Γ-HYPERMODULES: ISOMORPHISMS AND REGULAR RELATIONS

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The aim of this paper is to introduce the concept of Γ*-hypermodules. Some new characterizations of this hyperstructure are investigated. In particular, we construct three isomorphism theorems of* Γ*-hypermodules. Finally, the regular relations on* Γ*-hypermodules are obtained.*

Keywords: Γ-subhypermodule; Γ-hypermodule; isomorphism theorem; regular relation.
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1. Introduction

The theory of algebraic hyperstructures (or hypersystems) is a well established branch of classical algebraic theory. In the literature, the theory of hyperstructure was first initiated by Marty in 1934 [15] when he defined the hypergroups and began to investigate their properties with applications to groups, rational fractions and algebraic functions. Later on, many people have observed that the theory of hyperstructures also have many applications in both pure and applied sciences, for example, semi-hypergroups are the simplest algebraic hyperstructures which possess the properties of closure and associativity. Some review of the theory of hyperstructures can be found in [7, 9, 19], respectively.

The *Krasner hyperring* [13] is a well known type of hyperring, with the property that the addition is a hyperoperation and the multiplication is a binary operation. This concept has been studied in depth by many authors, for example, see [10]. The concept of hypermodule over a Krasner hyperring has been introduced and investigated by Massouros [16]. Zhan et al. [20] established three isomorphism theorems of hypermodules and derived the Jordan-Holder theorem for hypermodules. In [3], Anvariyeh and Davvaz introduced a new strongly regular equivalence relation on hypermodules so that the quotient is a module over a commutative ring. Further, Anvariyeh et al. [4] considered the fundamental relation θ defined on a hypermodule and proved some results in this respect. Also, they determined a family of subsets of a hypermodule M and gave sufficient conditions such that the geometric space is strongly transitive and the relation θ is transitive.

The concept of Γ-ring was introduced by Barnes [6]. This notion was discussed further by several researchers in connection with their radicals or ideals (see

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[12, 14, 17, 18]). The definition of the Γ-module was given for the first time by Ameri et al. in [2], studying some preliminary properties of them. Besides, some Γ-hyperstuctures have been studied by some researchers in the last years. Ameri et al. [1] considered the concept of hyperideal of Γ-hyperrings. Recently, Anvariyeh et al. [5] discussed the basic properties of Γ-hyperideals in Γ-semihypergroups. After that, Dehkordi et al. [11] investigated the ideals, homomorphisms and regular relations of Γ-semihyperrings.

In this paper, we introduce the concept of Γ-hypermodule. In particular, we prove three isomorphism theorems of Γ-hypermodules and finally, we discuss about the regular relations of Γ-hypermodules.

2. **Preliminaries**

A *hypergroupoid* is a non-empty set H together with a mapping $\circ : H \times H \to \mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ is the set of all the non-empty subsets of H.

A *quasicanonical hypergroup* (not necessarily commutative) is an algebraic structure $(H, +)$ satisfying the following conditions:

(i) for every $x, y, z \in H, x + (y + z) = (x + y) + z$;
(ii) there exists an element $0 \in H$ such that $0 + x = x$, for all $x \in H$;
(iii) for every $x \in H$, there exists a unique element $x' \in H$ such that $0 \in (x + x') \cap (x' + x)$ (we denote it by $-x$ and call it the opposite of x);
(iv) $z \in x + y$ implies that $y \in -x + z$ and $x \in z - y$.

The quasicanonical hypergroups are also called *polygroups.*

We note that, if $x \in H$ and A, B are non-empty subsets of H, then by $A + B$, $A + x$ and $x + B$ we mean that $A + B = \bigcup_{a\in A, b\in B} a + b, A + x = A + \{x\}$ and $x + B = \{x\} + B$, respectively. Also, for all $x, y \in H$, we have $-(-x) = x$, $-0 = 0$, and $-(x + y) = -y - x$.

A subhypergroup $A \subset H$ is said to be *normal* if $x + A - x \subseteq A$, for all $x \in H$. A normal subhypergroup A of H is called *left (right) hyperideal* of H if $xA \subseteq A$ ($Ax \subseteq A$, respectively), for all $x \in H$. Moreover A is said to be a *hyperideal* of H if it is both a left and a right hyperideal of H. A *canonical hypergroup* is a commutative quasicanonical hypergroup.

