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MODELLING OF AN IMPROVED METHOD OF PHASE DETECTION SCHEME FOR DISPLACEMENT OPTIC SENSORS

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În această lucrare este prezentată modelarea funcţionării unui senzor optic cu performanţe îmbunătăţite utilizat pentru măsurarea micilor deplasări şi vibraţii care se bazează pe o nouă metodă a detecţiei de fază. Metoda este de tip buclă deschisă şi este caracterizată de distorsiuni mici, raport semnal-zgomot bun şi preţ de cost mic. Considerând că dispozitivul este un receptor ideal s-au obţinut în cazul măsurării distanţelor valori minime de 0.74 μ*m corespunzătoare unui domeniu dinamic de* 6.6×10^{-7} *dB. A fost evaluată şi modelată densitatea de probabilitate funcţie de eroarea de fază pentru diferite valori ale numărului mediu de fotoelectroni generaţi de semnal şi de parametrul care caracterizează zgomotul de fază. Considerând că procesul de detecţie este stocastic staţionar de tip Gauss a fost evaluată densitatea spectrală de putere corespunzătoare. Rezultatele prezentate pot fi utilizate în cazul măsurării distanţelor şi vibraţiilor mici precum şi în detecţia seismelor.*

In this paper the modelling of an improved method for the measurement of small displacements and vibrations based on a novel method for overcoming DC drift in RF subcarrier phase detection scheme for fibre optic sensors is presented. The method works in open loop and is characterized by low distortions in the modulation process, good signal-to-noise ratio and rather low cost. Considering the receiver ideal, we obtained for the measurements of small distances a minimum of 0.74 μ*m with a* 6.6×10^{-7} *dB dynamic range. We evaluated the probability density and modelled it vs the phase error for different values of the average number of photoelectrons generated by the signal and the phase noise parameter. Considering a stationary Gaussian stochastic process the dependence of the corresponding power spectral density was also evaluated. The presented results can be used in the measurement of small distances, vibrations and seismic detection.*

Keywords: Optic sensor, Small displacement measurements, Phase detection scheme, Probability density, Phase error, Phase noise parameter, Power spectral density

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1. Introduction

In the last years, several papers presented optical sensors for displacements and vibrations measurement using different techniques [1]-[5]. The optical sensors used to measure positions and distances are attractive because they are non-contact. The principle of operation of the most sensitive optical sensors is based on optical interferometry. This offers displacement sensitivity much smaller than the wavelength of the radiation source.

Also, over the last 30 years, optical fiber technology proved to be useful in sensors offering numerous operational benefits. Fiber sensors proved to be cost competitive when compared to other established measurement approaches [5].

The method based on sub-carrier phase measurements of RF intensity modulated light is well known and was applied in the measurement of optical fibre length changes. In this case, a RF amplitude modulated light is launched into an optical fiber and detected using a photodiode receiver placed at the output of the optical fibre. After that, the obtained electrical signal is introduced into an electrical mixer together with a reference generator signal which produces the RF modulation. The signal obtained from the mixer after being low pass filtered produces a dc signal whose magnitude is determined by the phase difference between the signal from the receiver and the reference signal. In case the phase of the reference signal is constant, the changes in the phase of the optically derived signal generates a change in the output which may be produced for instance by straining the fibre to change its physical length or modifying its refractive index. It is possible to measure the changes in optical path length by measuring the changes in the modulation frequency required to maintain a null output from the mixer. For long term static measurements there is the possibility that the dc drift from the mixer generates spurious signals [4]. In this case it is possible to obtain a noise limited quasi-static resolution of about 20 μm and ac sensitivity of tens of nanometers [4].

Based on a novel method for overcoming DC drift in RF sub-carrier phase detection scheme for fibre optic sensors outlined in paper [5], we solved numerically the Fokker-Planck equation in the stationary regime and linear approximation in tha case of the above mentioned sensor and also, we evaluated the probability density and modelled it vs the phase error for different values of the average number of photoelectrons generated by the signal at the detector and the phase noise parameter [6]. Also, considering a stationary Gaussian stochastic process we evaluated the dependence of the corresponding power spectral density.

The paper is organized as follows: Section 2 presents the experimental set-up, results and performances of the devices used in measurement, Sections 3 is dedicated to the evaluation of sensor characteristics (magnitude of the displacement, signal-to-noise ratio) and to modelling of the probability density and the phase error for different values of the average number of photoelectrons generated by the signal and the phase noise parameter. Considering a stationary Gaussian stochastic process the dependence of the corresponding power spectral density was also evaluated. In Section 4 we emphasized the conclusions of this paper.

