DISTRIBUTED EVENT-TRIGGERED CONSENSUS CONTROL FOR MULTI-AGENT SYSTEMS

Fang QIU[1]

This paper studies the consensus problem for a class of multi-agent systems with single-integrator. The controller updating is event-driven and depend on the measurement error with the norm of state function and neighboring agent's state. It is shown that the controller updating for each agent only triggers at its own event time instants. Then, one distributed event-triggered control scheme is derived, which can reduce the frequency of the controller updating and the communication load. The proposed control protocols guarantee convergence to a ball centered at the average consensus, and events cannot exhibit Zeno behavior. Finally, a simulation example is demonstrated to verify the theoretical result.

Keywords: Multi-agent Systems, Event-triggered Control, Distributed Control, Consensus.

1. Introduction

Multi-agent systems have been used to solve a variety of problems efficiently, such as flocking, distributed sensor networks, unmanned aerial vehicle formation [1]–[5]. The consensus is one of the most fundamental research topic in multi-agent control community. In the recent decades, many excellent consensus methods have been reported, such as continuous control [6, 7] and periodic sampling control [8-10]. The consensus problem has been investigated extensively in the literature [6-31] and many references therein.

Considerable efforts have been made to achieve various kinds of cooperative behaviors by designing appropriate time-driven control protocols [11-13]. However, this mechanism requires high communication cost and expensive control action. In order to lower the cost, in recent years, the event-triggered mechanism has gradually caused extensive concern due to its advantage of saving unnecessary usage of energy [14]–[30]. The basic idea of event-triggered control is that when a certain control performance is still satisfactory then the execution of control tasks can be skipped and the transmission of the measured outputs or actuator signals can be cancelled. Following the thought of event-triggered control, in [18], a novel control strategy for multi-agent coordination with event-based broadcasting has been presented, which the trigger condition has a state-independent and exponentially decreasing threshold. In

---
[1] College of Science, Binzhou University, Shandong 256603, P. R. China
E-mail : rgbayqf@163.com

[19], a simple event-triggered mechanism is proposed to stabilize control tasks, which leads to guaranteed performance and relaxes traditional periodic execution. In [20], a distributed event-triggered feedback scheme is introduced to NCS that can be applied to linear or nonlinear system. In [21, 22], the event-triggered average consensus problem for linear system was investigated, where a state-independent triggering condition is utilized for decrease network communication. In [23-25], the authors studied event-based control strategies for multi-agent systems, where control updating depend on trigger condition which has a state-dependent threshold. In [26], the event-triggered consensus problem of distributed multi-agent systems has been studied by using state-dependent trigger function. However, the proof of the inequality (15) in [26] is incorrect. Event-triggered approach is also applied to stochastic systems [27, 28] and deals with state estimation [29]. Then, the control is driven by requiring state-dependent threshold of triggering condition for multi-agent systems.

In this paper, the control updating will be designed based on the event-triggered scheme for distributed multi-agent systems. The control updating of each agent are driven by properly defined events depending on the measurement error and the states of its neighboring agents. The primary contribution is that the sufficient condition requires reduced control efforts with continuous communication between neighboring agents. Using this strategy, an analytical method is proposed to guarantee the consensus for multi-agent systems and the exclusion of Zeno behavior.

The rest of this paper is organized as follows. Section 2 gives some basic theory on graphs and matrices. A novel event-triggered scheme is introduced in Section 3 to guarantee the exponential consensus. In Section 4, the simulation result is presented to illustrate the analysis results. Finally, Section 5 discusses conclusions.

2. Problem Statement

2.1 Basic Theory on Graphs and Matrices

An undirected graph $G = (V, E)$ consists of a node set $V = \{1, 2, \cdots, N\}$ representing N agents, and a set of edges $E \in V \times V$ corresponding to the communication links between agents. The adjacency matrix $A = \left(a_{ij}\right)$ is defined by $a_{ij} = 1$ if and only if there is an edge $(i, j) \in E$ in G; otherwise, $a_{ij} = 0$. In undirected graph, one has $(i, j) \in E \Leftrightarrow (j, i) \in E$. Then the adjacency matrix A is a symmetric matrix. If there is a path between any two nodes in an undirected graph G, G is called connected. The set $N_i = \left\{ j \in V : (i, j) \in E \right\}$ includes all neighbors of node i. Then, the degree D is a diagonal matrix $diag\left\{ d_1, d_2, \cdots, d_n \right\}$ in G, where d_i is the cardinality of agent $i's$ neighbor set N_i. The Laplacian matrix $L = (l_{ij})$ for the undirected graph G is

