# GRAVITATIONAL BREMSSTRAHLUNG IN HORAVA GRAVITY

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*The energy loss formula of the Horava gravity is used to derive the gravitational energy emitted during the classical collision between the charged particles in small-angle scattering approximation. The result is engaged to obtain the gravitational luminosity of a hot Hydrogen plasma. It is found that, like the Einstein gravity, the thermal luminosity has a temperature dependence of the form* $T^{3/2}$.

**Keywords:** gravitational bremsstrahlung, Horava gravity, small-angle approximation, gravitational luminosity, Hydrogen plasma.

## 1. Introduction

The recently proposed Horava gravity is essentially a non-relativistic (Lorentz violating) model of gravity because of a preferred time-foliation of space-time manifold supposed by the model [1]. At the large distances (low energies), the model is accompanied by a massless scalar field called "Khronon" which interacts with the matter. The presence of Khronon modifies the predictions of the Einstein gravity at cosmological scale [2, 3]. Also, the well-known quadrupole energy loss formula alterers because of the presence of scalar field [4]. By engaging the modified energy loss formula and comparing the results with the observed energy loss of the binary pulsars, constraints on the Horava gravity are obtained [4].
In this work, we consider the classical collision between the charged particles and use the energy loss formula of the Horava gravity to calculate the amount of gravitational energy emitted by the charge particles during the Coulomb scattering (gravitational bremsstrahlung radiation). The classical and quantum mechanical problem of gravitational bremsstrahlung and thermal gravitational luminosity of a plasma are studied within the context of the Einstein gravity in [5-10]. Here, we assume the small angle scattering approximation, taking the particles trajectory as a straight-line. Therefore, our calculations differ from [5-7] where the particle's trajectory considered to be a parabola as determined by the exact equation of motion. Then, we consider a hot plasma and obtain its gravitation luminosity by evaluating the ensemble average of the emitted energy taking place in a two-body collision.
In next section, we briefly discuss the quadrupole radiation in Horava gravity. In Sec. 3, the quadrupole tensor is calculated in small-angle approximation in terms of a general central potential. In Sec. 4, we obtain the emitted gravitational energy

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in a typical two-body collision. Finally, the problem of thermal luminosity of a hot Hydrogen plasma is calculated and discussed in Sec. 5.

## 2. Quadrupole Radiation in Horava Gravity

At the low energy, the action of Horava gravity is given by [4]

$$S_{\rm Hor} = -\frac{M^2}{2}\int d^4x\sqrt{-g}\big(R + K^{\mu\nu}{}_{\sigma\rho}\nabla_\mu u^\sigma \nabla_\nu u^\rho\big), \tag{1}$$

with

$$K^{\mu\nu}{}_{\sigma\rho} = \beta\delta^\mu_\rho\delta^\nu_\sigma + \lambda\delta^\mu_\sigma\delta^\nu_\rho + \alpha u^\mu u^\nu g_{\rho\sigma}. \tag{2}$$

where $\nabla_\mu$ stands for the covariant derivative and $g_{\alpha\beta}$ is metric tensor. The unite time-like vector is defined in terms of the Khronon field $\phi$ as

$$u_\mu = \frac{\partial_\mu\phi}{\sqrt{\partial_\nu\phi\partial^\nu\phi}}. \tag{3}$$

The modified version of the energy loss formula, based on the action (1) has the form [4]

$$\frac{dE_{\rm Hor}}{dt} = -\frac{1}{8\pi M^2}\Big(\frac{\mathcal{A}}{5}\dddot{Q}_{ij}\dddot{Q}^{ij} + \mathcal{B}\,\dddot{I}^i_i\dddot{I}^j_j\Big). \tag{4}$$

Here $M$ is a mass parameter and $\mathcal{A}$, $\mathcal{B}$ are some dimensionless constants depending on another three parameters $\alpha$, $\beta$ and $\lambda$, which appear in the action (1). By applying the above formula for the case of binary systems and comparing the result with the Hulse-Taylor binary, a bound on the parameters is found as [4]

$$\alpha \sim \beta \sim \lambda \lesssim 10^{-2}. \tag{5}$$

The Newton constant is related to $\alpha$ via

$$G_N = \frac{1}{4\pi M^2(2-\alpha)}. \tag{6}$$

The formula (4) reduces to the quadrupole formula of Einstein gravity, on setting $\mathcal{A} = 1$ and $\mathcal{B} = 0$.