2. Experimental results

Based on a phase detection scheme for overcoming DC drift in RF subcarrier optic sensors [4], we proposed an improved method to measure small displacements and vibrations.

The experimental set-up is presented in Fig. 1, where VCO represents a voltage controlled oscilator. The detection was performed using a low noise photoreceiver (diode).

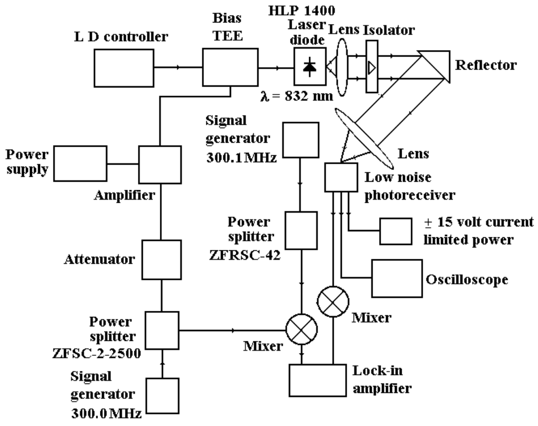


Fig. 1. Experimental set-up for phase detection (open loop).

In our experiments we used a laser diode (LD) Hitachi HLP 1400 which emits at a wavelength of λ= 827 nm. The spectral characteristics in the range 822 nm-830 nm of the laser diode Hitachi we for 60 mA (λ=826.85 nm) and 90 mA (λ=828.75 nm) injection currents, the modulated signals of the laser diode Hitachi HLP 1400 for 60 mA and respectively 90 mA injection currents are presented in paper [7]. For high frequencies (~10 MHz) the mode hoping effect is rather small in the case of the laser diode Hitachi HLP 1400.

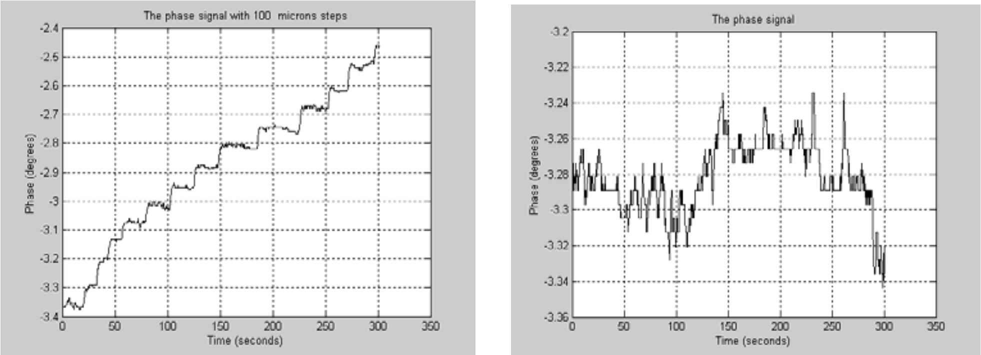
The phase signal registered from the Lock-in amplifier incremented by 100 μm steps is presented in Fig. 2 a) and respectively for the long time drift in Fig. 2 b). The drift, compare angles, is well below a step width of 100 μm. The oscilloscope traces show the modulation signal (and reference) in the time domain at two different positions, i.e. 0 mm (Fig. 3 a)) and 20 mm (Fig. 3 b)) apart. As can be seen the phase shift is visible.

3. Discussion of the modelling results

The operation of the experimental set-up presented in Fig. 1 can be analysed using the theoretical model outlined in paper [6].

Ideally the coherent optical signals have constant frequency and phase and for the coherent detection these parameters must be known at the receiver. The optical frequency may usually be considered as fixed, but in practice a phase estimator is needed in the receiver. The more realistic is the situation where phase noise is added to the signal.

Using an optical heterodyne system for detection, the received optical signal is converted to an electrical signal by a photodetector [6].



a) b)

Fig. 2. a), b). The phase signal with 100 μm steps (a) and with long time drift (b).