Definition 2.1. [13] A *hyperring* is an algebraic hyperstructure $(R, +, \cdot)$, which satisfies the following axioms:

(1) $(R, +)$ is a canonical hypergroup;
(2) Relating to the multiplication, $(R, \cdot)$ is a semigroup having zero as a bilaterally absorbing element, that is, $0 \cdot x = x \cdot 0 = 0$, for all $x \in R$;
(3) The multiplication is distributive with respect to the hyperoperation $''+''$ that is, $z \cdot (x + y) = z \cdot x + z \cdot y$ and $(x + y) \cdot z = x \cdot z + y \cdot z$, for all $x, y, z \in R$.

Definition 2.2. [6] Let M and Γ be two additive abelian groups. Then M is called a Γ-*ring* if there is a mapping $(a, \alpha, b) \longmapsto a\alpha b$ from $M \times \Gamma \times M$ to M satisfying the following conditions, for all $a, b, c \in M$ and $\alpha, \beta, \gamma \in \Gamma$:

(i) $a\alpha b \in M$;
(ii) $(a + b)\alpha c = a\alpha c + b\alpha c, a(\alpha + \beta)b = a\alpha b + a\beta b, a\alpha(b + c) = a\alpha b + a\alpha c$;
(iii) $a\alpha(b\beta c) = (a\alpha b)\beta c$.

In [2], Ameri et al. introduced the concept of Γ-module and investigated some basic properties.

Definition 2.3. [2] Let R be a Γ-ring. A left Γ-module is an additive abelian group M together with a mapping from $R \times \Gamma \times M$ to M (the image of the triple (r, γ, m) being denoted by $r\gamma m$), such that, for all $m, m_1, m_2 \in M$ and $\gamma, \gamma_1, \gamma_2 \in \Gamma, r, r_1, r_2 \in R$, the following relations hold:

(1) $r\gamma(m_1 + m_2) = r\gamma m_1 + r\gamma m_2$;
(2) $(r_1 + r_2)\gamma m = r_1\gamma m + r_2\gamma m$;
(3) $r(\gamma_1 + \gamma_2)m = r\gamma_1 m + r\gamma_2 m$;
(4) $r_1\gamma_1(r_2\gamma_2 m) = (r_1\gamma_1 r_2)\gamma_2 m$.

A right Γ-module is defined in an analogous manner. In this paper, a Γ-module means a left Γ-module.

In [1], Ameri et al. generalized it to Γ-hyperrings and obtained some related properties.

Definition 2.4. [1] Let $(R, \oplus)$ and $(\Gamma, \oplus)$ be two canonical hypergroups. Then R is called a Γ-*hyperring*, if there is a mapping $(x, \alpha, y) \longmapsto x\alpha y$ from $R \times \Gamma \times R$ to R satisfying the following conditions, for all $x, y, z \in R$ and for all $\alpha, \beta \in \Gamma$,

(i) $x\alpha y \in R$;
(ii) $(x \oplus y)\alpha z = x\alpha z \oplus y\alpha z, x\alpha(y \oplus z) = x\alpha y \oplus x\alpha z$;
(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

In the sequel, unless otherwise stated, $(R, \oplus, \Gamma)$ always denotes a Γ-hyperring.

Definition 2.5. [1] A subset A in R is said to be a *left (right)* Γ-*hyperideal* of R if it satisfies the following conditions:

(1) $(A, \oplus)$ is a normal subhypergroup of $(R, \oplus)$;
(2) $x\alpha y \in A$ ($y\alpha x \in A$, respectively), for all $x \in R$, $y \in A$ and $\alpha \in \Gamma$.

A is said to be a Γ- *hyperideal* of R if it is both a left and a right Γ-hyperideal of R.

3. Γ-hypermodules

In this section, we introduce the concept of Γ-hypermodules, which is a generalization of hypermodules and Γ-modules. We concentrate our study on the Γ-hypermodules and give three isomorphism theorems of them.

