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CONTRIBUTIONS TO THE STUDY OF THE PASSING THROUGH THE RESONANCE OF THE LINEAR SYSTEMS HAVING A FINITE NUMBER OF DEGREES OF FREEDOM

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În această lucrare se va studia problema regimului tranzitoriu, în condițiile legii de variație liniară a frecvenței forțelor perturbatoare, a unui sistem multimasă. Ecuațiile diferențiale care descriu mişcarea sistemului elastic studiat vor fi reduse la ecuații de ordinul doi. O astfel de reducere se poate realiza prin două moduri de considerare a forțelor de rezistență corespunzătoare ipotezelor lui Fogot şi ale lui E.S. Sorokin. Se va arăta că în cazul problemelor liniare utilizarea modului de rezolvare a lui Sorokin este mai indicată decât utilizarea modului de reprezentare a forțelor de frecare ca forțe de vâscozitate, forțe ce sunt proporționale cu viteza de deformare.

The unsteady passing of a multi-mass system under the law of linear variation of disturbance forces frequency will be studied in this paper. The differential equations, which describe the motion of the studied elastic system, will be reduced to the quadratic equations. Such a reduction may be achieved by two ways of considering the resistance forces corresponding to the Fogot's and E.S.Sorokin's hypothesis. In the case of linear problems, it will be shown that the application of Sorokin's solving way is more indicated than the representation way of friction forces as viscous forces, which are proportional to the deformation speed.

Keywords: absorption coefficient, disturbing moment, viscous damping, resonance.

1. Introduction

The unsteady passing of a multi-mass system will be studied under the law of linear variation of disturbance forces frequency. The differential equations, which describe the motion of the studied elastic system, will be reduced to the quadratic equations analyzed in the paper [1]. Such a reduction may be achieved by two ways of considering the resistance forces corresponding to the Fogot's and E.S.Sorokin's hypothesis [2]. In the case of linear materials, it will be shown that the application of Sorokin's solving way is more indicated than the representation

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way of friction forces as viscous forces, which are proportional to the deformation speed.

This consideration is explained not only by means of a good correspondence of Sorokin's hypothesis with experimental data but also by means of a reduced calculation volume.

2. Fogot's hypotheses method

The general form of the differential equations of the n-freedom degrees systems oscillations is

$$\sum_{k=1}^{n}\left(a_{ik}\ddot{q}_k + b_{ik}\dot{q}_k + c_{ik}q_k\right) = Q_i,\ i = \left(\overline{1,n}\right) \tag{1}$$

where a_{ik}, b_{ik}, c_{ik} are constant values while $q_i(t)$, $Q_i(t)$ are the coordinates and generalized forces, respectively.

By accepting the initial conditions:

$$x_i(p) \to q_i(t);\ F_i(p) \to Q_i(t) \tag{2}$$

there results from (1) the symbolical form

$$\sum_{k=1}^{n}\left(a_{ik}p^2 + b_{ik}p + c_{ik}\right)\cdot x_k(p) = F_i(p).\quad i = \left(\overline{1,n}\right) \tag{3}$$

Solving the system (3) one obtains:

$$x_i(p) = \frac{1}{\Delta(p)}\cdot\sum_{k=1}^{n}\Delta_{ik}(p)\cdot F_k(p), \tag{4}$$

where $\Delta(p)$ is the determinant of the system while $\Delta_{i1}(p)$, $\Delta_{i2}(p)$, … , $\Delta_{in}(p)$ are the complementary determinants corresponding to the *i* column elements.

If $\Delta(p)$ admits only complex roots with negative real part:

$$p_j = -\frac{1}{2}\mu_j + ik_j,\ \ \overline{p}_j = -\frac{1}{2}\mu_j - ik_j,\ \left(j = \overline{1,n}\right)\ \left(i = \sqrt{-1}\right) \tag{5}$$

than $\dfrac{1}{\Delta(p)}$ can be broken up into simple fractions in the form

$$\frac{1}{\Delta(p)} = \sum_{j=1}^{n} \frac{\gamma_j p + \delta_j}{p^2 + \mu_j p + \omega_j^2}, \tag{6}$$

where:

$$\omega_j^2 = \frac{\mu_j^2}{4} + k_j^2;\ \gamma_j = \frac{1}{\Delta'(p_j)} + \frac{1}{\Delta'(\overline{p}_j)};$$

$$\delta_j = -\left(\frac{p_j}{\Delta'(\overline{p}_j)} + \frac{\overline{p}_j}{\Delta'(p_j)}\right);\ \Delta'(p) = \frac{d}{dp}\left[\Delta(p)\right].$$