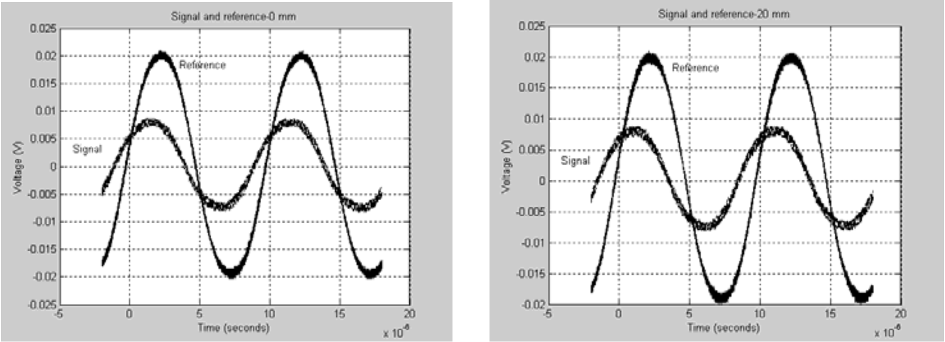
Based on the theoretical model presented in ref. [6] the signal-to-noise ratio, ρ at the decision point is given by:

$$\rho = \frac{E_1 - E_0}{\sigma_1 + \sigma_0} = \left(\int_0^T A^2(t)\,\mathrm{d}t \right)^{1/2} \tag{1}$$

where $E_{0,1}$ are the means of the decision variable, $\sigma_{0,1}$ are the standard deviations when the symbols “one” and “zero”, respectively are transmitted, T represents the pulse duration and A is the amplitude of the signal.

The variation of optical power delivered by the laser diode Hitachi HLP 1400 in the case of 60 mA and respectively 90 mA injection currents can be approximated by a quasi-sinusoidal signal of periods: 3.4×10^{-9} s (Fig. 3 a)) and 3.25×10^{-9} s (Fig. 5 3)).

Taking into account the phase shifts, $\Delta\varphi$ obtained experimentally (Figs. 4 and 5) we obtained for the measurements of the small distances ($\Delta x = \Delta\varphi \frac{\lambda}{2\pi}$, λ being the wavelength of the laser diode) the values 4.51 μm and respectively 0.74 μm.



a) b)

Fig. 3. a), b). The modulation signal in the time domain for 0 mm distance (a) and 20 mm distance (b).

Using a MATLAB programme we evaluated the errors (Fig. 3 a)) corresponding to the above mentioned small distances measurement. As can be seen from Fig. 4 they are in the range $1.45 \times 10^{-4} \div 0{,}19$ μm.

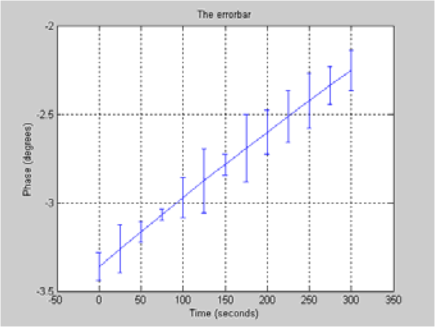


Fig. 4. The error bar graph corresponding to the data presented in Fig. 3 a).

Also, using the Eq. (1) and the processed (fitted) graph of the experimental results (Fig. 5 a), b)) we evaluated the amplitude and the period of the modulated signals of the laser diode Hitachi HLP 1400 and finally the signal-to-noise ratios and we obtained the following values: $\rho_1 = 6.9\times10^{-7}$ dB and respectively $\rho_2 = 6.6\times10^{-7}$ dB.

In order to emphasize the statistical properties of our sensor we solved the Fokker-Planck for the probability density, $p(\psi,t)$ corresponding to the function $\psi(t)$ at the moment t

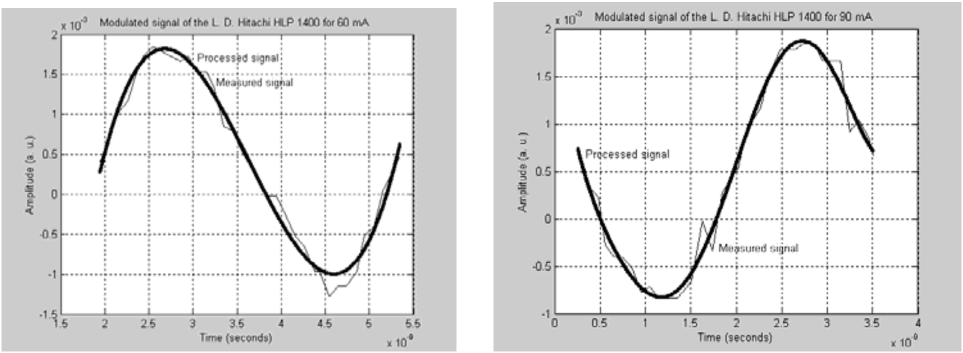
$$\frac{\partial p(\psi,t)}{\partial t} = \frac{\partial}{\partial\psi}\left[\frac{N\delta}{2} p(\psi,t)\sin\psi(t)\right] + \pi B_L \frac{\partial^2 p(\psi,t)}{\partial\psi^2} \tag{2}$$

in the case of stationary regime, $\left(\frac{\partial p}{\partial t} = 0\right)$ and linear approximation. The function $p(\psi,t)$ is defined in the interval $-\pi < \psi \le \pi$; than $p(-\pi) = p(\pi)$.