Definition 3.1. Let $(R, \oplus, \Gamma)$ be a Γ-hyperring and $(M, \oplus)$ be a canonical hypergroup. M is called a Γ-*hypermodule* over R if there exists a mapping f : $R \times \Gamma \times M \to M$ (the image of (r, α, m) being denoted by $r\alpha m$) such that, for all $a, b \in R, m_1, m_2 \in M$, and $\alpha, \beta \in \Gamma$, we have (i) $a\alpha(m_1 \oplus m_2) = a\alpha m_1 \oplus a\alpha m_2$; (ii) $(a \oplus b)\alpha m_1 = a\alpha m_1 \oplus b\alpha m_1$;

(iii) $a(\alpha \oplus \beta)m_1 = a\alpha m_1 \oplus a\beta m_1$; (iv)$(a\alpha b)\beta m_1 = a\alpha(b\beta m_1)$.

Throughout this paper, R and M are a Γ-hyperring and a Γ-hypermodule, respectively, unless otherwise specified.

Example 3.2. Let $(R, +, \Gamma)$ be a Γ-ring and G be a Γ-subsemigroup of the Γ-semigroup (R, Γ) which satisfies, for all $a, b \in R$, the condition $a\alpha G\gamma b\beta G = a\alpha\gamma b\beta G$. Let $(M, +, \Gamma)$ be a Γ-module. Define the following equivalence on M:

$x\rho y \Leftrightarrow \exists t \in G$ and $\alpha \in \Gamma$ such that $x = y\alpha t$.

Define the hyperoperation $\oplus$ on M/ρ:

$R/G \times \Gamma \times M/\rho \to M/\rho$ by $(\overline{r}, \alpha, \overline{x}) \mapsto \overline{r\alpha x}$, for all $\overline{r} \in R/G, \overline{x} \in M/\rho$ and $\alpha \in \Gamma$. Then M/ρ is a Γ-hypermodule.

Example 3.3. Let $M = \mathbb{R}^2$ and $(\Gamma, \oplus)$ be a canonical hypergroup with the following property: for all $a \in \mathbb{R}$ and $\alpha \in \Gamma$,

$$a\alpha(x,y) = \begin{cases} \{(u,v)\} \in \mathbb{R}^2 | x\alpha u = y\alpha v\}, & \text{if} \quad (x,y) \in \mathbb{R}^2 - \{(0,0)\}, \\ \{(0,0)\} & \text{otherwise} \end{cases}$$

Then $\mathbb{R}^2$ is a Γ-hypermodule.

Definition 3.4. A subset A in M is said to be a Γ-*subhypermodule* of M if it satisfies the following conditions:

(1) $(A, \oplus)$ is a subhypergroup of $(M, \oplus)$;
(2) $r\alpha x \in A$, for all $x \in R$, $\alpha \in \Gamma$ and $x \in A$.

A Γ-subhypermodule A of M is called *normal* if $x + A - x \subseteq A$, for all $x \in M$.

Definition 3.5. If M and M' are Γ-hypermodules, then a mapping $f : M \longrightarrow M'$ such that $f(x \oplus y) = f(x) \oplus f(y)$ and $f(r\alpha x) = r\alpha f(x)$, for all $r \in R$, $\alpha \in \Gamma$ and $x \in M$, is called a Γ-*hypermodule homomorphism.*

Clearly, a Γ-hypermodule homomorphism f is an isomorphism if f is injective and surjective. We write $M \cong M'$ if M is isomorphic to M'.

The following proposition is obvious, therefore the proof is omitted.

Proposition 3.6. Let $f : M \longrightarrow M'$ be a Γ-hypermodule homomorphism, then the kernel $Kerf = \{x \in M | f(x) = 0\}$ is a Γ-subhypermodule of M.

If A is a normal Γ-subhypermodule of M, then we define the relation A^* by

$$xA^*y(modA) \Longleftrightarrow (x-y) \bigcap A \neq \emptyset.$$

The following proposition is obtained exactly from the definitions.

Proposition 3.7. Let A be a normal Γ-subhypermodule of M, then

(1) A^* is an equivalence relation.
(2) If $A^*[x]$ is the equivalence class of the element $x \in M$, then $A \oplus x = A^*[x]$.
(3) For all $x, y \in M, A^*[A^*[x] \oplus A^*[y]] = A^*[x] \oplus A^*[y]$.
(4) For all $r \in R, \alpha \in \Gamma$ and $x \in M, A^*[A^*[r\alpha x]] = A^*[r\alpha x]$.

Let A be a normal Γ-subhypermodule of M, then set $[M : A^*] = \{A^*[x] | x \in M\}$.