If the relation (7) is introduced

$$\psi_{ij}(p) = \left(\gamma_j p + \delta_j\right) \cdot \sum_{k=1}^{n} \Delta_{ik}(p) \cdot F_k(p), \tag{7}$$

and by means of the relation (6), the relation (4) becomes

$$x_i(p) = \sum_{j=1}^{n} \frac{\psi_{ij}(p)}{p^2 + \mu_j p + \omega_j^2}, \tag{8}$$

or

$$x_i(p) = \sum_{j=1}^{n} Z_{ij}(p), \tag{9}$$

where every term can be considered as an image of solving the quadratic differential equations

$$\ddot{\zeta}_{ij}(t) + \mu_j \dot{\zeta}_{ij}(t) + \omega_j^2 \zeta_{ij}(t) = \varphi_{ij}(t), \tag{10}$$

that is

$$Z_{ij}(p) \to \zeta_{ij}(t);\ \psi_{ij}(p) \to \varphi_{ij}(t). \tag{11}$$

If the initial conditions are null, the equations (10) become

$$\zeta_{ij}(t) = \frac{1}{k_j} \cdot \int_0^t \varphi_{ij}(\tau) e^{-\frac{1}{2}\mu_j (t-\tau)} \cdot \sin k_j (t-\tau) d\tau, \tag{12}$$

while

$$q_i(t)=\sum_{j=1}^{n}\frac{1}{k_j}\int_0^t \varphi_{ij}(\tau)\cdot e^{-\frac{1}{2}\mu_j(t-\tau)}\cdot \sin k_j(t-\tau)d\tau\ .\ i=\left(\overline{1,n}\right) \tag{13}$$

If the system is operated by generalized forces varying according to the law

$$Q_k(t)=Q_{ok}\exp\left(-i\cdot\frac{\varepsilon_k t^2}{2}\right), \tag{14}$$

the integral (13) is reduced to probability integrals with the complex argument

$$q_m(t)=\frac{(1+i)\sqrt{\pi}}{4}\cdot\sum_{j=1}^{n}\frac{Q_{oj}}{\sqrt{\varepsilon_j}}\left[W(u_j)+W(V_j)-W(u_{oj})\cdot e^{u_{oj}^2-u_j^2}-\right.$$
$$\left.-W(V_{oj})\cdot e^{V_{oj}^2-V_j^2}\right]\cdot e^{-\frac{1}{2}\varepsilon_j t^2} \tag{15}$$

where

$$u_j=\frac{1-i}{2\sqrt{\varepsilon_j}}\left(\varepsilon_j t-k_j+i\frac{\mu_j}{2}\right),\ V_j=\frac{-1+i}{2\sqrt{\varepsilon_j}}\left(\varepsilon_j t+k_j+i\frac{\mu_j}{2}\right).$$

3. Sorokin's method

The differential equations system is easily obtained from the forced oscillation equations (1), ignoring the resistance forces and introducing instead of the E elasticity modulus the complex modulus in the form $\left(1\pm i\frac{\psi}{2\pi}\right)E$ where Ψ is an energy absorption coefficient. Thus, it is obtained

$$\sum_{k=1}^{n}\left[a_{jk}\cdot\ddot{q}_k+\left(1\pm i\cdot\frac{\psi}{2\pi}\right)c_{jk}\cdot q_k\right]=Q_j(t), \tag{16}$$

or in the symbolical form

$$\sum_{k=1}^{n}\left[a_{jk}\cdot p^2+\left(1\pm i\cdot\frac{\psi}{2\pi}\right)c_{ik}\right]\cdot x_k(p)=F_i(p)\,.\,i=\left(\overline{1,n}\right) \tag{17}$$