defined as $L = D - A$. For undirected graphs, the Laplacian matrix L is symmetric and positive semi-definite, that is, $L = L^T > 0$. Then, its eigenvalues are real and it have exactly one zero eigenvalue which can be ordered as $0 = \lambda_1(G) \le \lambda_2(G) \le \cdots \le \lambda_N(G)$, where $\lambda_2(G)$ is the smallest nonzero eigenvalue. The vector $\mathbf{1}$, with all entries equal to 1, is an eigenvector of L associated with eigenvalue 0, i.e. $L\mathbf{1} = \mathbf{0}$.

The following definition and lemma will be very useful in the remainder of this paper.

Definition 1. A hybrid system H is Zeno, if for some execution ε of H, there exists a finite constant τ_∞ such that $\lim\limits_{i \to \infty} \tau_i = \sum\limits_{i=0}^{\infty} (\tau_{i+1} - \tau_i) = \tau_\infty$. The execution ε is called a Zeno execution.

Remark 1. The definition of a Zeno execution results in the types of Zeno behavior. For an execution ε that is Zeno, ε is Chattering Zeno: If there exists a finite C such that $\tau_{i+1} - \tau_i > \mathbf{0}$ for all $i > C$.

Lemma 1([30]). Suppose L is the Laplacian matrix of an undirected and connected graph G. Then, for all $t \ge 0$ and all vectors $v \in R^n$, the following holds true
$$\left\| e^{-Lt}v \right\| \le -e^{-\lambda_2(G)t} \left\| v \right\|.$$

2.2 Problem formulation

The dynamics associated with each agent $i \in V = \{1, 2, \cdots, N\}$ is described by the following equation:
$$\dot{x}_i(t) = u_i(t), \quad i \in V, \tag{1}$$

where $x_i(t) \in R$ is the state and $u_i(t) \in R$ is the control input of the ith agent. With the stack vectors $x(t) = (x_1(t), x_2(t), \cdots, x_N(t))^T$ and $u(t) = (u_1(t), u_2(t), \cdots, u_N(t))^T$, the multi-agent system (1) can be written as $\dot{x}(t) = u(t)$, $x(0) = x_0 \in R^N$.

It is well known that the continuous distributed control law
$$u_i(t) = - \sum_{j \in N_i} (x_i(t) - x_j(t)), \tag{2}$$

drives each agent of the system to asymptotically converge to the average of the agents' initial conditions, i.e., $x(t) \to \dfrac{1}{N} \sum\limits_{i \in V} x_i(0)$, as $t \to \infty$. The closed-loop system can be written as $\dot{x}(t) = Lx(t)$. Based on local information, agent i does not use its true state $x_i(t)$ but the last broadcast value $\hat{x}_i(t)$ given by $\hat{x}_i(t) = x_i(t_k^i), t \in [t_k^i, t_{k+1}^i)$, and

$t_0^i, t_1^i, \cdots$ is the sequence of event times of agent i. The next event-triggering instant t_k^i is defined by the trigger function $f_i\left(t, e_i(t), x_i(t)\right) > 0$, that is,

$$t_{k+1}^i = \inf\left\{t : t > t_k^i, f_i\left(t, e_i(t), x_i(t)\right) > 0\right\}, k = 0, 1, \cdots, \infty. \qquad (3)$$

For each $i \in V$ and $t \geq 0$, the measurement error is defined as

$$e_i(t) = \hat{x}_i(t) - x_i(t). \qquad (4)$$

Then, this can be expressed by

$$\dot{x}(t) = -L\hat{x}(t) = -L(x(t) + e(t)) \qquad (5)$$

with $e(t) = \left(e_1(t), e_2(t), \cdots, e_N(t)\right)^T$.

Let $a(t) = \dfrac{1}{N} \sum_{i=1}^{N} x_i(t)$ be the average of all states. Then, the derivative of $a(t)$ is

$$\dot{a}(t) = \frac{1}{N} \sum_{i=1}^{N} \dot{x}_i(t) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}^T \dot{x}_i(t) = -\frac{1}{N} \sum_{i=1}^{N} \mathbf{1}^T L\dot{x}_i(t) = 0.$$

Thus, $a(t) = a(0) = a$ for all $t \geq 0$ and the state $x(t)$ can be decomposed

$$x(t) = a\mathbf{1} + \delta(t), \qquad (6)$$

where $\delta(t)$ is the disagreement vector of the multi-agent system, that is, $\mathbf{1}^T \delta(t) = \mathbf{0}$.

