CONSISTENT CONTROL FOR SECOND ORDER MULTI-UAV SYSTEM BASED ON EVENT-TRIGGERED MECHANISM

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Based on the event-triggered mechanism, the consistency control problem for the second-order multi-UAV systems with the fixed directed topology is investigated. In order to further reduce the number of system triggers, the event-triggered mechanism whose threshold value depends on the combination of the real-time state error in the multi-UAV systems and the decay time function is proposed. The decay function is used as a dynamic parameter to give the control strategy under the event-triggered mechanism. Information interaction and control input updates are performed between UAVs when the change in UAV state errors satisfies the trigger condition. Compared with the static event-triggered mechanism, the event-triggered mechanism proposed in this paper can further reduce the number of event triggers. Consistency control is proved by Lyapunov theory and it is shown that the system does not have Zeno behavior. Finally, the effectiveness of the control strategy is verified through simulation.

Keywords: event-triggered mechanism; multi-UAV system; consistent control; zeno behavior

1. Introduction

In nature, some organisms tend to move in groups, such as birds migrating or fish feeding, etc. Through an in-depth analysis for the behavioral mechanisms of biological groups, it can be found that organisms can move autonomously through the information obtained from other individuals at a local scale, ultimately achieving a common goal. Inspired by this phenomenon, Multi-UAV has been linked by communication networks to form multi-agent systems, which emulate biological communities to achieve clustered missions.

In recent years, the multi-agent system has become a research hotspot in the field of control, of which the study for the multi-UAV system occupies a very important position [1-4]. Compared to a single UAV, the multi-UAV can solve more complex problems and increase efficiency. Along with the development of UAVs, the number of UAVs in a multi-UAV system has increased from a few to dozens or even more, which brings up the issue of cooperative control.

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The cooperative control of multi-UAV system mainly includes issues such as UAV models, communication connections and cooperative control algorithms. The greater the number of UAVs in the system, the more complex the control and information interaction between the UAVs will become. The consistent control algorithms can describe the information interaction process between UAVs very well. The consistent control problem is the basis of the cooperative control for multi-UAV systems, and is also a hot topic of the current research [5-8].

The consistent control for multi-UAV system requires effective information sharing between UAVs. However, system network bandwidth and computing power are often limited, which cannot guarantee the information interaction needs of all UAVs. When the number of UAVs exceeds the limit of the system, communication congestion and other problems will occur, which will cause multi-UAV system failures, or even crashes. In order to save communication and computational resources, the number of communications and the amount of computations should be reduced on the premise of system consistency. The linear sampled data controllers for multi-agent system were studied in [9]. In [10], the consistent protocol for multi-agent linear systems under sampling pulse control is presented, and the consistent problem of second-order multi-agent systems with time-varying topologies under this control protocol is investigated. However, no disturbance is considered in the state in the paper.

In addition to the sampling control approach, the event-triggered mechanism has more significant applications for consistent control of multi-UAVs [11-13]. The event-triggered mechanism is based on pre-determined trigger conditions. If the control task satisfies the trigger condition, the system executes the trigger task and carries out information transfer between neighbouring UAVs or controller update. There are continuous event triggers and discrete event triggers. It can also be divided into dynamic event triggers and asynchronous event triggers. In addition, according to the different objects targeted by the event triggering conditions, it can be divided into edge-based event-triggered mechanism and point-based event-triggered mechanism. Edge-based event triggering is based on the connection of edges between UAVs. When the event triggering conditions are met, the UAVs on the edge interact with each other. On the other hand, point-based event triggering is specific to each UAV in the system. When the trigger condition is met, the UAV interacts with all its neighbours. In [11], the consensus criteria related with the general algebraic connectivity of communication topology under event-triggered mechanism are derived. In [12], a distributed event-triggered fault-tolerant consistency protocol is proposed, which is used to transform the fault-tolerant consistency of distributed multi-agent systems into the stability of time-delay systems. However, these literatures do not consider the output disturbances in models.