In Eq. (2) B_L represents the spectral bandwidth of the signal (~ 10^5 Hz in our case), N is an electron intensity and

$$\delta = \left(\frac{2m}{\pi B_L T}\right)^{1/2}, \tag{3}$$

a parameter depending on m (the average number of photoelectrons in the input) and $B_L T$ (the phase noise parameter).



a) b)

Fig. 5. a), b). The modulated signals of the laser diode Hitachi HLP 1400 in the case of: a) 60 mA and respectively b) 90 mA injection currents.

Taking into account the values of the the signal-to-noise ratios calculated above and the values of the corresponding parameters: $\delta_1 = 1.2 \cdot 10^{-3}$ and respectively $\delta_2 = 1.17 \cdot 10^{-3}$ we obtained the numerical solution of Eq. (2) (Fig. 6) which is in good agreement with other results [6].

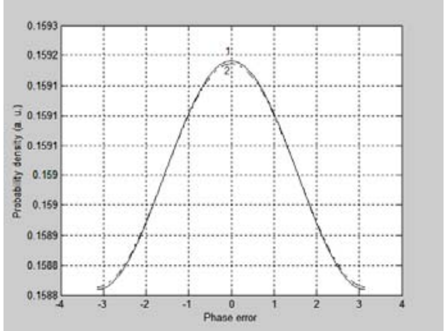
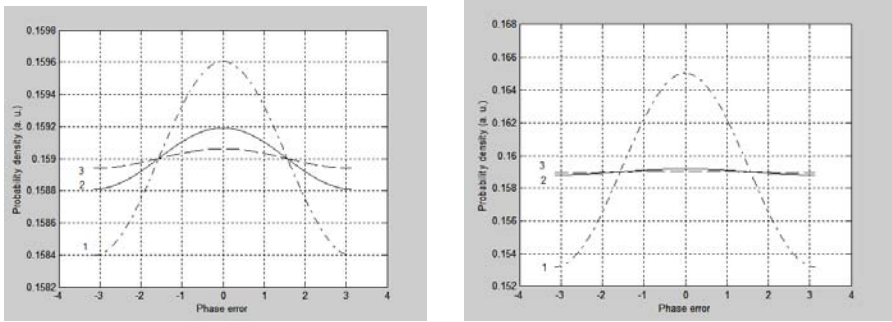


Fig. 6. The probability density vs phase error for signal-to-noise ratios: 6.9× 10^{-7} dB, curve (1) and respectively 6.6× 10^{-7} dB curve (2).

Also, we obtained the numerical solutions of Eq. (2) for several values of the parameters: $\delta_1 = 3.8 \cdot 10^{-3}$ (curve 1), $\delta_2 = 1.2 \cdot 10^{-3}$, (curve 2), $\delta_3 = 3.8 \cdot 10^{-4}$, (curve 3), for ρ_1=6.9× 10^{-7} dB and respectively $\delta_1 = 3.72 \cdot 10^{-3}$ (curve 1), $\delta_2 = 1.17 \cdot 10^{-3}$, (curve 2), $\delta_3 = 3.72 \cdot 10^{-4}$, (curve 3), for ρ_2=6.6× 10^{-7} dB (Figs. 7 a), b).



a) b)

Fig. 7 a), b). The probability density vs phase error for signal-to-noise ratios: a) ρ_1 and b) ρ_2 , respectively.

From Figs. 7 a), b) one can see that the smaller value of the the signal-to-noise ratios determine a higher probability density.

Maintaining constant the values of the parameter δ: $1.2 \cdot 10^{-3}$ and $1.17 \cdot 10^{-3}$ (corresponding to the signal-to-noise ratios: ρ_1 and ρ_2, respectively), then the average number of photoelectrons in the input, we obtained the numerical solution of Eq. (2) for several values of the phase noise parameter: $1.5 \cdot 10^{-2}$, $1.5 \cdot 10^{-3}$ and $1.5 \cdot 10^{-4}$ (Figs. 8 a), b)). As can be seen from Figs. 8 a), b) in this case the diminution of the phase noise parameter determines smaller values of probability density (0.1609, 0.1592 and 0.1589 respectively).