From (6), we can get $\delta(t) = x(t) - a\mathbf{1}$ that

$$\dot{\delta}(t) = \dot{x}(t) = -L(x(t) + e(t)) = -Le(t) - L\delta(t). \qquad (7)$$

Then, it follows that

$$\delta(t) = e^{-Lt}\delta(0) - \int_0^t Le(\tau)e^{-L(t-\tau)}d\tau, \qquad (8)$$

where $\delta(0)$ is the initial value of the disagreement vector.

3. Consensus Analysis

Next, a sufficient condition on the consensus is presented with triggering condition where continuous communications between neighboring agents are required.

Theorem 1. Consider the multi-agent systems (1) with control strategy (2) and the event-triggering instants determined by (3). Suppose the triggering function is designed as

$$f_i(t, e_i(t), x_i(t)) = |e_i(t)| - c_0 \sqrt{\sum_{j \in N_i} |x_j(t) - x_i(t)|^2} \tag{9}$$

with $c_0 \in \left(0, \dfrac{\lambda_2(G)}{\sqrt{2}(\sqrt{\|L\|})^3}\right)$. Then, for all initial conditions $x(0) \in R^N$, the disagreement vector $\delta(t)$ of the closed-loop system will achieve exponential convergence to a ball centered at the rendezvous with radius Δ , where $\Delta = \dfrac{\sqrt{2}c_0 \left(\sqrt{\|L\|}\right)^3 \sqrt{N}|a|}{\lambda_2(G) - \sqrt{2}c_0 \left(\sqrt{\|L\|}\right)^3}$. Furthermore, the closed-loop system does not exhibit Zeno behavior.

Proof. From the triggering condition (9), the upper bound of measurement error $e(t)$ is given by

$$\|e(t)\| = \sqrt{\sum_{i=1}^{N} |e_i(t)|^2} \leq c_0 \sqrt{\sum_{i=1}^{N} \sum_{j \in N_i} |x_j(t) - x_i(t)|^2}$$
$$= c_0 \sqrt{2x^T(t)Lx(t)} \leq \sqrt{2}c_0 \sqrt{\|L\|}\|x(t)\|$$
$$\leq \sqrt{2}c_0 \sqrt{\|L\|}\left(\|\delta(t)\| + \sqrt{N}|a|\right) . \tag{10}$$

It follows from Eq. (8) and Lemma 1 that the disagreement vector is bounded by

$$\|\delta(t)\| = \left\|e^{-Lt}\delta(0)\right\| + \int_0^t \left\|e^{-L(t-\tau)}Le(\tau)\right\|d\tau$$
$$\leq e^{-\lambda_2(G)t}\|\delta(0)\| + \int_0^t e^{-\lambda_2(G)(t-\tau)}\|Le(\tau)\|d\tau$$
$$\leq e^{-\lambda_2(G)t}\|\delta(0)\| + \sqrt{2}c_0(\sqrt{\|L\|})^3 \int_0^t e^{-\lambda_2(G)(t-\tau)}\left(\|\delta(\tau)\| + \sqrt{N}|a|\right)d\tau$$
$$\leq e^{-\lambda_2(G)t}\|\delta(0)\| + \sqrt{2}c_0(\sqrt{\|L\|})^3 \int_0^t e^{-\lambda_2(G)(t-\tau)}\|\delta(\tau)\|d\tau$$
$$+ \sqrt{2}c_0(\sqrt{\|L\|})^3 \sqrt{N}|a| \int_0^t e^{-\lambda_2(G)(t-\tau)}d\tau . \tag{11}$$

Let $\sqrt{2}c_0 \left(\sqrt{\|L\|}\right)^3 = A, \sqrt{2}c_0 \left(\sqrt{\|L\|}\right)^3 \sqrt{N}|a| = B$. Then, from (11), we can derive

$$\|\delta(t)\| \leq e^{-\lambda_2(G)t}\|\delta(0)\| + A\int_0^t e^{-\lambda_2(G)(t-\tau)}\|\delta(\tau)\|d\tau + B\int_0^t e^{-\lambda_2(G)(t-\tau)}d\tau . \tag{12}$$

