U.P.B. Sci. Bull., Series D, Vol. 73, Iss. 4, 2011 ISSN 1454-2358

MONEY MARKET MODELS AND STOCK DISCRET TIME MARKET MODELS

Ion STANCIOIU[1], Marian –Sorin IONESCU[2]

În acest articol, se prezintă şi se discută unele modele de pe piaţa monetară actual utilizate în literatura de specialitate, precum şi posibilitatea de extindere a lor ulterioară.

Preţurile pieţei depind de opţiunile şi deciziile luate de un număr relativ mare de agenţi care acţionează în condiţii de incertitudine, specifice pietelor monetare. Prin urmare, se propune să se trateze preţurile de active ca fiind variabile aleatorii. Vom prezenta evoluţiile modelelor care reduc riscul de a investi în piaţa de valori actuală reliefând faptul ca o creştere a investiţiilor se va face simţită doar pe termen lung.

O problemă centrală, in prezenta abordare, folosindu-ne de modele predictive elaborate cu ajutorul performanţele din trecutul economic al evolutiei pieţii, este aceea ca acţiunile viitoare ale pieţei nu pot imita trecut.

Situaţiile financiare neîntalnite pe piaţă în trecut (inedite), combinate cu o nouă tehnologie de creare şi dezvoltare de piete, generează noi moduri de comunicare şi schimburi de informaţii, interacţiunile între tranzacţionarea instrumentelor derivate şi de tranzacţionare de pe piaţa financiară, prin intermediul unor metode matematice specifice trecutului economic, difera in mod radical de aplicaţiile prezente.

Dacă piaţa poate fi structurată şi dezvoltată pe principii fundamentale ştiinţifice, atunci se pot elabora modele universal valabile şi funcţionale pentru orice tip de piaţă abordat .Ne aşteptam astfel ca modele elaborate, vor genera profituri ridicate si vor evita perioadele temporare de intaindere prelungita, ce genereaza pierderi.

Obiectivul principal rămâne insa cel al maximizarea profiturilor generate.

In this paper common stock and money market models are investigated, extended and discussed. Market prices depend on the choices and decisions made by a great number of agents acting under condition of uncertainty. It is therefore proposed to treat the prices of assets as random. We report models developments that reduce the risk of investing in stock market, while increasing long-term returns. A central problem with developing predictive models based on past market performance is the future market actions may not mimic the past. Financial situations never encountered by the market in the past, combined with new market-making technology, new modes of information communication and exchange, the interactions between derivatives trading and trading in other financial market, making mathematical methods that would have worked in the past fail in the present. If there is any fundamental nature acting in the market, then it should be possible to invent models that work well under all conditions, in all markets. We would expect such models to return high profits in bullish market phases, and neatly sidestep the biggest losses in bearish phases. The main objective remains the maximization of the profits.

[1] Professor , Departement of Management, University POLITEHNICA Bucharest , Romania
[2] PhD student, University POLITEHNICABucharest, e-mail: sorin_ionescu07@yahoo.com

Keywords: money market, primary securities, derivative securities, underlying securities, investment strategies, optimization of the risks.

1. Money Market Account

A *money market account* (MMA) is like a series of zero coupon bonds, maturing after only one day (or whatever periods we have in our futures contracts|it might be for example a week.) If I deposit the amount 1euro day 0, the balance of my account day 1 is euro *er*1 , where *r*1 is the short interest rate from day 0 to day 1. The rate *r*1 is known already day 0. The next day, day 2, the balance has grown to euro *er*1+*r*2 , where *r*2 is the short interest rate from day 1 to day 2; it is random as seen from day 0, and its outcome is determined day 1. The next day, day 3, the balance has grown to euro *er*1+*r*2+*r*3 , where *r*3 is the short interest rate from day 2 to day 3; it is random as seen from day 0 and day 1, and its outcome occurs day 2, and so on. Day *T* the balance is thus euro *er*1+*¢¢¢*+*rT* which is a *random variable.* In order to simplify the notation, we introduce the symbol *R*(*t; T*) = *rt*+1 + *¢ ¢ ¢* + *rT* .