Unlike the previous case, the determinant Δ(s) of this system has real roots

$$\pm S_1,\ \pm S_2,\ \ldots\ \pm S_n,$$

where

$$S^2 = -\frac{p^2}{1 \pm i \cdot \dfrac{\psi}{2\pi}}. \tag{18}$$

Thus the calculation of a lot of roots simplifies. For each S value two p values correspond. From (18) one obtained

$$p_j = \left(-\frac{\psi}{2\pi} + i\right) S_j,\ \overline{p}_j = \left(-\frac{\psi}{2\pi} - i\right) S_j$$

Removing p_j, $\overline{p}_j$ which has no material meaning and considering $\sqrt{1 \pm i\dfrac{\psi}{2\pi}} \approx 1 \pm i\dfrac{\psi}{4\pi}$ one obtained from (17)

$$x_j(p) = \frac{1}{\Delta(S)} \sum_{k=1}^{n} \Delta_{jk}(S) \cdot \frac{F_k(p)}{K}, \tag{19}$$

where $K = 1 \pm i\dfrac{\psi}{2\pi}$.

Since the roots equation $\Delta(S) = 0$ are $\pm S_1,\ \pm S_2,\ \ldots\ \pm S_n$, its breaking up $\dfrac{1}{\Delta(S)}$ into simple fractions will be

$$\frac{1}{\Delta(S)} = \sum_{m=1}^{n} \frac{\gamma_m S + \delta m}{S^2 - S_m^2}, \tag{20}$$

in which $\gamma_m = \dfrac{1}{\Delta'(S_m)} + \dfrac{1}{\Delta'(-S_m)}$; $\delta_m = S_m\left[\dfrac{1}{\Delta'(S_m)} - \dfrac{1}{\Delta'(-S_m)}\right]$.

Since Δ(S) is an S even function, there results that:

$$\Delta'(-S) = -\Delta'(S),\ \gamma_m = 0\,,\ \delta_m = \frac{2S_m}{\Delta'(S_m)}.$$

Further, we noted

$$\psi_{mj}(p) = \delta_m \sum_{k=1}^{n} \Delta_{jk}(S) \cdot F(p)\,.\, n, j = \overline{1,n} \tag{21}$$

If (20) is introduced into (19), there results

$$x_j(p) = \sum_{m=1}^{n} \frac{\psi_{mj}(p)}{S^2 - S_m^2} \cdot \frac{1}{K}, \tag{22}$$

or

$$x_j(p) = -\sum_{m=1}^{n} \frac{\psi_{mj}(p)}{p^2 + S_m^2\left(1 \pm \dfrac{\psi}{2\pi}\right)} = -\sum_{m=1}^{n} Z_{mj}(p). \tag{23}$$

From (23) one obtains:

$$\ddot{\zeta}_{mj}(t) + \left(1 \pm i\frac{\psi}{2\pi}\right) S_m^2 \cdot \zeta_{mj}(t) = \varphi_{mj}(t). \tag{24}$$

The final solution is

$$q_j(t) = -\sum_{m=1}^{n} \frac{1}{S_m} \cdot \int_0^t \varphi_{mj}(\tau) \cdot e^{-\frac{\psi}{4\pi} S_m (t-\tau)} \cdot \sin S_m (t-\tau) d\tau\,, \tag{25}$$

where (26) was introduced

$$\varphi_{mj}(t) \leftarrow \psi_{mj}(p) = \delta_m \sum_{k=1}^{n} \Delta_{jk}(S) \cdot F_k(p).\ \left(m, j = \overline{1,n}\right) \tag{26}$$

In the case of the two methods, the similarity between the solutions (26) and solutions (13) is noted.

4. Application in the case of the passing through the resonance of a three – mass system that performs twist oscillations.

If the case of the viscous damping is considered, the differential equations system will be

$$\begin{cases} J_1\ddot{\varphi}_1 + b_1(\dot{\varphi}_1 - \dot{\varphi}_2) + c_1(\varphi_1 - \varphi_2) = 0 \\ J_2\ddot{\varphi}_2 + b_1(\dot{\varphi}_2 - \dot{\varphi}_1) + b_2(\dot{\varphi}_2 - \dot{\varphi}_3) + c_1(\varphi_2 - \varphi_1) + c_2(\varphi_2 - \varphi_3) = m_2(t), \\ J_3\ddot{\varphi}_3 + b_2(\dot{\varphi}_3 - \dot{\varphi}_2) + c_2(\varphi_3 - \varphi_2) = 0 \end{cases} \quad (27)$$

where J_i – inertia moments of the wheels; c_j – elasticity coefficient of the shaft sector j, $m_2(t)$ disturbing moment acting on the mass 2; b_j – damping coefficient in the *j* sector; φ_j rotation angle (figure 1).