Since $c_0 \in \left(0, \dfrac{\lambda_2(G)}{\sqrt{2}\left(\sqrt{\|L\|}\right)^3}\right)$, we have $\dfrac{A}{\lambda_2(G)} < 1$. Therefore, there is a positive

constant $\rho \in (0, \lambda_2(G))$ such that $\dfrac{A}{\lambda_2(G) - \rho} < 1$. Hence, we can claim that

$$\|\delta(t)\| \le \|\delta(0)\| e^{-\rho t} + \Delta, \tag{13}$$

where $\Delta = \dfrac{\sqrt{2} c_0 \left(\sqrt{\|L\|}\right)^3 \sqrt{N}\, |a|}{\lambda_2(G) - \sqrt{2} c_0 \left(\sqrt{\|L\|}\right)^3}$.

In order to prove (13), for any $\eta > 1$ and $t > 0$, we first show that the following inequality is true

$$\|\delta(t)\| < \eta \|\delta(0)\| e^{-\rho t} + \Delta = v(t). \tag{14}$$

Otherwise, there will be a $t^* > 0$ such that $\|\delta(t^*)\| = v(t^*)$ and $\|\delta(t)\| < v(t)$ for $t \in (0, t^*)$. From (12), we have

$$
\begin{aligned}
v(t^*) = \|\delta(t^*)\| &\le e^{-\lambda_2(G)t^*}\|\delta(0)\| + A\int_0^{t^*}\|\delta(\tau)\|e^{-\lambda_2(G)(t^*-\tau)}d\tau + B\int_0^{t^*}e^{-\lambda_2(G)(t^*-\tau)}d\tau \\
&= e^{-\lambda_2(G)t^*}\|\delta(0)\| + A\int_0^{t^*}\left(\eta\|\delta(0)\|e^{-\rho\tau}+\Delta\right)e^{-\lambda_2(G)(t^*-\tau)}d\tau + \frac{B}{\lambda_2(G)} - \frac{B}{\lambda_2(G)}e^{-\lambda_2(G)t^*} \\
&= e^{-\lambda_2(G)t^*}\|\delta(0)\| + \frac{A\eta\|\delta(0)\|}{\lambda_2(G)-\rho}e^{-\lambda_2(G)t^*}\left(e^{(\lambda_2(G)-\rho)t^*}-1\right) \\
&\quad + \left(\frac{A\Delta}{\lambda_2(G)} + \frac{B}{\lambda_2(G)}\right)\left(1 - e^{-\lambda_2(G)t^*}\right) \\
&< \eta\|\delta(0)\|\left(e^{-\lambda_2(G)t^*} + \frac{A}{\lambda_2(G)-\rho}\left(e^{-\rho t^*}-e^{-\lambda_2(G)t^*}\right)\right) \\
&\quad + \left(\frac{A\Delta}{\lambda_2(G)} + \frac{B}{\lambda_2(G)}\right)\left(1 - e^{-\lambda_2(G)t^*}\right) \\
&< \eta\|\delta(0)\|e^{-\rho t^*} + \frac{A\Delta + B}{\lambda_2(G)} \\
&< \eta\|\delta(0)\|e^{-\rho t^*} + \Delta = v(t^*).
\end{aligned}
\tag{15}
$$

Obviously, this is a contradiction which implies the inequality (14) holds for any $\eta > 1$. Then, the inequality (13) holds by denoting $\eta \to 1$. Since $c_0 \in \left(0, \dfrac{\lambda_2(G)}{\sqrt{2}(\sqrt{\|L\|})^3} \right)$, we get $\lambda_2(G) - \sqrt{2}(\sqrt{\|L\|})^3 > 0$. Then, it follows that $\Delta > 0$. Consequently, $\|\delta(t)\|$ converges exponentially to a ball with radius Δ as $t \to \infty$. Then, the state of multi-agent systems can converge exponentially to a ball centered at the average consensus.