Define a hyperoperation $\boxplus$ and an operation $\odot_\alpha$ on $[M : A^*]$ by

$$A^*[x] \boxplus A^*[y] = \{A^*[z] | z \in A^*[x] \oplus A^*[y]\},$$

$$A^*[x] \odot_\alpha A^*[y] = A^*[x\alpha y],$$

for all $r \in R$, $\alpha \in \Gamma$ and $x \in M$.

From the above discussion, we can get the following result.

Theorem 3.8. $([M : A^*], \boxplus, \odot_\alpha)$ *is a* Γ-*hypermodule.*

Next, we establish three Isomorphism Theorems of Γ-hypermodules.

Theorem 3.9 (First Isomorphism Theorem). Let f be a Γ-hypermodule homomorphism from M_1 into M_2 with the kernel K, such that K is a normal Γ-subhypermodule of M_1. Then we have $[M_1 : K^*] \cong Imf$.

Proof. Define $\rho : [M_1 : K^*] \to Imf$ by considering $\rho(K^*[x]) = f(x)$, for all $x \in M_1$. Then ρ is clearly well-defined. Moreover, if we suppose that xK^*y, then $x-y \bigcap K \neq \emptyset$. This means that there exists $z \in x-y \bigcap K$, and therefore $f(z) = 0$. It follows now that $0 = f(z) \in f(x) - f(y)$, and so $f(x) = f(y)$. This shows that ρ is surjective. In order to show that ρ is injective, we let $f(x) = f(y)$. Then we have $0 \in f(x-y)$, and this means that there exists $z \in x-y$ with $z \in Kerf$. Thus, $x - y \bigcap K \neq \emptyset$, which implies that $K^*[x] = K^*[y]$, and hence ρ is indeed injective. Moreover, we can deduce the following equalities:

(1)

$$\rho(K^*[x] \boxplus K^*[y]) = \rho(\{K^*[z] | z \in K^*[x] \oplus K^*[y]\}) = \{f(z) | z \in K^*[x] \oplus K^*[y]\}$$

$$= f(K^*[x]) \oplus f(K^*[y]) = f(x) \oplus f(y) = \rho(K^*[x]) \oplus \rho(K^*[y]);$$

(2)

$$r \odot_\alpha \rho(K^*[x]) = \rho(K^*[r\alpha x]) = f(r\alpha x) = r\alpha f(x) = r\alpha\rho(K^*[x])$$

.

Hence, ρ is an isomorphism.

From the above theorem, we can easily get the following two theorems.

Theorem 3.10 (Second Isomorphism Theorem). If A and B are Γ-subhypermodules of M with B normal in M, then we have

$$[A : (A \bigcap B)^*] \cong [(A+B) : B^*].$$

Theorem 3.11. (Third Isomorphism Theorem). If A and B are normal Γ-subhypermodules of M such that $A \subseteq B$, then $[B : A^*]$ is a normal Γ-subhypermodule of $[M : A^*]$ and

$$[[M : A^*] : [B : A^*]] \cong [M : B^*].$$

Finally, we consider the Jordan-Holder Theorem of Γ-hypermodules. The proof is similar to that of Theorem 2.15 in [20] and we omit here the details. First we introduce the following notion:

Definition 3.12. A *finite chain* of $n+1$ Γ-hypermodules of M is a composition series of M with the length n, $M = A_0 \supset A_1 \supset A_2 \supset \ldots \supset A_n = 0$, where $[A_{i-1} : A_i^*]$ is simple ($i = 1, 2, \ldots, n$), that is, every term of the chain is maximal in its predecessor.

Theorem 3.13. (Jordan-Holder Theorem). If a Γ-hypermodule M has some composition series, then any two of them are equivalent, which means that the composition quotient Γ-hypermodules are isomorphic in pairs, though they may occur in different orders in the sequences.

4. Regular relations

Let M be a Γ-hypermodule and θ be an equivalence relation on M. Then one may extend θ to the subsets of M by $\overline{\theta}$ and $\overline{\overline{\theta}}$ as follows.

