3D X-RAY IMAGE COMPOSITION

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3D image reconstruction algorithms fall into one of the three major categories of methods: analytical reconstruction - the filtered backprojection (FBT) method, iterative reconstruction - algebraic reconstruction techniques (ART), statistical image reconstruction techniques (SIRT) and hybrid methods.

The analytical methods are based on filtered backprojection (FBP) are currently and widely used on radiology scanners because of their computational efficiency and numerical stability.

These mathematical models are applied in X-ray imaging, thermograms, multispectral scanning and many more. Presented study is oriented on X-ray image reconstruction applied in the field of Cultural Heritage investigations, field in which it becomes one of the most important source of information.

Keywords: X-ray, tomography, Radon, Hough, Iteration, image reconstruction.

1. Introduction

Besides other data obtained in 2D and reconstructed in 3D images, X-ray scanning is a widely used technique in Cultural Heritage investigations because of the importance of the information it gives us. Practically by using one of the reconstruction techniques we can reproduce from the 2D images, the 3D image of the studied object, being able to observe by a noncontact manner, all the inside of an object.

In art, it can be used X-ray scanning to reveal hidden defects, precious conservation hints, and even previous paintings under the visible layer. It can also identify certain use of some pigments. In historical artifacts imaging, it can give important information about the technology used by artists, previous conservations, degraded areas and causes of the degradation.

Radiology can be further associated with non-destructive photonic techniques for a better characterization of analyzed object. Either if its 3D laser scanning [1], multispectral imaging, thermal imaging, laser induced fluorescence [2] or even laser Doppler vibrometry. In recent years a special attention was given to technique's portability, since a large number of cultural goods are immovable (for example collections that are not allowed to leave the museum facility and all the analysis and measurement should be done within the premises of the institution) [3].

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Special attention can be given to other 3D imaging technique [4], as laser scanning, in order to associate the result and map it onto a 3D model or laser Doppler vibrometry, a method used to detect and identify hidden defects of mural paintings or statues [5]. Also, at a lower resolution, possibility to inspect the interior of an object can be done with VHF electromagnetic radiation, with frequency higher than several GHz and which can go with milimetric resolution [6].

X-ray methods do not generate three dimensional images of an object directly. It provides 1 or 2 dimensional projections of the studied object. Hence, images have to be reconstructed from a set of projections. Real-world application needs image reconstruction from X-ray absorption projections obtained by measuring the radiation attenuation by crossing through a physical object at different angles. Digitalized projections are collected by X-ray devices connected to computers and after that acquisition a virtual image of the object is reconstructed using different mathematical reconstruction methods. Energy of any given beam (not only X-ray beam) is absorbed depending on what it cross on its way between the source to the detector. This projection can be represented as an integral. Projection does not carry enough information to reconstruct an image, but it is a good starting point to build using mathematical methods to complete an image who approximate good enough the studied object.

2. 3D image reconstruction algorithm

The adopted image reconstruction method and procedure has an essential impact on image accuracy, on image quality, on radiation dose, on image usability, and financial and computational costs. For example, if the data processing takes too long, for some applications can be a serious impediment. Or, for a given cost and available devices it is advisable to obtain reconstructed images with the lowest possible noise without major sacrificing image resolution, accuracy, readability (for the specific object and scope) and quality. Also, reconstructions that improve image quality can be used to reduce costs, or the device limitations, or computing time, or the radiation dose.

Reconstruction algorithms fall into one of the three major categories of methods:

1. analytical reconstruction: the so called filtered backprojection (FBT) method,
2. iterative reconstruction: the so called algebraic reconstruction techniques (ART), or the iterative statistical image reconstruction techniques (SIRT) and
3. hybrid methods who combines different analytical and iterative methods.

The analytical methods are based on filtered backprojection (FBP) are currently and widely used on radiology scanners because they are computational and time efficient and have high numerical stability.

These methods requires a reconstruction algorithm (or filter) and a stop procedure, who contains the procedure and the most important parameters who can be altered and can impact the image quality, accuracy and readability. In general, it is impossible to avoid reasonable compromise between resolution, noise, quality, readability for each algorithm. For example, a smooth algorithm produce lower noise images but reduce resolution. A sharp algorithm produce images with higher resolution, but increased image noise and more phantoms.