In order to exclude Zeno behavior, we need to show that the inter-event times are lower-bounded by a strictly positive time τ. For undirected and connected graph G, Laplacian matrix L is symmetric and positive semi-definite. Then, there exists a matrix P such that $L = P^T P$ [32]. Thus, we can derive

$$
\begin{aligned}
\frac{d\left(\dfrac{\|e(t)\|}{\|Px(t)\|} \right)}{dt} &= \frac{\dfrac{2e^T(t)\dot{e}(t)}{\|e(t)\|}\|Px(t)\| - \|e(t)\|\dfrac{2(Px(t))^T P\dot{x}(t)}{2\|Px(t)\|}}{\|Px(t)\|^2} \\
&= \frac{e^T(t)\dot{e}(t)\|Px(t)\|}{\|Px(t)\|^2\|e(t)\|} - \frac{\|e(t)\|(Px(t))^T P\dot{x}(t)}{\|Px(t)\|^3} \\
&= \frac{-e^T(t)\dot{x}(t)}{\|Px(t)\|\|e(t)\|} - \frac{\|e(t)\|(Px(t))^T P\dot{x}(t)}{\|Px(t)\|^3} \\
&\leq \frac{\|e(t)\|\|\dot{x}(t)\|}{\|Px(t)\|\|e(t)\|} + \frac{\|e(t)\|\|Px(t)\|\|P\|\|\dot{x}(t)\|}{\|Px(t)\|^3} \\
&= \left(1 + \frac{\|e(t)\|\|P\|}{\|Px(t)\|} \right)\frac{\|\dot{x}(t)\|}{\|Px(t)\|} = \left(1 + \frac{\|e(t)\|\|P\|}{\|Px(t)\|} \right)\frac{\|L(x(t)+e(t))\|}{\|Px(t)\|} \\
&= \left(1 + \frac{\|e(t)\|\|P\|}{\|Px(t)\|} \right)\frac{\|P^T P(x(t)+e(t))\|}{\|Px(t)\|} \\
&= \|P\|\left(1 + \|P\|\frac{\|e(t)\|}{\|Px(t)\|} \right)^2.
\end{aligned}
\tag{16}
$$

Let $y(t) = \dfrac{\|e(t)\|}{\|Px(t)\|}$. Then, we have $\dot{y}(t) \leq \|P\|\left(1 + \|P\|y(t)\right)^2$ so that $y(t)$ satisfies the bound $y(t) \leq \phi(t, \phi_0)$, where $\phi_0 = \phi(0, \phi_0)$ and $\phi(t, \phi_0)$ is the solution of

$$\dot{\phi}(t,\phi_0) = \|P\|\left(1+\|P\|\,y(t)\right)^2 \qquad (17)$$

Hence the inter-event times $t_{i+1}-t_i$ are bounded from below by the time τ that satisfies $\phi(\tau,0) = \dfrac{\tau\|P\|}{1-\tau\|P\|^2}$. On the other hand, the event is triggered as soon as (9) crosses zero. Therefore, one has

$$\|e(t)\| = c_0\sqrt{2x^T(t)Lx(t)} = \sqrt{2}c_0\sqrt{(Px(t))^T Px(t)} = \sqrt{2}c_0\|Px(t)\| \qquad (18)$$

Then, the triggering condition can be rewritten as

$$y(t) = \frac{\|e(t)\|}{\|Px(t)\|} \le \sqrt{2}c_0 \qquad (19)$$

Thus, one can obtain $\phi(\tau,0) = \sqrt{2}c_0$. So, from $\dfrac{\tau\|P\|}{1-\tau\|P\|^2} = \sqrt{2}c_0$, we can get a

lower bound on the inter-event times $\tau = \dfrac{\sqrt{2}c_0}{\|P\|+\sqrt{2}c_0\|P\|^2}$, which implies that agent i

will not exhibit Zeno-behavior. The proof of Theorem 1 is thus completed.

Remark 1. Theorem 1 has proposed novel event-triggered communication and control strategies for the multi-agent average consensus problem. In [21], the consensus problem of a class of multi-agent systems with single-integrator is investigated. But the proof of the inequality (15) in [21] is incorrect which can be seen in the following

$$\left(\lambda_2(G)-c_0\left(\sqrt{\|L\|}\right)^3-\|L\|\right)\int_0^t e^{-\lambda_2(G)(t-\tau)}\|\delta(\tau)\|d\tau$$

$$\le \frac{\lambda_2(G)\|\delta(t)\|-\left(\lambda_2(G)\|\delta(0)\|+c_0\left(\sqrt{\|L\|}\right)^3\sqrt{N}\,|a|\right)e^{-\lambda_2(G)}+c_0\left(\sqrt{\|L\|}\right)^3\sqrt{N}\,|a|}{\lambda_2(G)}.$$