Based on the data presented in Fig. 5 a) and considering a stationary Gaussian stochastic process the dependence of the corresponding power spectral density [6]

$$S(\omega_0 - \omega) = \frac{2\frac{4m}{\delta T}}{\left(\frac{4m}{\delta T}\right)^2 + (\omega_0 - \omega)^2} \frac{1}{\delta} \tag{4}$$

vs angular frequency difference,

$$\omega_0 - \omega = \frac{4\pi}{T\delta} \sin\psi + \frac{2}{\delta}\sqrt{\frac{2m}{T}} n_1 \tag{5}$$

where ω_0 is the frequency of the signal is presented in Fig. 9 for several values of the parameters: $\delta_1 = 3.8 \cdot 10^{-3}$ (curve 1), $\delta_2 = 1.2 \cdot 10^{-3}$, (curve 2), $\delta_3 = 3.8 \cdot 10^{-4}$, and respectively (curve 3).

As can be seen from Fig. 9 the power spectral density reaches its minimum in the case of resonance, the two maxima obtained in the case of curve 2 being outside of the operation bandwidth.

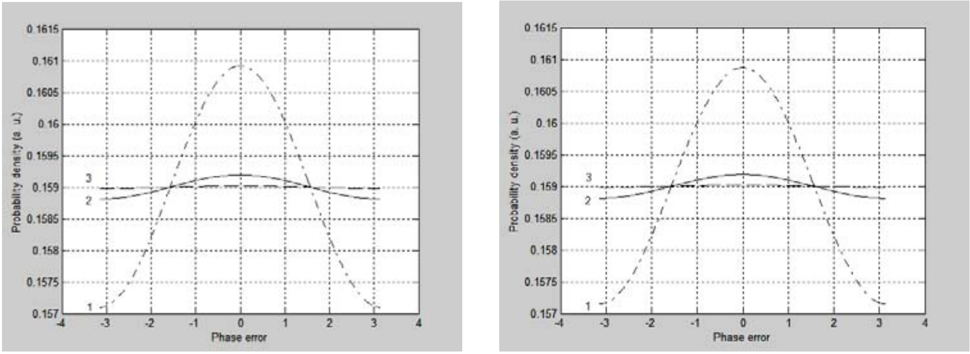
4. Conclusions

Based on a method to overcome the DC drift in RF subcarrier phase detection schemes used in fibre optic sensors we proposed an improved method (*open loop*) for the measurement of small displacements and vibrations [5].

For two values of the injection currents of the laser diode Hitachi HLP 1400 used in the experimental work and based on the theoretical model of the ideal receiver we obtained for the measurements of small distances values of 4.51

μm and respectively 0.74 μm and for the signal-to-noise ratios: 6.9×10^{-7} dB and respectively 6.6×10^{-7} dB.

In order to emphasize the noise statistics we solved numerically the Fokker-Planck equation in the stationary regime and linear approximation and also, we evaluated the probability density and modelled it vs the phase error for different values of the average number of photoelectrons generated by the signal and the phase noise parameter [6].



a) b)

Fig. 8 a), b). The probability density vs phase error for signal-to-noise ratios: a) ρ_1 and b) ρ_2, respectively.

Considering a stationary Gaussian stochastic process the dependence of the corresponding power spectral density was also evaluated.

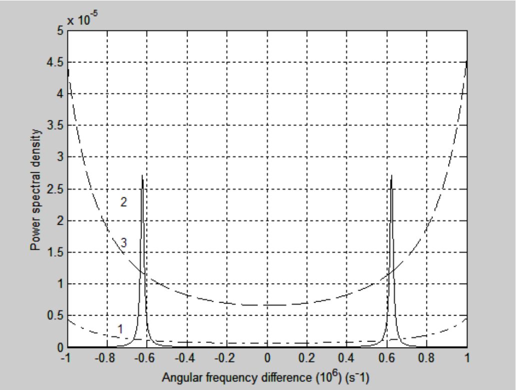


Fig. 9. The dependence of the power spectral density vs angular frequency difference for: $\delta_1 = 3.8 \cdot 10^{-3}$ (curve 1), $\delta_2 = 1.2 \cdot 10^{-3}$, (curve $2 \times 25 \cdot 10^{-4}$), and $\delta_3 = 3.8 \cdot 10^{-4}$, (curve 3).

We concluded that the smaller value of the the signal-to-noise ratios determine a higher probability density and that the diminution of the phase noise parameter determines smaller values of probability density. The theoretical results are in good agreement with other ones [6].

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