The choice of the proper reconstruction algorithm is the task of the expert or device operator and must be based on the prior experience and the specific application. For example, smooth algorithms are currently used to reduce image noise and improve the image for low contrast objects. Radiation dose associated with low structure contrast objects is usually higher than that for other examinations based on the inherent contrast of the object structure. Sharper algorithms are currently used in examinations who require to evaluate high density structures to obtain better resolution. And lower radiation dose must be used in the evaluation of objects with high contrast structures.

In addition to the usual reconstruction algorithms applied during image reconstruction, they are also many available noise reduction techniques, operating initially on the projection data, or finally on image. These methods involve non-linear de-noising or deblurring algorithms, combined into the basic reconstruction algorithm for the operation facility. For some applications these methods perform very well to reduce image noise and blur while maintaining high-contrast resolution. Using these methods too aggressively, can change usability, the noise texture, can sacrifice the image low-contrast detectability and can affect image readability. Hence, careful evaluation of these algorithms and procedures must and should be performed by experts and operators for each task very carefully.

Iterative reconstruction methods, has been intensively used in the early years of image reconstruction and abandoned because of the computing limitations, but has been re-evaluated recently based on the increased computing power of the modern computers, but also based on the necessities for better image quality, better resolution, better readability, and also because of the diversification of applications of x-ray imagery. The attention for x-ray (and not only x-ray) scanning increase also because it has many other advantages compared with analytic reconstruction techniques. While analytic methods are widely used for image reconstruction, the iterative reconstruction methods offer distinct advantages than analytic counterparts when data are incomplete, inconsistent, and rather noisy. They are also widely used for deblurring images. Key physical parameters like focal spot, X-ray beam energy and spectrum, photon statistics, detector geometry can be easier and precisely incorporated into iterative reconstruction algorithms, acquiring lower noise and higher resolution compared with the images obtained with analytical reconstruction. Moreover, iterative reconstruction methods can reduce image artifacts. The recent studies on iterative reconstruction methods demonstrates the high potential of these methods and of the hybrid methods compared with only analytical-based reconstruction algorithms to improve the quality, applicability, readability, resolution and many other. Because of to the inherent difference in data handling between analytical reconstruction and iterative reconstruction methods, images from different reconstruction methods may have a different appearance (like noise texture, or resolution). Hence, a careful evaluation of the technique and reconstruction parameter optimization is required before an iterative reconstruction algorithm can be accepted into practice.

Iterative methods can have several advantages over direct methods. These methods can incorporate some prior knowledge, including system geometry, detector response, object constraints, and they also permit modeling data noise. Also, an assumption underlying FBP is that x-ray sources are monoenergetic; in practice, there is a nonuniform distribution of photons of different wavelengths, and hence, different energies, that leads to a phenomenon physicists call "beam hardening". In practice, X-ray beams produced in scanners are polyenergetic with a relatively wide energy spectrum. Moreover the attenuation coefficients are beam energy and spectrum dependent. Low energy x-rays, which are more easily attenuated, are called soft X-rays. The more penetrating high energy x-rays, are called as hard X-rays. The beam hardening phenomenon is the process of increasing the average energy level of an X-ray beam, as it passes through the scanned object. The explanation of this phenomenon is that, as a polyenergetic beam passes through an object, the lower-energy parts of its spectrum attenuate more rapidly than the higher-energy parts of the spectrum.

The degree to which a given X-ray beam is hardened in passing through matter depends on both the initial X-ray energy and spectrum and the material composition of the scanned object. For a fixed initial X-ray energy and spectrum and object material type, the beam hardening is a monotone increasing function depending on the distance. In other words, the attenuation coefficient depends on the thickness of traversed material.

Different methods to compensate for the effects of beam hardening have been proposed, such as pre-filtering; post-reconstruction; and incorporating a polyenergetic acquisition model. Some iterative methods, such as statistical image reconstruction techniques (SIRT), can model polyenergetic x-ray sources and thus account for beam hardening in the reconstruction. They use a statistical model, in order to estimate the attenuation coefficient.

Statistical image reconstruction techniques are based on modeling assumptions that incorporate the stochastic nature of physical measurements. As with other reconstruction algorithms, the basic idea in SIRT is to find the distribution of the energy dependent attenuation coefficient given by the measurements. In FBP, usually monoenergetic x-ray beams are assumed, and therefore the issue of beam hardening is not considered. Statistical methods allow us to assume polyenergetic sources, and thereby reduce the negative effects of beam hardening artifacts.