Obviously, $\lambda_2(G)-c_0\left(\sqrt{\|L\|}\right)^3-\|L\| < 0$ implies

$$\int_0^t e^{-\lambda_2(G)(t-\tau)}\|\delta(\tau)\|d\tau$$

$$\ge \frac{\lambda_2(G)\|\delta(t)\|-\left(\lambda_2(G)\|\delta(0)\|+c_0\left(\sqrt{\|L\|}\right)^3\sqrt{N}\,|a|\right)e^{-\lambda_2(G)}+c_0\left(\sqrt{\|L\|}\right)^3\sqrt{N}\,|a|}{\lambda_2(G)\left(\lambda_2(G)-c_0\left(\sqrt{\|L\|}\right)^3-\|L\|\right)}.$$

Then, the inequality (15) in [21] is incorrect which results in the following conclusion to be wrong. In this paper, this error has been corrected and the ideal result is obtained to guarantee the consensus for multi-agent systems.

4. Simulation Example

In this section, a simulation example is provided to illustrate the effectiveness of theoretical result.

Example. Consider a network of four agents whose Laplacian matrix is given by

$$L = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix} \qquad (20)$$

According to the control strategy proposed in Theorem 1, we give $c_0 = 0.08$ satisfying $c_0 \in (0, 0.0884)$. By using controller (2) and the triggering function (9), the four agents will achieve consensus which can be shown in Fig.1.
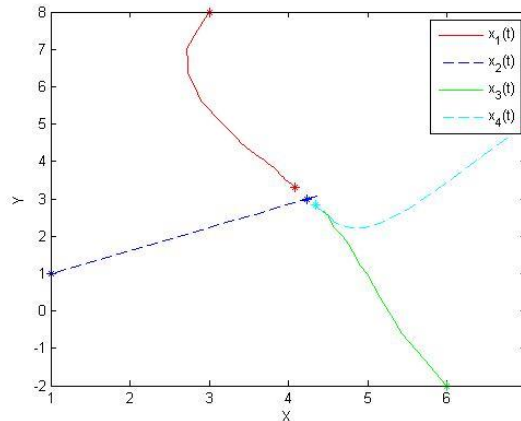

Fig.1 State trajectories of all agents with event-triggered control

Fig.2 shows the evolution of the error norm of each agent which embodies the distributed event-triggered scheme. When the error norm will reach the threshold $c_0 \sqrt{\sum_{j \in N_i} |x_j(t) - x_i(t)|^2}$, the error norm of each agent is reset to 0.
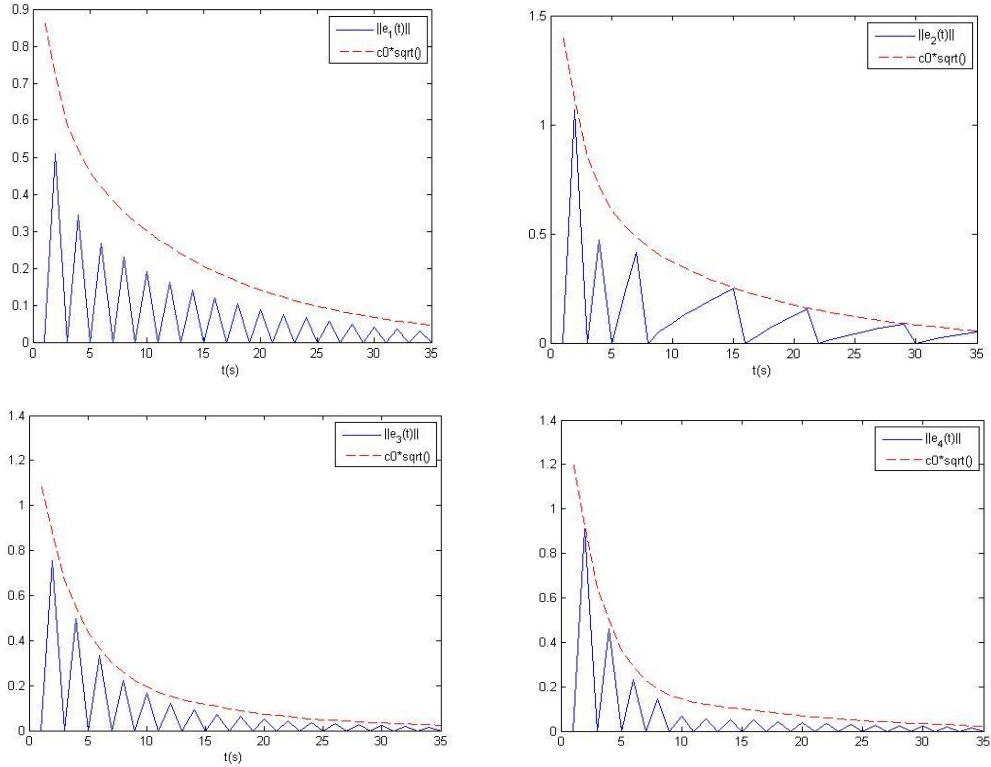
Fig.2 Evolution of the error norm of agent 1-agent 4
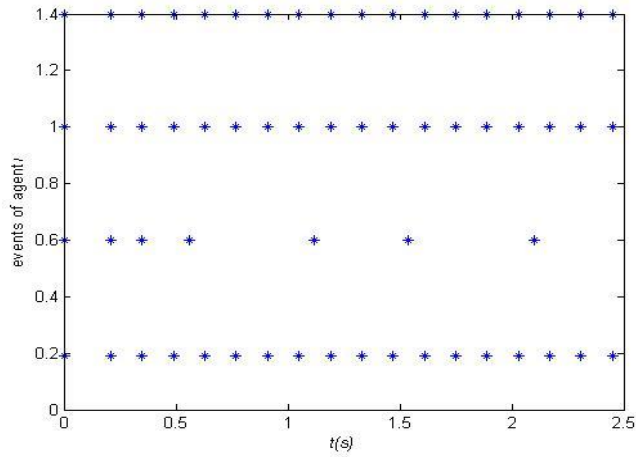