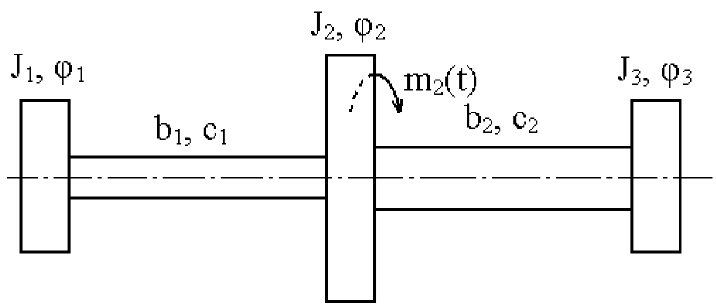

Fig. 1.

If we denote $\varphi_2 - \varphi_1 = \psi_1$; $\varphi_2 - \varphi_3 = \psi_2$, after several calculations, the symbolical form becomes:

$$\begin{cases} \psi_1(p) = -\dfrac{M_2(p)}{J_2} \cdot \dfrac{p^2 + a_1 p + d_1}{\Delta(p)}, \\ \psi_2(p) = \dfrac{M_2(p)}{J_2} \cdot \dfrac{p^2 + a_2 p + d_2}{\Delta(p)}. \end{cases} \quad (28)$$

There will be introduced initial null conditions and the notations

$$a_1 = \frac{b_2}{J_3};\ a_2 = \frac{b_1}{J_1};\ d_1 = \frac{c_2}{J_3};\ d_2 = \frac{c_1}{J_1};\ \Delta(p) = \Delta_1(p)\cdot\Delta_2(p);$$
$$\Delta_1(p) = (p - p_1)\cdot(p - \bar{p}_1);\ \Delta_2(p) = (p - p_2)\cdot(p - \bar{p}_2). \tag{29}$$

p_1, $\bar{p}_1$, p_2, $\bar{p}_2$ – associate complex roots of the equation $\Delta(p) = 0$ given by the relations (5).

Further, the coordinate $\psi_1(t)$ is calculated; thus solution (28) has the form

$$\psi_1(p) = -\frac{M_2(p)}{J_2}\cdot\left[\frac{A_1}{p - p_1} + \frac{\bar{A}_1}{p - \bar{p}_1} + \frac{A_2}{p - p_2} + \frac{\bar{A}_2}{p - \bar{p}_2}\right], \tag{30,a}$$

where

$$A_1 = \frac{p_1^2 + a_1 p_1 + d_1}{2ik_1\cdot\Delta_2(p_1)};\ A_2 = \frac{p_2^2 + a_2 p_2 + d_2}{2ik_2\cdot\Delta_1(p_2)}. \tag{30,b}$$

while $\bar{A}_1, \bar{A}_2$ are associated with A_1 and A_2.

The expression $m_2(t)$

$$m_2(t) = m_0 \exp\left(-i\frac{\varepsilon t^2}{2}\right); \tag{31}$$

is introduced into (30) and having the new variables $u_j = x_j + iy_j$, $v_j = x'_j + iy'_j$, $i = (1,2)$ one obtained

$$\psi_1(t) = \frac{m_0}{2J_2}(1 - i)\sqrt{\frac{\pi}{\varepsilon}}\left[A_1 W(v_1) - \bar{A}_1 W(u_1) + A_2 W(v_2) - \bar{A}_2 W(u_2)\right]\exp\left(-i\frac{\varepsilon t^2}{2}\right). \tag{32}$$