2 .Financial market processes

Here we consider a financial market consisting of N+1 financial assets, where one of these assets called bond or money-market, is risk free, while the remaining N assets, called stocks are risky. A financial market is defined as M=(r, b, δ, σ, A, S(0)):

-a probability space (Ω, F, P)

-a time interval [0, T]

-a D-dimensional Brownian process $W(t)=(W_1(t) \ldots W_D(t))`$, o<=t<=T adapted to the augumented filtration {F(t) ;0<=t<=T}

-a measurable risk-free market rate process r(t) from $L_1[0, T]$

-a measurable mean rate of return process $b:[0, T]xR^N \rightarrow R$ from $L_2[0, T]$

-a measurable dividend rate of return process $\delta:[0, T]xR^N \rightarrow R$ from $L_2[0, T]$

-a measurable volatility process $\sigma:[0, T]xR^{NxD}$ such that $\Sigma\Sigma\int_0^T \sigma^2_{n, d}(s)ds<\infty$

-a measurable finite variation, singularly continuous stochastics A(t)

-the initial condition given by $S(0)=(S_0(0) \ldots S_N(0))`$

A share of a bond (money market) has a price $S_0(t)>0$ at time t with $S_0(0)=1$, is continuous, {F(t), 0<=t<=T} adapted, and has finite variation.

Stock prices are modeled as begin similarly to that of bonds, except of a randomly fluctuating component (called volatility). As a premium for the risk originating from these random fluctuations, the mean rate of return of a stock is higher than that of a bond.

3. Money Market Models and Stock

We suppose that m risky assets are traded and we refer them to as stocks. Their prices at time n=0, 1, 2, … are denoted by $S_1(n)$, …, $S_m(n)$.. In addition, we suppose that investors have at their disposal a risk-free asset, that is, an investment in the money market. We take the initial level of risk-free investment to be one unit of home currency, A(0)=1. The money market account can be manufactured using bonds, we shall frequently refer to a risk-free investment as a position in bonds, finding it convenient to think of A(n) as a bond price at time n.

The risky positions in assets number 1, …, m are denoted x_1, …, x_m, respectively, and the risk-free position by y. The wealth of an investor holding such positions at time n will be:

$$V(n)=\Sigma_{j=1}^{m}x_jS_j(n)+yA(n) \qquad (1)$$

Our task is to build a corresponding mathematical model, a crucial stage being concerned with the properties of the mathematical objects involved. This is done below by specifying a number of assumptions, the purpose of which is to find a compromise between the complexity of the real world and limitations and simplifications of the mathematical model, imposed it order to make it tractable. The assumptions reflect our current position on this compromise.

4. Extended Models

Securities such as stock, which are traded independently of other assets, are called *primary securities*. By contrast, *derivative securities*, are legal contracts conferring certain financial rights or obligations upon holder, contingent on the prices of other securities, referred to as the *underlying securities*. An underlying security may be a primary security, as for a forward contract on stock, but it may also be derivative security, as in the case of an option on futures. A derivative security cannot exist in its own right, unless the underlying securities are traded. Derivative securities are also referred to as contingent claims, because their value is contingent on the underlying securities.

The holder of a long forward contract on a stock is committed to buy the stock for the forward price at a specified time of delivery, no matter how much the actual stock prices turns out to be at the time. The value of the forward position is contingent on the stock. It will become positive if the market price of stock turns out to be higher than the forward price on delivery. If the stock prices turn out to be lower than the forward price, then the value of the forward position will be negative.

The setting of our study, involving portfolio of risky stocks and the money market account, will be extended to include risky securities of various other kinds in addition to stock. To cover real-life situations, we need to include derivative securities such as forwards or options, but also primary securities such as bonds of various maturities, the future prices of which may be random.

In the following, we list the main assumptions that are made for our mathematical model.

Assumption 1: Positivity of Prices

All stock and bond prices are strictly positive,

S(n)>0 and A(n)>0, for n=0, 1, 2, …

Assumption 2: Randomness

The future stock prices $S_1(n)$, …, $S_m(n)$ are random variables for any n=1, 2, 3, …. The future prices A(n) of the risk-free security for any n=1, 2, 3,… are known numbers.

Assumption 3: Divisibility, Liquidity and Short Selling

An investor may buy, sell and hold any number x_k of stock shares of each kind k=1, 2, …, m and take any risk –free position y, whether integer or fractional, negative, positive or zero. In general,

x_1, …, x_m, y from R

Assumption 4: Solvency

The wealth of an investor must be non-negative at all times,

V(n)>=0, for n=0, 1, 2, …

Assumption 5: Discret Unit Prices

For each n=0, 1, 2, … the shares prices $S_1(n)$, …, $S_m(n)$ are random variables taking only finitely many values.

5. Investment Strategies

The position held by an investor in the risky and risky-free assets can be altered at any time step by selling some assets and investing the proceeds in other assets. In real economic life cash, it can be taken out of portofolio for consumption or injected from other sources. Nevertheless, we shall assume that no consumption or injection of funds takes place in our models to keep things as simple as possible.

Decisions made by any investor of when to alter his portofolio and how many assets to buy one sell are based on the information currently available. We are going to exclude the unlikely possibility that investors could foresee the future, as well as the somewhat more likely (but not legal), one that they will act on insider information. All the historical information about the market up to and including the time instant when a particular trading decision is executed will be freely available.