## 3. quadrupole Tensor in Small-Angle Approximation

The quadrupole tensor $Q_{ij}$ of a single particle is related to the mass moment $I_{ij} = mx_ix_j$ via

$$Q_{ij} = I_{ij} - \frac{1}{3}I^k_k\delta_{ij}, \qquad (i,j = 1,...3). \tag{7}$$

The time derivatives of the mass moment and quadrupole tensor are

$$\dddot{I}_{ij} = m\big(\dddot{x}_i x_j + 3\ddot{x}_i\dot{x}_j + 3\dot{x}_i\ddot{x}_j + x_i\dddot{x}_j\big), \tag{8}$$

$$\dddot{Q}_{ij} = \frac{m}{3}\big[3\dddot{x}_i x_j + 9\ddot{x}_i\dot{x}_j + 9\dot{x}_i\ddot{x}_j + 3x_i\dddot{x}_j - 2(3\dot{\mathbf{x}}\cdot\ddot{\mathbf{x}} + \mathbf{x}\cdot\dddot{\mathbf{x}})\delta_{ij}\big], \tag{9}$$

from which we get [5, 6]

$$\dddot{I}^i_i = 2m\big(\dddot{\mathbf{x}}\cdot\mathbf{x} + 3\ddot{\mathbf{x}}\cdot\dot{\mathbf{x}}\big), \tag{10}$$

$$\begin{aligned}\dddot{Q}_{ij}\dddot{Q}^{ij} &= \frac{2m^2}{3}\big(3\dddot{\mathbf{x}}\cdot\dddot{\mathbf{x}}\mathbf{x}\cdot\mathbf{x} + 18\dddot{\mathbf{x}}\cdot\ddot{\mathbf{x}}\dot{\mathbf{x}}\cdot\mathbf{x} + 18\dddot{\mathbf{x}}\cdot\dot{\mathbf{x}}\ddot{\mathbf{x}}\cdot\mathbf{x} + \dddot{\mathbf{x}}\cdot\mathbf{x}\dddot{\mathbf{x}}\cdot\mathbf{x} \\ &+ 27\ddot{\mathbf{x}}\cdot\ddot{\mathbf{x}}\dot{\mathbf{x}}\cdot\dot{\mathbf{x}} + 9\ddot{\mathbf{x}}\cdot\dot{\mathbf{x}}\ddot{\mathbf{x}}\cdot\dot{\mathbf{x}} - 12\dddot{\mathbf{x}}\cdot\mathbf{x}\ddot{\mathbf{x}}\cdot\dot{\mathbf{x}}\big).\end{aligned} \tag{11}$$

Let us consider the classical two-body problem where the particles are subjected to the central potential $U(r)$. The particles have masses $m_1$ and $m_2$. To achieve the explicit form of the terms $\ddot{\mathbf{x}}$ and $\dddot{\mathbf{x}}$, appeared in (10) and (11) in terms of the potential, one starts form the classical equation of motion

$$\mu\ddot{\mathbf{x}} = -\frac{\partial U}{\partial r}\widehat{e}_r, \qquad \widehat{e}_r = \frac{\mathbf{x}}{r}. \tag{12}$$

where $r = |\mathbf{x}|$. The relative distance between the particles is $\mathbf{x} = \mathbf{x_1} - \mathbf{x_2}$ and $\mu$ is the reduced mass. From (12) one immediately finds

$$\mu\dddot{\mathbf{x}} = -\frac{\partial^2 U}{\partial r^2}\dot{r}\widehat{e}_r - \frac{\partial U}{\partial r}\dot{\phi}\widehat{e}_\phi. \tag{13}$$

where we have used $\dot{\widehat{e}}_r = \dot{\phi}\widehat{e}_\phi$. Assuming that the collision takes place on the $X-Y$ plane and using the polar coordinate for the position and velocity of the particles as $\mathbf{x} = r\widehat{e}_r$ and $\dot{\mathbf{x}} = \dot{r}\widehat{e}_r + r\dot{\phi}\widehat{e}_\phi$, one obtains for the several terms appearing in (10) and (11)

$$\mu^2\dddot{\mathbf{x}}\cdot\dddot{\mathbf{x}} = \left(\frac{\partial^2 U}{\partial r^2}\right)^2\dot{r}^2 + \left(\frac{\partial U}{\partial r}\right)^2\dot{\phi}^2, \tag{14}$$

$$\mu^2\dddot{\mathbf{x}}\cdot\ddot{\mathbf{x}} = \frac{\partial^2 U}{\partial r^2}\frac{\partial U}{\partial r}\dot{r}, \tag{15}$$

$$\mu\dddot{\mathbf{x}}\cdot\dot{\mathbf{x}} = -\frac{\partial^2 U}{\partial r^2}\dot{r}^2 - \frac{\partial U}{\partial r}r\dot{\phi}^2, \tag{16}$$

$$\mu\dddot{\mathbf{x}}\cdot\mathbf{x} = -\frac{\partial^2 U}{\partial r^2}r\dot{r}, \tag{17}$$