Let A, B be non-empty subsets of M. Define

$A\overline{\theta}B \Leftrightarrow \forall a \in A, \exists b \in B$ such that $a\theta b$ and $\forall b \in B, \exists a \in A$, such that $b\theta a$

$A\overline{\overline{\theta}}B \Leftrightarrow \forall a \in A, \forall b \in B$, one has $a\theta b$,

where by $a\theta b$, we mean $(a, b) \in \theta$.

An equivalence relation θ on M is called *regular*(respectively *strongly regular*) if, for all $a, b, x \in M, r \in R$ and $\alpha \in \Gamma$, we have

(i) $a\theta b \Rightarrow (a+x)\overline{\theta}(b+x)$ and $(x+a)\overline{\theta}(x+b)$

(respectively, $a\theta b \Rightarrow (a+x)\overline{\overline{\theta}}(b+x)$ and $(x+a)\overline{\overline{\theta}}(x+b)$);

(ii) $a\theta b \Rightarrow (r\alpha a)\overline{\theta}(r\alpha b)$ (respectively, $a\theta b \Rightarrow (r\alpha a)\overline{\overline{\theta}}(r\alpha b)$).

Recall the results from [11], we can get the following results.

Theorem 4.1. Let M be a Γ-hypermodule and θ a regular relation on M. Then $M/\theta = \{\theta(a)|a \in M\}$ is a Γ-hypermodule with respect to the following hyperoperations: $\theta(a) \boxplus \theta(b) = \{\theta(c)|c \in \theta(a) \oplus \theta(b)\}$,$\theta(a) \odot_\alpha \theta(b) = \theta(a)\alpha\theta(b)$.

Proof. Firstly we show that $\{\theta(c)|c \in \theta(a)\oplus\theta(b)\} = \theta(a\oplus b)$ and $\theta(a)\alpha\theta(b) = \theta(a\alpha b)$.

Let $x \in a \oplus b$, then $x \in \theta(a) \oplus \theta(b)$. Hence $\theta(a \oplus b) \subseteq \{\theta(c)|c \in \theta(a) \oplus \theta(b)\}$. On the other hand, let $x \in \theta(a)\oplus\theta(b)$; then there exist $x_1 \in \theta(a)$ and $x_2 \in \theta(b)$ such that $x \in x_1 \oplus x_2$. Since θ is a regular relation and having $x_1\theta a$ and $x_2\theta b$, it follows that $(x_1 \oplus x_2)\overline{\theta}(a \oplus b)$. Hence, since $x \in x_1 \oplus x_2$, there exists $c \in a \oplus b$ such that $\theta(x) = \theta(c)$, which implies that $\{\theta(c)|c \in \theta(a) \oplus \theta(b)\} \subseteq \theta(a \oplus b)$. In the same way, one proves that $\theta(a)\alpha\theta(b) = \theta(a\alpha b)$.

Secondly, we show that the hyperoperations $\boxplus$ and $\odot_\alpha$ are well-defined. Let $a, b, a_1, b_1 \in M, r \in R$ and $\alpha \in \Gamma$ such that $\theta(a) = \theta(a_1)$ and $\theta(b) = \theta(b_1)$. Since θ is a regular relation and $a\theta a_1$, $b\theta b_1$, it follows that $(a \oplus b)\overline{\theta}(a_1 \oplus b_1)$. This means that, for any $u \in a\oplus b$, there exists $v \in a_1 \oplus b_1$ such that $\theta(u) = \theta(v) \in \theta(a_1 \oplus b_1)$. thereby $\theta(a \oplus b) \subset \theta(a_1 \oplus b_1)$. The converse inclusion may be proven in a similar way.

Finally, by a similar procedure, one proves that $\theta(a) = \theta(a_1)$ implies that $(r\alpha a)\overline{\theta}(r\alpha a_1)$ and thus $\theta(r\alpha a) = \theta(r\alpha a_1)$.

Now, the conditions of Definition 3.1 follow directly.

Corollary 4.2. Let M be a Γ-hypermodule and θ a strong regular relation on M. Then $M/\theta = \{\theta(a)|a \in M\}$ is a Γ-module with the above operations.

Definition 4.3. If M and M' are Γ and Γ'-hypermodules, respectively, then (φ, f) is called a (Γ, Γ')-*hypermodule homomorphism* if, for all $r \in R$, $\alpha \in \Gamma$ and $x, y \in M$, the following relations are satisfied: $\varphi(x \oplus y) = \varphi(x) \oplus \varphi(y)$, $\varphi(r\alpha x) = rf(\alpha)\varphi(x)$ and $f(x \oplus y) = f(x) \oplus f(y)$.