The variables x_j, y_j, x'_j, y'_j are determined by means of the relations

$$x_j = h_j\dot{\lambda}_j(\xi_j - 1) + \frac{h_j}{2}, y_j = -h_j\dot{\lambda}_j(\xi_j - 1) + \frac{h_j}{2}$$
$$x'_j = -h_j\dot{\lambda}_j(1 + \xi_j) - \frac{h_j}{2}, y'_j = h_j\dot{\lambda}_j(1 + \xi_j) - \frac{h_j}{2}. \quad (j = \overline{1,2}) \tag{33}$$

5. Numerical application

The calculation of the system non-steady process is performed having the numerical data $J_1 = 19{,}6 Nms^2$, $J_2 = 1{,}96 Nms^2$, $J_3 = 3{,}92 Nms^2$, $b_1 = 14{,}7 Nms$, $b_2 = 19{,}6 Nms$, $c_1 = 38{,}2 \cdot 10^4\, Nm$, $c_2 = 88{,}2 \cdot 10^4\, Nm$, $\varepsilon = 74{,}91 s^{-2}$. The following equation is obtained for determining the roots

$$p^4 + 23{,}25 p^3 + 8{,}95048 \cdot 10^5 p^2 + 34{,}9375 \cdot 10^4 p + 585 \cdot 10^8 = 0 . \text{(b)}$$

The relations (5) are applied and one obtains

$$\begin{matrix} \mu_1 = 2{,}448\, s^{-1} & k_1 = 266\, s^{-1} \\ \mu_2 = 20{,}802\, s^{-1} & k_2 = 908\, s^{-1} \end{matrix} . \qquad \text{(c)}$$

The expressions of the coefficients given by (30,b) are:

$$\begin{cases} 2A_1 k_1 = -0{,}425 \cdot 10^{-3} - i \cdot 0{,}2046 \\ 2A_2 k_2 = 0{,}145 \cdot 10^{-2} - i \cdot 0{,}7953 \end{cases} \qquad \text{(d).}$$

The dynamic coefficient λ_1 will be determined using the relation $\lambda_1 = \left| \frac{\psi_1(t)}{\psi_1^{st}} \right|$, where

$$\psi_1^{st} = -\frac{m_0}{J_2} \cdot \frac{J_1 \cdot J_2}{(J_1 + J_2 + J_3) \cdot c_1} . \qquad \text{(e)}$$

In tables 1 and 2, the W(Z) probability integrals values are given for different values of time $(\xi_1 = \varepsilon t / k_1)$.

Table 1

ξ_1	$W(u_1)$	$W(v_1)$	$W(u_2)$	$W(v_2)$
0,9	0,2009 - *i*0,1518	0,0092 – *i*0,0092	0,0070 – *i*0,0070	0,0041 – *i*0,0042
1,0	0,9207 + *i*0,0705	0,0092 – *i*0,0092	0,0077 – *i*0,0075	0,0041 – *i*0,0042
1,05	0,2461 + *i*1,7392	0,0089 – *i*0,0090	0,0079 – *i*0,0076	0,0041 – *i*0,0041
1,07	-1,2643 + *i*1,3160	0,0089 – *i*0,0090	0,0079 – *i*0,0076	0,0040 – *i*0,0041
1,10	-0,1902 – *i*1,1245	0,0087 – *i*0,0088	0,0082 – *i*0,0079	0,0040 – *i*0,0041
1,107	0,6178 – *i*0,8109	0,0086 – *i*0,0086	0,0082 – *i*0,0079	0,0040 – *i*0,0041
1,135	-0,8912 + *i*0,9513	0,0086 – *i*0,0086	0,0082 – *i*0,0079	0,0040 – *i*0,0041
1,15	0,8454 + *i*0,5058	0,0085 – *i*0,0085	0,0083 – *i*0,0080	0,0040 – *i*0,0040
1,157	0,5289 – *i*0,6601	0,0085 – *i*0,0085	0,0083 – *i*0,0080	0,0040 – *i*0,0040
1,177	-0,6647 + *i*0,8417	0,0084 – *i*0,0084	0,0084 – *i*0,0081	0,0039 – *i*0,0040
1,20	0,7450 + *i*0,1182	0,0082 – *i*0,0082	0,0085 – *i*0,0082	0,0039 – *i*0,0040

