Contact stress of primary support and surrounding rock

measuring points number	θ (rad)	contact stress $P(\theta_i)$ (kPa)
1	0.08727	165.52
2	0.62832	46.66
3	0.52360	20.8

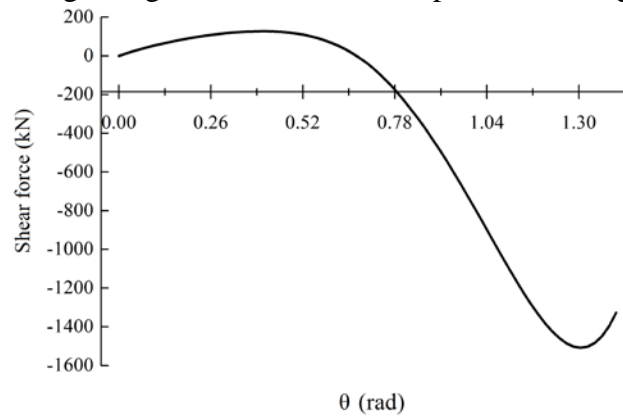The results regarding internal forces are represented in figures 4–6.
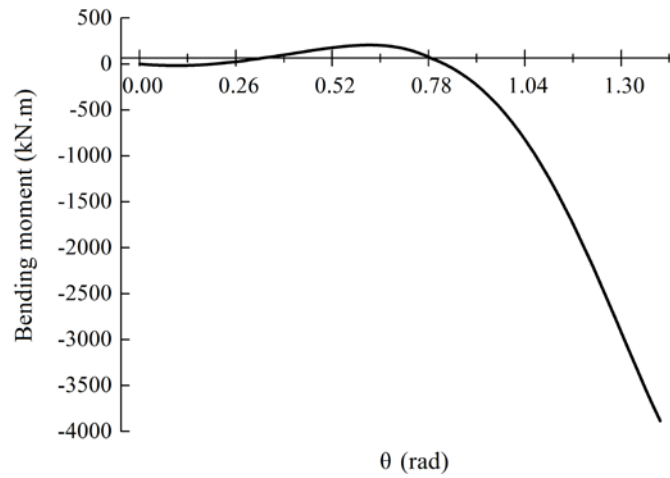

Fig. 4. Lining shear force

Fig. 5. Lining bending moment

According to the analysis results of Fig. 4 and Fig. 5, the shear force and bending moment are small in the AB segmental arc thus the load state is determined by axial and shear forces. The maximum shearing force is located at the middle of AB arc with the value of 127.27kN. The maximum bending moment is close to the point B and the value is 206.65kN·m.

To the other segmental arc BG, shearing force and bending moment are negative and the values shows an increase tendency from point B to point G. Shearing force and bending moment values of point G are 1.33MN and 3.88MN·m, respectively.
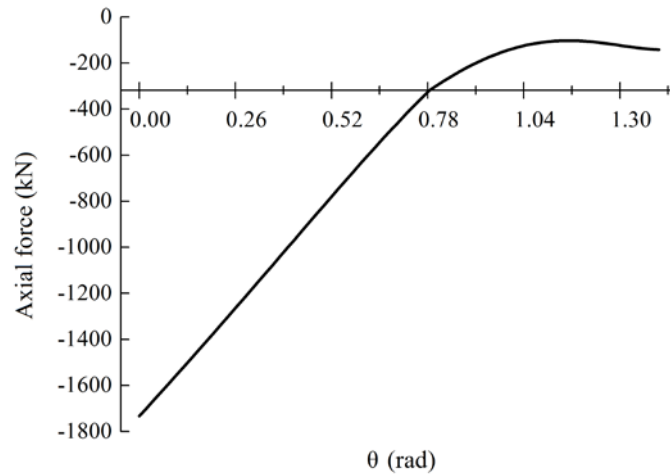

Fig. 6. Lining axial force

According to the analysis results of fig.6, segmental arc BG and AB are both in axial compression state and the axial force indicate a decrease tendency from point A to point G, the maximum axial force is located in point A and the value is 1732.25kN.
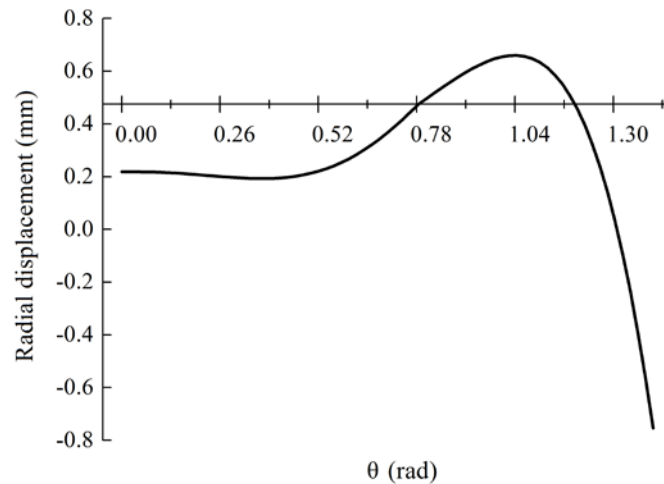
Fig. 7. Lining radial displacement

The radial displacements variation for AB and BG arches is shown in Fig. 7. The maximum value of 0.66 cm is located in segmental arc BG. Displacement toward the surrounding rock occurs on arc BG between points 3 and G, having the maximum value.

The internal forces were solved to evaluate the security of supporting structure. The maximum axial stress of the lining is located at point A and the value is 3.46MPa, and it is far less than the axial compressive strength of C30 concrete (15MPa).

The maximum shearing stress is located at point 3 and the value is 3.02MPa. The maximum bending stress is located at point G and the value is 182.4MPa, which is higher than the design value. Because of the space truss-concrete of rebar and the surrounding rock, bending strength of the supporting structure concrete is enhanced. Therefore, supporting structure of the tunnel wouldn't be not damaged during the excavation process.

6. Conclusions

Based on the initial parameter method and field monitoring results of surrounding rock pressure, theory of circular beam on elastic foundation is employed to derive the analytical solution of supporting structure internal forces. Combining the monitored data of rock pressure, solution of lining structure internal force of a section is back analyzed, and safety assessment of the lining weak parts is implemented. Analysis results indicate that, bending stress and shear stress of the lining at BG segmental arc are higher, indicating that rigorous monitoring and measurement works in the tunnel construction should be applied.

Considering the geometric feature of tunnel section and the measurement point arrangement of surrounding rock pressure, the analytical solution for the

supporting structure internal forces of multi-center circular arc tunnel proposed in this paper has a strong feasibility in tunnel construction. The conclusions obtained in this paper, could provide theoretical supports for the tunnel section design and the optimization of tunnel construction technology, and thus to provide theoretical references for the construction safety of tunnel.

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