$$\mu\ddot{\mathbf{x}}\cdot\mathbf{x} = -\frac{\partial U}{\partial r}r, \tag{18}$$

$$\mu^2\ddot{\mathbf{x}}\cdot\ddot{\mathbf{x}} = \left(\frac{\partial U}{\partial r}\right)^2, \tag{19}$$

$$\mu\ddot{\mathbf{x}}\cdot\dot{\mathbf{x}} = -\frac{\partial U}{\partial r}\dot{r}, \tag{20}$$

which on inserting in (10) and (11) yield

$$\dddot{Q}_{ij}\dddot{Q}^{ij} = \frac{8}{3}\left[\left(\frac{\partial^2 U}{\partial r^2}r + 3\frac{\partial U}{\partial r}\right)^2\dot{r}^2 + 12\left(\frac{\partial U}{\partial r}\right)^2 r^2\dot{\phi}^2\right], \tag{21}$$

$$\dddot{I}^i_i\dddot{I}^j_j = 4\left(\frac{\partial^2 U}{\partial r^2}r + 3\frac{\partial U}{\partial r}\right)^2\dot{r}^2. \tag{22}$$

Now, suppose $m_1 \ll m_2$. If the velocity $v$ of the particle "1" is so large, its deviation from the straight-line motion becomes negligible and on can assume a constant velocity for scattered particle [11, 12]. So, for the particle "1" moving along the $X$-axis, with the particle "2" located at the origin, we have

$$\mathbf{x} = vt\widehat{e}_x + b\widehat{e}_y, \tag{23}$$

$$r^2 = v^2t^2 + b^2, \tag{24}$$

where $b$ denotes the impact parameter. From (23) and (24) for a particle moving on a straight-line, we have

$$\phi = \tan^{-1}\Big(\frac{b}{vt}\Big), \tag{25}$$

yielding

$$\dot{\phi} = -\frac{bv}{r^2}, \tag{26}$$

$$\dot{r} = \frac{v^2 t}{r}, \tag{27}$$

which modifies (21) and (22) to

$$\dddot{Q}_{ij}\dddot{Q}^{ij} = \frac{8v^2}{3}\Bigg[\Big(\frac{\partial^2 U}{\partial r^2}+\frac{3}{r}\frac{\partial U}{\partial r}\Big)^2 v^2t^2 + 12\Big(\frac{1}{r}\frac{\partial U}{\partial r}\Big)^2 b^2\Bigg], \tag{28}$$

$$\dddot{I}^i_i\dddot{I}^j_j = 4t^2v^4\Big(\frac{\partial^2 U}{\partial r^2}+\frac{3}{r}\frac{\partial U}{\partial r}\Big)^2. \tag{29}$$

## 4. **Energy Loss in Coulomb Scattering**

Now, we assume $U(r)$ to be the Coulomb potential, i.e. $U = \frac{e_1e_2}{r}$. With the help of (4), (28) and (29), one easily finds

$$\dot{E}_{\rm Hor} = -\frac{1}{8\pi M}\frac{v^2e_1^2e_2^2}{r^6}\Big[\frac{8}{15}(v^2t^2+12b^2)\mathcal{A} + 4v^2t^2\mathcal{B}\Big]. \tag{30}$$

Therefore, the emitted energy during the collision becomes

$$|\Delta E_{\rm Hor}| = \frac{v^2e_1^2e_2^2}{8\pi M}\int_{-\infty}^{\infty}\frac{8}{15}\frac{v^2t^2+12b^2}{(v^2t^2+b^2)^3}\mathcal{A} + \frac{4v^2t^2}{(v^2t^2+b^2)^3}\mathcal{B}dt,$$
$$= \frac{\pi v e_1^2e_2^2G_N}{b^3}\Big(1-\frac{\alpha}{2}\Big)\Big(\frac{37}{15}\mathcal{A}+\frac{1}{2}\mathcal{B}\Big), \tag{31}$$

where we have used

$$\int_{-\infty}^{\infty} dt\frac{t^2}{(v^2t^2+b^2)^3} = \frac{\pi}{8v^3b^3}, \tag{32}$$

$$\int_{-\infty}^{\infty} dt\frac{1}{(v^2t^2+b^2)^3} = \frac{3\pi}{8vb^5}. \tag{33}$$

and exploited (6) to write the final result in terms of the Newton constant. We find that, the result for the Einstein gravity is recovered by assuming $\mathcal{A}=1$ and $\mathcal{B}=\alpha=0$, for which the emitted energy becomes

$$|\Delta E_{\rm Ein}| = \frac{37}{15}\frac{\pi v e_1^2e_2^2G_N}{b^3}. \tag{34}$$