Theorem 4.4. Let M_1 be a Γ_1-hypermodule and θ be a regular relation on M_1. Then $\pi : M_1 \to M_1/\theta$ is a canonical Γ-hypermodule homomorphism. Suppose that M_2 is a Γ-hypermodule and $(\phi, f) : M_1 \to M_2$ is a (Γ_1, Γ_2)-hypermodule homomorphism. Then the relation $\theta = \{(a, b) \in M_1 \times M_2 \mid \phi(a) = \phi(b)\}$ is a regular relation on M_1 and there exists a (Γ_1, Γ_2)-hypermodule homomorphism $(\psi, id) : M_1/\theta \to M_2$ such that $\psi\pi = \phi$.

Proof. Let us consider $r \in R$, $\alpha \in \Gamma$ and $a, b \in M$. Then

$$\pi(x \oplus y) = \{\pi(t)|t \in x \oplus y\} = \{\theta(t)|t \in x \oplus y\} = \theta(x) \boxplus \theta(y) = \pi(x) \boxplus \pi(y)$$

and

$$\pi(r\alpha x) = \theta(r\alpha x) = r\alpha\theta(x) = r\alpha\pi(x).$$

Hence, π is a canonical Γ-hypermodule homomorphism.

Clearly, θ is an equivalence relation. In order to show that θ is a regular relation, we set $a\theta a_1$ and $b\theta b_1$; then $\phi(a) = \phi(a_1)$ and $\phi(b) = \phi(b_1)$. Hence, for every $\alpha \in \Gamma_1, r \in R$, we have $\phi(a \oplus b) = \phi(a_1 \oplus b_1)$ and $\phi(r\alpha a) = \phi(r\alpha a_1)$, which implies that $(a \oplus b)\overline{\theta}(a_1 \oplus b_1)$ and $(r\alpha a)\overline{\theta}(r\alpha a_1)$.

Define $\psi : M_1/\theta \to M_2$ by $\psi(\theta(a)) = \phi(a)$. Since $\theta(a) = \theta(b)$ if and only if $\phi(a) = \phi(b)$ if and only if $\psi(\theta(a)) = \psi(\theta(b))$, it follows that ψ is well-defined.

For all $a, b \in M, r \in R$ and $\alpha \in \Gamma$, we have

$$\psi(\theta(a) \boxplus \theta(b)) = \{\psi(\theta(a \oplus b))\} = \psi\{\theta(t) \mid t \in a \oplus b\} = \psi((\theta(a)) \oplus \psi(\theta(b))$$

and

$$\psi(r\alpha\theta(a)) = \psi(\theta(r\alpha a)) = \phi(r\alpha a) = rf(\alpha)\phi(a) = rf(\alpha)\psi(\theta(a)).$$

Thus, (ψ, id) is a (Γ_1, Γ_2)-hypermodule homomorphism.

Theorem 4.5. Let θ be a regular relation on a Γ-hypermodule M and $(\phi, f) : M_1 \to M_2$ be a (Γ_1, Γ_2)-hypermodule homomorphism such that $\theta \subseteq \{(a, b) | \phi(a) = \phi(b)\}$. Then there exists a unique (Γ_1, Γ_2)-hypermodule homomorphism $(\psi, id) : M_1/\theta \to M_2$ such that $\psi\pi = \phi$, where $\pi : M_1 \to M_1/\theta$ is a canonical Γ-hypermodule homomorphism.

Proof. Define $\psi : M_1/\theta \to M_2$ by $\psi(\theta(a)) = \phi(a)$. As in the proof of Theorem 4.4, one proves that ψ is well-defined and a (Γ_1, Γ_2)-hypermodule homomorphism. Finally, it is obvious that ψ is a unique homomorphism such that $\psi\pi = \phi$.

5. Conclusions

In this paper, we have considered the Γ-hypermodules as a generalization of the notion of Γ-module. In particular, we have given three isomorphism theorems of Γ-hypermodules and we have discussed about regular relations on Γ-hypermodules.

In our future research concerning the Γ-hypermodules, the following topics could be considered: 1) To define n-ary Γ-hypermodules; 2) To define soft Γ-hypermodules.

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