In statistical methods, a physical, statistical acquisition model is assumed first. Then a statistical model is used to estimate the attenuation coefficient. At the end, the estimation found is optimized, by applying an iterative method.

Different techniques are used to reduce the incidence of beam hardening artifacts in x-ray reconstructions:

1. Pre-filtering: a physical device is used to ensure that the x-ray beams used arecloser to be truly monoenergetic, making the assumption of monoenergeticx-ray beams more reasonable.
2. Post-reconstruction: this is a standard post-processing method used since 1978. This method relies on assumptions about the material characteristics to provide corrections to the measured sinogram data. The reconstruction is done in two stages: an approximate material distribution is assumed at first, and the corresponding beam hardening artifacts are then reduced.
3. Incorporating a polyenergetic acquisition model. Statistical image reconstruction techniques for x-ray scanning can be developed based on physical models that account for polyenergetic sources. In this case, since the reconstruction algorithm is built upon a polyenergetic acquisition model, the beam hardening phenomenon is taken into account.

The greatest challenge for iterative reconstruction has always been, and still is, and has affected its use in radiology imaging. Meanwhile, methods, software and hardware are tested and improved to accelerate iterative reconstruction. Taking advantage of the advances in computational theory and technology, iterative reconstruction are now used in some specific applications or to improve the analytical methods and may be incorporated into routine practice in the future.

Radon transform

The Radon transform is named after Johann Radon in his work in 1917who showed in pure theoretic way, with no association to applications, how to describe a function in terms of its integral projections. The mapping from the function onto the projections is the Radon transform. The inverse Radon transform corresponds to the reconstruction of the function from the integral projections obtained by measuring the attenuation of X-ray radiation that passes through a physical object at different angles.

It will be used the following notations and definitions: x and y will denote the spatial coordinates; I(x) is the D-dimensional image containing the N-dimensional shapes; p denotesthe vector containing the curve parameters; c(p) is the member of a class of shapes described by the parameter vector p; c(s; p) are the coordinates of a point belonging to the shape c(p); C(x; p) will be the set of constraint functions that define a shape.
The number of constraint functions depends on the dimensionality of the shape. It is needed D – N constraints to describe a N-dimensional shape. For a point on the shape, the constraint functions will be zero. The template C(x,p) is also called the kernel that defines the shape given by p as an image with spatial coordinates x. It can be modeled the image I as a sum of several of these templates.
Observe that the parameters subset contains also the location of the shape (like the center of a sphere), hence we will write p = {q, xo}, with xo the location parameter of the shape and q the remainder of the parameters.
The original formulation of the Radon [7] transform is as follows:

$$R\{I\}(d,\phi) = \int_R I(d\ cos\phi - s\ sin\phi, d\ sin\phi + s\ cos\ \phi)ds \quad (1)$$

Even that initially it was a pure theoretical result, the Radon transform is mostly known for its role in radiology scanning. It is used to model the process of acquiring projections of the original object using X-rays. Given the projection data, the inverse Radon transform, can be applied to reconstruct the original object. The Radon transform can also be used for shape and pattern recognition. We can reformulate the Radon transform for a simpler use:

$$R\{I\}(d,\phi) = \int_{(x,y)} I(x,y)dxdy = \int_{R^R} I(x,y)\delta(x\ cos\phi + y\ sin\phi - d)dxdy \quad (2)$$

It is easy to generalize the Radon transform to arbitrary shapes c(p). We can use another equivalent formulation, useful for some applications:

$$R_{c(P)}\{I\}(P) = \int_{x\ on\ c(P)} I(x)dx = \int_{R^N} I(c(s;p)) \left\|\frac{\partial c}{\partial s}\right\| \mathrm{ds} = \int_{R^D} I(x)\delta\big(c(x;p)\big)dx \quad (3)$$

Other formulation expresses the Radon transform as a volume integral, a form that is particularly practical in image analysis.

Imagine now that there is a shape in the image with parameter set a. When p≠a, the Radon transform will evaluate to some finite number which is proportional to the number of intersections between the shapes c(p) and c(a). When p = a, the Radon transform yields a large response namely a peak in the parameter space. This response is proportional to the N-dimensional hyper-volume of the shape. We can now interpret the Radon transform as follows: it provides a function from image space to the parameter space spanned by the parameters p. The function P(p) created in this parameter space, contains peaks for those p for which the corresponding shape c(p) is present in the image. Shape detection is therefore reduced to the simpler problem of peak detection.