Fig.3 Event time of the four agents with $c_0 = 0.08$.

Fig.3 shows the event-triggered time instant for each agent during [0, 2.5]. The distributed event-triggered scheme reduces the frequency of information transmission and control updating.

5. Conclusion

This paper studies the event-based consensus of distributed multi-agent systems by using the algebraic graph theory and matrix theory. One sufficient condition has been presented to guarantee the consensus under the trigger function. The trigger condition can reduce the number of controller updating with continuous communication between neighboring agents. Finally, the simulation result is presented to support the effectiveness of the theoretical result.

Acknowledgments

R E F E R E N C E S

[1]. *R. Olfati-Saber, R. M. Murray.* Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. Autom. Control, 2004, 49: 1520–1533.

[2]. *Y. Tang, H. Gao, W. Zou, J. Kurths.* Distributed synchronization in networks of agent systems with nonlinearities and random switchings, IEEE Trans. Cybern., 2013, 43 (1): 358–370.

[3]. *Y. Tang, Z. Wang, H. Gao, H. Qiao, J. Kurths.* On controllability of neuronal networks with constraints on the average of control gains, IEEE Trans. Cybern., 2014, 44 (12): 2670–2681.

[4]. *J. Lu, Z. Wang, J. Cao, D. W. C. Ho, J. Kurths,* Pinning impulsive stabilization of nonlinear dynamical networks with time-varying delay, Int. J. Bifurcat. Chaos, 2012, 22 (7):1250176.

[5]. *W. Xu, J. Cao, M. Xiao, D. W. C. Ho, G. Wen.* A new framework for analysis on stability and bifurcation in a class of neural networks with discrete and distributed delays, IEEE Trans. Cybern., 2015, 45 (10): 2224–2236.

[6]. *N. Huang, Z. Duan, Y. Zhao.* Consensus of multi-agent systems via delayed and intermittent communications. IET Control Theory Appl., 2015, 9: 62–73.

[7]. *L. Z. Ren, W, Lin, X. Fu.* Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols. IEEE Trans. Autom. Control, 2013, 58: 1786–1791.

[8]. *Y. Gao, L. Wang, G. Xie, B. Wu.* Consensus of multi-agent systems based on sampled-data control. Int. J. Control, 2009, 82: 2193–2205.

[9]. *Z. Tang, T. Huang, J. Shao.* Leader-following consensus for multi-agent systems via sampled-data control. IET Control Theory Appl., 2011, 5: 1658–1665.

[10]. *Y. Cao, W. Ren.* Sampled-data discrete-time coordination algorithms for double-integrator dynamics under dynamic directed interaction. Int. J. Control, 2010, 83: 506-515.