We compare this result with a similar expression for the gravitational scattering, i.e. $U = -G_N\frac{m_1m_2}{r}$ in small-angle approximation [13]

$$|\Delta E_{\rm Ein}| = \frac{37}{15}\frac{\pi v m_1^2m_2^2G_N^3}{b^3}. \tag{35}$$

which coincides with (34) on replacing $m_1^2 m_2^2 G_N^2 \to e_1^2 e_2^2$. So, according to (31), beside a numerical factor, the bremsstrahlung energy in Horava gravity has the same form of the Einstein gravity

$$\Delta E_{\text{Hor}} \sim \frac{v e_1^2 e_2^2 G_N}{b^3}. \tag{36}$$

## 5. Gravitational Luminosity of a Hot Plasma

Let us consider a hot electrically neutral plasma with the ion and electron density $n_i$ and $n_e$ respectively. To obtain the gravitational luminosity of the plasma with the gravitational bremsstrahlung as a mechanism for the loss of its energy, we multiply (31) with the electron flux $vn_e$, the ion density, and integrate over the impact parameter $b$ [11, 12]. So, we obtain the energy loss per volume $V$ as

$$\frac{d\mathcal{E}_{\text{Hor}}}{dV} = 2\pi n_i n_e v \int_{b_{min}}^{\infty} db\, |\Delta E_{\text{Hor}}(b)| b. \tag{37}$$

The semiclassical cut-off on the impact parameter is $b_{min} = \frac{\hbar}{mv}$, which is imposed by the Heisenberg uncertainty relation [11, 12]. For the Hydrogen plasma, we have $n_e = n_i$ and $e_1 = -e_2 = e$. Hence, from (31) and (37) the luminosity becomes

$$\frac{d\mathcal{E}_{\text{Hor}}}{dV} = 2\pi^2 \frac{me^4 n_e^2 G_N}{\hbar c^5}\Big(1 - \frac{\alpha}{2}\Big)\Big(\frac{37}{15}\mathcal{A} + \frac{1}{2}\mathcal{B}\Big) v^3. \tag{38}$$

We have restored the speed of light, $c$ in (38) for the sake of dimensional consistency. Taking the thermal average of this expression, yields the thermal luminosity of the plasma. In a hot plasma, where the ratio of Coulomb energy to thermal energy is negligible, it behaves like an ideal gas [11, 12]. So, averaging the electron speed in (38) over a thermal distribution of speeds, gives rise to the gravitational luminosity. At temperature $T$, the thermal average of an ensemble obeying the Maxwell-Boltzman statistic is

$$\langle f(\mathbf{v})\rangle = \Big(\frac{m\beta}{2\pi}\Big)^{\frac{3}{2}} \int d^3v e^{-\frac{\beta}{2}mv^2} f(\mathbf{v}), \quad \beta = \frac{1}{kT}. \tag{39}$$

where $k$ denotes the Boltzman constant and $f(\mathbf{v})$ is a function of particle's velocity. For $f(\mathbf{v}) = v^3$ we have

$$\langle v^3\rangle = \frac{2}{\pi^2}\Big(\frac{2\pi}{\beta m}\Big)^{\frac{3}{2}}. \tag{40}$$

Thus, from (38) and (40) we obtain the thermal luminosity in Horava gravity

$$\Big\langle \frac{d\mathcal{E}_{\text{Hor}}}{dV} \Big\rangle = 8\sqrt{2\pi^3}\frac{me^4 n_e^2 G_N}{\hbar c^5}\Big(\frac{kT}{m}\Big)^{\frac{3}{2}}\Big(1 - \frac{\alpha}{2}\Big)\Big(\frac{37}{15}\mathcal{A} + \frac{1}{2}\mathcal{B}\Big). \tag{41}$$

We compare (41) to the plasma luminosity in Einstein gravity, first derived by Weinberg using the quantum field theoretic methods [7-9]

$$\Big\langle \frac{d\mathcal{E}_{\text{Ein}}}{dV} \Big\rangle \sim \frac{me^4 n_e^2 G_N}{\hbar c^5}\Big(\frac{kT}{m}\Big)^{\frac{3}{2}}. \tag{42}$$

Thus, as (41) and (42) imply, in both theories the dependence of thermal luminosity on the physical parameters is same. In particular, dependence on the temperature is of the form $T^{\frac{3}{2}}$, in contrast to well-known Stephan-Boltzamnn law in electromagnetic radiation. One must note that the $T^{\frac{3}{2}}$ dependence of (42) is a consequence of the semiclassical cut-off. Utilizing the classical cut-off of the form $b_{min} = \frac{e^2}{m_e v^2}$ yields a temperature dependence of the form $T^2$. See [5-9].

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