The third formulation of the Radon transform in equation (4) demonstrates an important reason for using distributions (generalized functions). In this formulation, it can be recognized the form of a linear integral operator, also known as a Fredholm operator, L_C with kernel C:

$$(L_C I)(p) = \int_{R^D} C(p,x) I(x) dx \tag{4}$$

Hence, if we allow for the kernel C to be a distribution, the Radon transform can be treated as any other linear transformation. In fact, using distributions, the identity operator, using the Dirac delta distribution as well as differential and integral operators, using derivatives and primitives of the Dirac delta, can be described in integral form. Dirac introduced these in order to develop a continuous equivalent to matrix algebra in his work on quantum mechanics.

In case of a Radon transform, the kernel C is of the form:

$$C(p,x) = \delta\big(c(x;p)\big) \tag{5}$$

In terms of shape detection, the role of the operator L_C is to compute the inner product between the image and a template C for a given parameter set p. Here it can be seed see the connection between the Radon transform and template matching. Often, the parameters p consist of the position of the shape xo and the actual shape parameters q. In this case the kernel has a special (shift-invariant) structure:

$$C(\{q,x_0\}x) = C(\{q,x_0+d\},x+d) \text{ (6)for any d}$$

The operator L_C now reduces to a set of convolutions:

$$(L_C I)(q,x_0) = (K_C(q) * xI)(x_0) \tag{7}$$

$$\text{with } K_C(q,x) = C(\{q,x\},0) \tag{8}$$

This implies a large speed-up: using the convolution property of the Fourier transform, each convolution reduces to a multiplication in the Fourier domain.

Hough Transform

They are two techniques for curve detection: the first use the Radon transform. The second one use a transform due to Hough [8], which become very popular because its applications. Many authors have noted that the Radon and Hough transforms are very closely related [15].

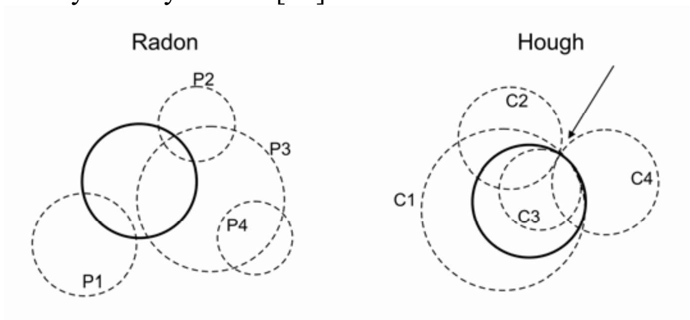

Fig. 1

The Hough transform was originally defined as a shape and pattern recognition tool, to detect, in black and white images, straight lines, and is clearly discrete. It is straightforward to generalize the Hough transform to other, more complex shapes, and grey-value images, and we will describe it shortly in this form. It is defined an N-dimensional storage array, each dimension corresponding to one of the parameters defining the shape. Each element of this array contains the number of votes for the presence of a shape with the corresponding parameters. The votes are obtained turn by turn by considering each point in the input image. Now we select which shapes could potentially be a member of this point, with grey value g, see Figure 1. We increment the vote for each of these shapes with h. Of course, if a shape with parameters p exists in the image, all pixels that are part of it will give a vote for it, yielding a large peak in the accumulator array. The Hough transform, like the Radon transform, associate to image space a parameter space.

Let's explore the relation between the Hough transform and the Radon transform. The Radon transform is a mapping and a mapping can be approached from different points of view.

The first one is the reading paradigm. In this view we consider how a data point in the destination space is obtained from the data in the source space. This is the usual way the Radon transform is interpreted.

The second is the writing paradigm. In this view it is to consider how a data point in the source space maps onto data points in the destination space. This is what the Hough transform does, even though in a discrete setting. Following this picture, the Hough transform is essentially a discretization of the Radon transform.

The mathematical formalism for the two methods is parallel and given by equation (4) with kernel functions of the form δ(C(x; p)). The mathematical formalism admit two different interpretations from computational point of view.
Consider reading paradigm given by Radon. For each p, we pick all the values of I(x), then apply the template weights K(x; p), and sum everything.
Consider now writing paradigm given by Hough. We initialize the entire function P(p) by zero. For each point x in the input image we have to determine its contribution, weighted with K(x; p), to each of the points in P(p) and then update P(p).