Table 2

ξ_1	$W(u_2)$
3,3282	0,2433 + i0,0754
3,4135	0,4715 + i0,2332
3,4477	0,2953 + i0,5384
3,4545	0,2119 + i0,5803
3,5210	-0,1152 + i0,1689
3,5484	-0,1016 + i0,2979
3,5705	-0,0941 + i0,0731

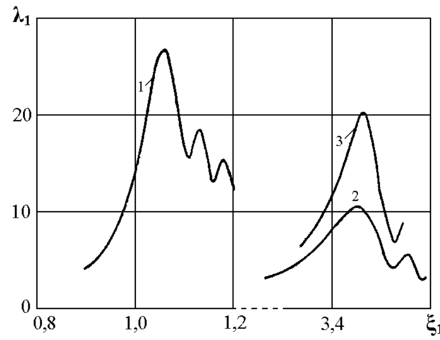

Fig. 2.

By varying ξ_2 from 0.975 to 1.046, the system is in the area of the second resonance (ξ_1 is modified accordingly from 3.3282 to 3.5705). During that time the influence of the functions $W(u_1)$, $W(v_1)$ and $W(v_2)$ is unimportant, they have sizes of degrees $W(u_1) \approx -0{,}0075 + 0{,}0075i$, $W(v_1) \approx 0{,}0040 \cdot (1 - i)$, $W(v_2) \approx 0{,}0027 \cdot (1 - i)$.

Besides, and from the tables 1 and 2, as well, there results that, in the area (f) of the first resonance, $W(u_1)$ has the most important value while in the area of the second resonance, it is the magnitude of $W(u_2)$ which has the most important value. In this way it is confirmed the thesis often used in practical calculations that at the moment when the resonance has a certain natural frequency, the forms of the non-resonance vibrations (with values of the natural frequency differing a lot from the resonance frequency) have a small influence on the magnitude of the amplitude of the system resonance vibrations. The charts of the dynamic coefficient λ_1 when passing through the first and the second resonance (figure 2) show the existence of the same characteristics as in case of the passing through the resonance of the linear system having one freedom degree: overlying and reducing the highest amplitude, strokes attenuation aspect etc. By considering the frictions according to Sorokin, the system of equations will have the form

$$\begin{cases} J_1\ddot{\varphi}_1 + \left(1 \pm i\dfrac{\psi}{2\pi}\right)c_1(\varphi_1 - \varphi_2) = 0, \\ J_2\ddot{\varphi}_2 + \left(1 \pm i\dfrac{\psi}{2\pi}\right)c_1(\varphi_2 - \varphi_1) + \left(1 \pm i\dfrac{\psi}{2\pi}\right)c_2(\varphi_2 - \varphi_3) = m_2(t), \\ J_3\ddot{\varphi}_3 + \left(1 \pm i\dfrac{\psi}{2\pi}\right)c_2(\varphi_3 - \varphi_2) = 0, \end{cases} \tag{34}$$

or it will be in the symbolic form under null initial conditions (after introducing $\psi_1 = \varphi_1 - \varphi_2$, $\psi_2 = \varphi_2 - \varphi_3$,)

$$\begin{cases} \left(\frac{c_1}{J_1} + \frac{c_1}{J_2} - S^2 \right) \psi_1(p) - \frac{c_2}{J_2} \psi_2(p) = -\frac{1}{KJ_2} M_2(p), \\ -\frac{c_1}{J_2} \psi_1(p) + \left(\frac{c_2}{J_2} + \frac{c_2}{J_3} - S^2 \right) \psi_2(p) = \frac{1}{KJ_2} M_2(p), \end{cases} \tag{35}$$

where $S^2 = -p^2/K$, $K = 1 \pm i\psi/2\pi$.