[11]. *W. Xu, J. Cao, W. Yu, J. Lu.* Leader-following consensus of nonlinear multi-agent systems with jointly connected topology. IET Control Theory Appl., 2014, 8 (6): 432–440.

[12]. *S. Yang, J. Cao, J. Lu.* A new protocol for finite-time consensus of detail-balanced multi-agent networks. Chaos, 2012, 22 (4): 043134.

[13]. *S. Yang, Q. Liu, J.Wang.* Distributed optimization based on a multiagent system in the presence of communication delays. IEEE Trans. Syst. Man Cybern., Syst., to be published, doi: 10.1109/TSMC.2016.2531649.

[14]. *X. Meng, T. Chen.* Event based agreement protocols for multi-agent networks. Automatica, 2013, 49 (7): 2125–2132.

[15]. *H. Zhang, G. Feng, H. Yan, Q. Chen.* Observer-based output feedback event-triggered control for consensus of multi-agent systems. IEEE Trans. Ind. Electron, 2014, 61 (9): 4885-4894.

[16]. *W. Xu, D. W. C. Ho, L. Li, J. Cao.* Event-triggered schemes on leader-following consensus of general linear multiagent systems under different topologies. IEEE Trans. Cybern., 2017, 47(1):212–223.

[17]. *W. P. M. H. Heemels, M. C. F. Donkers.* Model-based periodic event-triggered control for linear systems. Automatica, 2013, 49 (3): 698–711.

[18]. *G. S. Seyboth, D. V. Dimarogonas, K. H. Johansson.* Event-based broadcasting for multi-agent average consensus. Automatica, 2013, 49 (1): 245–252.

[19]. *P. Tabuada.* Event-triggered real-time scheduling of stabilizing control tasks. IEEE Trans. Autom. Control, 2007, 52 (9): 1680–1685.

[20]. *X. Wang, M. D. Lemmon.* Event-triggering in distributed networked control systems. IEEE Trans. Autom. Control, 2011, 56: 586–601.

[21]. *J. Zhang, G. Feng.* Event-driven observer-based output feedback control for linear systems. Automatica, 2014, 50: 1852–1859.

[22]. *D. V. Dimarogonas, E. Frazzoli, K. H. Johansson.* Distributed event triggered control for multi-agent systems. IEEE Trans. Autom. Control, 2012, 57: 1291–1297.

[23]. *E. Garcia, Y. Cao, H. Yu, P.* Antsaklis, D. Casbeer. Decentralised event-triggered cooperative control with limited communication. Int. J. Control, 2013, 86: 1479–1488.

[24]. *W. Zhu, Z. Jiang, G. Feng.* Event-based consensus of multi-agent systems with general linear models. Automatica, 2014, 50: 552–558.

[25]. *Y. Fan, L. Liu, G. Feng, Y. Wang.* Self-triggered consensus for multi-agent systems with Zeno-free triggers. IEEE Trans. Autom. Control, 2015 (60): 2779-2784.

[26]. *H. Zhang, R. H. Yang, H. C. Yan, Q. J. Chen.* Distributed event-triggered control for consensus of multi-agent systems. J. Frankl Inst., 2015, 352: 3476–3488.

[27]. *B. Wang, X. Meng, and T. Chen,* "Event based pulse-modulated control of linear stochastic systems. IEEE Trans. Autom. Control, 2014, 59 (8): 2144–2150.

[28]. *L. Zou, Z. Wang, H. Gao, X. Liu.* Event-triggered state estimation for complex networks with mixed time delays via sampled data information: The continuous-time case. IEEE Trans. Cybern., 2015, 45 (12): 2804–2815.

[29]. *H. Dong, Z. Wang, F. E. Alsaadi, B. Ahmad.* Event-triggered robust distributed state estimation for sensor networks with state-dependent noises. Int. J. Gen. Syst., 2014, 44 (2): 254–266.

[30]. *G. Seyboth, D. V. Dimarogonas, K. H. Johansson.* Control of multi-agent systems via event-based communication. In Proc. 18th IFAC World Congress, Milan, Italy, 2011, pp. 10086–10091.

[31]. *W. W. Yu, G. H. Wen, G. R. Chen, J. D. Cao.* Distributed cooperative control of multi-agent systems. Wiley, 2016

[32]. *H. G. Werner. Linear Algebra.* Springer-Verlag, NewYork, 1981.