Using this interpretation it is clear that if the input data is sparse, the Hough paradigm offers an immediate reduction in computation time and if the interest is only in a view points in parameter space, then the Radon paradigm is to be preferred. So, we can benefit from both methods and both mathematical formalism.

The equivalence of the Radon transform, Hough transform and template matching has been discussed by several authors. Stockman and Agrawala [9], and Sklansky [10] have used arguments similar to those above to demonstrate the equivalence of the Hough transform and template matching. The formulation by Gel'fand et al. [11] of the Radon transform in terms of the Dirac delta function is in fact a form of template matching. Deans [12] was the first who establish the equivalence of the Radon and Hough transforms, as well as the first to bring the work of Gel'fand et al. to the attention of the field of image analysis.

Also, Princen [13] et al. have given a continuous formulation of the Hough transform, using an interesting approach that is perhaps more in the spirit of the Hough frame of mind. At the basis for their formulation are the constraint functions C. For any given point x in the input space, the constraints C(x; p) trace out a manifold in the parameter space spanned by the parameters p. Multiple points x give rise to multiple manifolds. These will intersect each other at the point p_0 in parameter space, and thus identifying the curve. The mathematical formulation of this principle is given by the familiar relation (4), unifying this approach with the others given above. Their claim that the Radon and Hough transforms are not equivalent, seems to be based on comparing the continuous Radon transform to the original discrete Hough transform, rather than comparing the continuous definitions, and not recognizing that the Radon transform can be written in the form of equation (4), despite using the Dirac delta in their own formulation of the Hough transform.

Iteration methods

Iteration methods are methods which compute a sequence of progressively accurate iterates to approximate the solution of the linear system of equations $Ax = b$.
Iterative methods for $Ax = b$ begin with an approximation to the solution,$x^{(0)}$, then provide a series of improved approximations $x^{(0)}, x^{(1)}, \dots, x^{(k)}, \dots$ that converge to the exact solution. For applications in image reconstruction, this approach is appealing because it can be stopped as soon as the approximations $x^{(k)}$ have converged to an acceptable precisionϵ, which might be something as 10^{-3} , 10^{-4} or even smaller. With a direct method, stopping early is not an option; because the process of elimination and back-substitution has to be completed, or else abandoned altogether and provide no result. But, by far, the main attraction of iterative methods, is that for certain problems, especially for those where the matrix A is large and sparse, they are much faster than direct methods and with much lower computational. On the other hand, iterative methods can be unreliable; for some problems they may confront to very slow convergence, or they may not converge at all.
Such methods are very usefully for solving large linear systems as the systems of image reconstruction are. In this case, the matrix is almost always too large to be stored even in the computer memory, making a direct method too difficult or impossible to use.
Very important also, the operations cost and hence computing time of $\frac{2}{3}n^3$steps for Gaussian elimination is too large for most large systems.
With iteration methods, the operations cost can often be reduced to something of cost $O(n^2)$ or even less. Even when a special form for A can be used to reduce the cost of elimination, iteration will be faster.
The general procedure for iterative methods is as follows. Rewrite $Ax = b$ as $Nx = b + Px$ with $A = N - P$ a picked splittingof A, where N is chosen to be nonsingular, and usually we select it such that the equation $Nz = f$ is easy solvable for any f. For example we choose N such that it is easy to invert it.
The iteration method is based on constructing $x^{(k)}$ by the formula

$$Nx^{(k+1)} = b + Px^{(k)}, \qquad k = 0,1,2, \dots$$

Applying the general convergence theorem [14] we have that $x^{(k)}$ converge for any b and all initial guesses $x^{(0)}$ if and only if all eigenvalues $\mu$of the matrix $M = N^{-1}P$satisfy $|\mu| < 1$. This is the basis of deriving splitting $A = N - P$ that leads to different convergent iteration methods whose main step is to choose a comfortable N.

6. Discussions

The iterative method is the most accurate algorithm, but it also requires, as the resolutions increased, a powerful computational station, as used in CT. It is easy to notice that the fastest method, considering the quality of the image reconstructed, is a hybrid of Radon and Hough transforms, the two being complementary with each other. As an addition, iterative method is perhaps the only solution when we have a very limited set of data (e.g. on human subjects the exposures are limited and the data are few), but also can be used to refine certain zones in which Radon and Hough transforms couldnot form a clear image.

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