We introduce the following definitions.

Definitions

1: A portfolio is a vector ($x_1(n)$, …, $x_m(n)$)indicating the numbers of shares and bonds held by an investor between times n-1 and n. A sequence of portfolios indexed by n=1, 2, … is called an investment strategy. The wealth of an investor or the *value of the strategy* at time n≥1 is:

$V(n)=\Sigma_{j=1}^{m}x_j(n)S_j(n)+y(n)A(n)$

At time n=0 the initial wealth is given by :

$V(0)=\Sigma_{j=1}^{m}x_j(1)S_j(0)+y(1)A(0)$

The contests of a portfolio can be adjusted by buying or selling some assets at any time step, as long as the current value of the portfolio remains unaltered.

2: An investment strategy is called *self-financing* if the portfolio constructed at time n≥1 to be held over the next time step n+1 is financed entirely by the current wealth V(n), that is:

$\Sigma_{j=1}^{m}x_j(n+1)S_j(n)+y(n+1)A(n)=V(n)$

3: An investment strategy is called *predictable* if for each n=0, 1, 2, …, the portfolio ($x_1(n+1)$, …, $x_m(n+1)$, y(n+1)) constructed at time n depends only on the nodes of the tree market scenarios reached up to and including time n.

4: A strategy is called *admissible* if it is self-financing, predictable, and for each n=0, 1, 2, 3, … V(n)≥0, with probability 1.

Proposition:We shows that the position taken in the risk-free asset is always determined by the current wealth and the positions in risky assets.

Given the initial wealth V(0) and a predictable sequence ($x_1(n)$, …, $x_m(n)$), n=1, 2, … of positions in risky assets, it is always possible to find a sequence y(n) of risk-free positions such that ($x_1(n)$, …, $x_m(n)$, y(n)) is predictable self-financing investment strategy.

Proof: Put $y(1)=\{V(0)-x_1(1)S_1(0)- \ldots -x_m(1)S_m(0)\}/\{A(0)\}$

And then compute

$V(1)=x_1(1)S_1(1)+ \ldots +x_m(1)S_m(1)+y(1)A(1)$.

Next,

$y(2)=\{V(1)-x_1(2)S_1(1)- \ldots -x_m(2)S_m(1)\}/\{A(1)\}$

$V(2)=x_1(2)S_1(2)+ \ldots +x_m(2)S_m(2)+y(2)A(2)$, and so on. This clearly defines a self-strategy. The strategy is predictable because y(n+1) can be expressed in terms of stock and bond prices up to time n.

Example: For m=2, suppose that :

$S_1(0)=60$,	$S_1(1)=65$,	$S_1(2)=75$
$S_2(0)=20$,	$S_2(1)=15$,	$S_2(2)=25$
$A(0)=100$,	$A(1)=110$	$A(2)=121$,

in a certain market scenario. At the time "0" initial wealth V(0)=3,000 euros is invested in a portfolio consisting of $x_1(1)=20$ shares of stock number one, $x_2(1)=65$ shares of stock number two, and y(1)=5 bonds. Our notational convention is to use 1 rather than 0 as the argument in $x_1(1)$, $x_2(1)$ and y(1) to reflect the fact that this portfolio will be worth V(1)=20x65+65x15+5x110=2,825 euros. At that time the number of assets can be altered by buying or selling some of them, as long as the total value remains 2,825 Euros.

We could form a new portfolio consisting of $x_1(2)$=15 shares of stock one, $x_2(2)$=94 shares of stock two, and y(2)=4 bonds, which will be held during the second time step. The value of this portfolio will be V(2)=15x75+94x25+4x121=3,959 euros, at time 2, when the position in stock and bonds can be adjusted once again, as long as total value remains 3,959 euros, and so on. If no adjustments are made to the original portfolio, then it will worth 2,825 euros at time 1and 3,730 euros at the time 2.

For the same stock and bond prices, suppose that an initial wealth of V(0)=3,000 euros is invested by purchasing $x_1(1)$=18.22 shares of the first stock, short selling $x_2(1)$=-16.81 shares second stock, and buying y(1)=22.43 bonds. The time 1 value of this portfolio will be V(1)=18.22x65-16.81x15+22.43x110=3,399.45 euros. The investor will benefit from the drop of the price of the shorted stock. We illustrate in this example, the fact that portfolios containing fractional or negative numbers of assets are allowed.

We do not impose any restrictions on the numbers $x_1(n)$, …, $x_m(n)$, y(n). The fact that they can take non-integer values is referred to as *divisibility*. Negative $x_j(n)$ means that stock number j is *sold short* (a short position is taken in stock j), negative y(n) corresponds to borrowing cash (taking a short position in the money market). The absence of any bounds on the size of these numbers means that the market is *liquid*, that is, any number of assets of each type can be purchased or sold at any time. Investors are required to pay a certain percentage of the short sale as a security deposit to cover possible loss. If their losses exceed the deposit must be closed. The deposit creates a burden on the portfolio, particularly if it earns no interest for the investors.