Solving the system (35) with respect to $\psi_1(p)$, one obtained

$$\psi_1(p) = -\frac{M_2(p)}{KJ_2 \cdot \Delta(S)} \cdot \left(\frac{c_2}{J_3} - S^2 \right), \tag{36}$$

or

$$\psi_1(p) = -\frac{M_2(p)}{KJ_2} \cdot \left(\frac{B_1}{S - S_1} + \frac{B_2}{S + S_1} + \frac{C_1}{S - S_2} + \frac{C_2}{S + S_2} \right), \tag{37}$$

where $\pm S_j (j = 1,2)$ are the roots of the equation Δ(s) = 0, while B_1, B_2, C_1, C_2 are given by

$$B_1 = -B_2 = \frac{1}{\Delta'(S_1)} \left(\frac{c_2}{J_3} - S_1^2 \right), \; C_1 = -C_2 = \frac{1}{\Delta'(S_2)} \left(\frac{c_2}{J_3} - S_2^2 \right). \tag{38}$$

Before obtaining the final form, (37) is written under the form

$$\psi_1(p) = \frac{M_2(p)}{J_2} \cdot \left(\frac{2S_1 B_1}{p^2 + S_1^2 K} + \frac{2S_2 C_1}{p^2 + S_2^2 K} \right)$$

or

$$\psi_1(p) = -\frac{M_2(p)}{J_2} \cdot \left(\frac{A_1}{p - p_1} + \frac{\overline{A}_1}{p - p_1} + \frac{A_2}{p - p_2} + \frac{\overline{A}_2}{p - p_2} \right), \tag{39}$$

where

$$\begin{cases} A_1 = iB_1;\ A_2 = iC_1;\ \overline{A}_1, \overline{A}_2 \ \text{are conjugate with } A_1 \text{ and } A_2 \\ p_j = -\dfrac{\mu_j}{2} + ik_j, \overline{p}_j = -\dfrac{\mu_j}{2} - ik_j, \mu_j = \dfrac{\psi}{2\pi} S_j, k_j = S_j, \text{j} = \left(\overline{1,2}\right) \end{cases} \quad (40)$$

The results (30) and (39) are the same. Thus, the solving way of (30) will be the same if we introduce the coefficients (40) into (30). Unlike the previous case, instead of a fourth-power equation there is obtained a biquadratic equation

$$s(S) = S^4 - S^2\left[\frac{c_1(J_1 + J_2)}{J_1 \cdot J_2} + \frac{c_2(J_2 + J_3)}{J_2 \cdot J_3}\right] + c_1 c_2 \frac{J_1 + J_2 + J_3}{J_1 \cdot J_2 \cdot J_3} = 0. \quad \text{(g)}$$

In the general case, $\Delta(S)$ is an S even function, because the determinant of the equation $\Delta(S) = 0$ can be two-times reduced. As noted above, there is one more characteristic given by the fact that the roots $\pm S_j$ ($j = 1,2,3...$) are real (R) and not associate complex. All these simplify very much the calculations. The absorption coefficient of the vibrations power is chosen using the condition of coefficients coincidence $\mu_1 = \frac{\psi}{2\pi} S_1 = 2{,}448 s^{-1}$. There are found the roots $S_1 = 266.45 s^{-1}$, $S_2 = 907{,}75 s^{-1}$ and then $\psi/2\pi = 0{,}9188 \cdot 10^{-2}$.

The coefficients (40) will be

$$2k_1 A_1 = 0{,}00145 - 0{,}2046i,\ 2k_2 A_2 = 0{,}00166 - 0{,}7953i. \quad \text{(h)}$$

For the studied case, in figure 2 it is shown only a curve (3) for passing through the second resonance. As a result of choosing the coefficient ψ, the passing curves through the first resonance are practically overlapping for both cases.

6. Conclusions

As we expected, the maximum of the curve (3) is higher than the one of the curve (2). The explanation is that the friction forces considered in this study do not depend on the vibration frequency. They have remained smaller than the forces, which are proportional with the distortion speed within vibrations with higher frequency. In the same time, the maximum of the dynamic coefficient of the first resonance is higher than the two maximum of the second resonance. That happens in the case of the constant amplitude m_0 of the disturbing moment. When m_0 depends on frequency, then the relation between the maximum value of the coefficient λ_1 is changed and may result that the maximum amplitude of vibrations in the second resonance is higher than the first one. Thus, for example with $m_2(t)=m_0\left(\frac{\varepsilon t}{k_2}\right)^2 \exp\left(-i\frac{1}{2}\varepsilon t^2\right)$ and with other similar conditions, the maximum coefficients will be $\lambda_{1t}=2{,}34$ for the first resonance and $\lambda_{1t}=19{,}35$ for the second one.

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