We continue assuming that stock prices follow in the first part of our example.

Suppose that 20 shares of the first stock are sold short, $x_1(1)$=-20. The investor will receive 20x60=1,200 Euros in cash, but has to pay a security deposit of, say 50%, that is 600 Euros. One time step later he will suffer a loss of (20x65-1,200) Euros=100 Euros. This is subtracted from the deposit and the position can be closed by withdrawing the balance of (600-100) Euros=500 Euros. On the other hand, if 60 shares of the second stock are shorted, that is, $x_2(1)$=-60, then the investors will make a profit of, (1,200-60x15) Euros=300 Euros, after one time step. In both cases, the final balance should be reduced by 600x0.1=60 euros, the interest that would have been earned on the amount deposited, had it been invested in the money market.

An investor constructing a portfolio at time n has no knowledge of future stock prices. Investment decisions can be based only on the performance of the market to date, this is reflected in the definition of investment strategy, called predictable.

Finding the number of bonds y(1) and y(2) held by an investor during the first and second steps of a predictable self-financing investment strategy with initial value V(0)=200 euros and risky asset positions is done as following:

$x_1(1)=35.24$, $x_1(2)=-40.50$,

$x_2(1)=24.18$, $x_2(2)=10.13$,

if the prices of assets follow the same scenario. Also one can find the time 1 value V(1) and time 2 value V(2) of this strategy.

Once again, we suppose that the stock and bond prices follow the same scenario, if an amount V(0)=100 Euros were invested in a portfolio with $x_1(1)=-12$, $x_2(1)=31$ and y(1)=2, then it would lead to insolvency, since the time 1 value of this portfolio is negative, V(1)=-12x65+31x15+2x10=-95 Euros.

Such a portfolio would be impossible to construct in practice, which excluded assumption with solvency. No short position will be allowed unless it can be closed at any time and in any scenario. The wealth of an investor must be non-negative at all times.

7. Conclusion

In this work we have analyzed common stock and money market models with main goal of develop mathematical models and calculations based on some clear assumptions. We have also discussed the impact on investment strategies. Decisions made by any investor of when to alter his portfolio and how many assets to buy can be based on our mathematical analysis that is illustrated with some basic case study and practical calculations.

These proposed mathematical models could be implemented in the future in various programming languages and used to assist the decision aiming at the profit maximization with an optimization of the risks.

R E F E R E NC E S

[1] *F. Alizadeh and D.Goldfarb*, Second-order cone programming. Mathematical Programming, 95:1 (2003), 3-51.

[2] *F. Andreson, H.Mausser, D.Rosen, and S.Uryasev*, Credut risk optimization with conditional value-at-risk criterion, Mathematical Programming B, 89 (2001), 273-91.

[3]*V.S.Bawa, S.J. Brown, and R.W.Klein*, Estimation Risk and Optimal Portofolio Choice, Amsterdam:North Holland, 1979.

[4] *J.R.Birge and F.Louveaux*, Introduction to Stochastic Programming, New York, Springer, 1997.

[5] *F.Black and R.Litterman*, Global portofolio optimization. Financial Analysts Journal, 48:5 (1992), 28-43.
[6] *M.Capinski and T.Zastawniak*, Mathematics for Finance: An Introduction to Financial Engineering, Springer, 2003.
[7] *J.Cox, S.Ross, and M.Rubinstein*, Option pricing: a simplified approach. Journal of Financial Economics, 7:3 (1979), 229-63.
[8] *G.Cornuejols and R.Tutuncu*, Optimization methods in Finance, CAMBRIDGE University Press 2009
[9] *J.Gondzio and R.Kouwenberg*, High performance computing for asset liability management. Operations Research, 49 (2001), 879-91.
[10] *H.Lang,* Lectures on Financial mathematics, KTH Math, 2010
[11] *S.M.Ross*, Introduction to Stochastic Dynamic Programming, Academic Press, 1983.
[12] *S. Shreve*, Stochastic Calculus and Finance, Copyright; Steven E. Shreve, 1996.
[13] *J.F.Sturm*, UsingSeDuMi 1.02, a mathlab toolbox for optimizatzion oversymmetric cones. Optimization Methods and Software, 11-12 (1999), 625-53
[14] *M.J. Todd*, Semidefinite optimization.Acta Numerica, 10(2001), 515-60
[15] *S.Uryasev*, Conditional value-at-risk:Optimization algorithms and applications.Financial Engineering News, 14(2000), 1-6