Since the sampling time of event-triggered control depends on whether the trigger conditions can be satisfied, it can effectively reduce the number of control task executions, save communication and computational resources, and solve the problem of network congestion to a certain extent. Event-triggered control strategies have been extensively studied in multi-UAV systems. In [13], event-triggered control was used for consistency control in the multi-agent non-linear system. In [14-17], the different event-triggered mechanisms were applied to the linear multi-agent systems and the consistency under control strategies was demonstrated. Based on event-triggered strategies, in [18-22], control studies were conducted for multi-agent systems with fixed directed topologies. Currently, in most studies, for example, in [14], [16], [19], etc., the fixed-parameter type of event-triggered mechanism is used. This type of event-triggered mechanism depends only on the state error change during the triggering process for the threshold change, and it will potentially fluctuate uncertainly with the state error change.

In order to cope with the problem of the limited communication resources and computational power in multi-UAV, based on the event-triggered mechanism, the consistency control problem for the second-order multi-UAV systems is investigated in this paper. The main contributions of the paper are as follows. Firstly, the threshold of the event-triggered mechanism is considered in combination with the real-time state error and the time decay function, and the decay function is used as a dynamic parameter. A consistency control strategy is given to enable a multi-UAV model with disturbances in both the position and velocity states to achieve a consistent state. The control algorithm is also analysed and proved through matrix theory and Lyapunov theory. Secondly, in this paper, the decay function is used as the time-dynamic parameter. In the event triggering process, the threshold change not only depends on the state error change, but also limits the threshold continuously with the event triggering process, so as to avoid unnecessary event triggering when the threshold becomes larger or smaller with the state error. The number of event triggering is further reduced compared to the fixed-parameter event-triggered mechanism. Finally, the validity of the control strategy is verified by simulation.

2. System Statement

The multi-UAV system with N nodes is considered in this paper. The fixed topology graph G of this system contains only one directed spanning tree $G=(V,E,W)$. $V=\{v_1,v_2,v_3,\ldots,v_N\}$ is the set of nodes of the graph G. E denotes the set of directed edges. W is the weight matrix between the UAVs in the graph G, whose elements w_{ij} denote the weights between UAV *i* and UAV *j*. N_i is the neighbors set of node i. When the directed communication topology graph G contains a directed spanning tree, the corresponding Laplacian matrix is denoted by

L. L has one eigenvalue of zero and all the real parts of the other non-zero eigenvalues of this matrix are positive. According to the [22], the N eigenvalues of L are $0=\lambda_1(L)\le\cdots\le\lambda_N(L)=\lambda_{\max}(L)$ in ascending order.

Considering the second-order dynamics model for the i_{th} UAV in the fixed directed topology system:

$$\begin{cases}\dot{x}_i(t)=v_i(t)+h_i(t)\\ \dot{v}_i(t)=u_i(t)+f_i(t)\end{cases} \tag{1}$$

where, $x_i(t)$ is the position state. $v_i(t)$ is the velocity state. $u_i(t)$ is the control input. $h_i(t)$ and $f_i(t)$ are disturbances.

Assumption 1 There exist scalars $\beta_1>0,\beta_2>0$, such that the disturbance terms $h_i(t)$ and $f_i(t)$ satisfy the following inequalities

$$\begin{cases}\left\|h_i(t)-h_1(t)\right\|\le\beta_1\left\|x_i(t)-x_1(t)\right\|\\ \left\|f_i(t)-f_1(t)\right\|\le\beta_2\left\|v_i(t)-v_1(t)\right\|\end{cases} \tag{2}$$

$\beta_1\le\beta,\beta_2\le\beta$, β is scalars.

For the actual system, considering the different effects of disturbances on the velocity state and position state, different parameters are chosen to represent in Assumption 1.

Definition 1 In the multi-UAV system, the position and velocity state information satisfies the equation (3), and then system (1) is capable of achieving consistency.

$$\begin{cases}\lim\limits_{t\to\infty}\left\|x_i(t)-x_j(t)\right\|=0\\ \lim\limits_{t\to\infty}\left\|v_i(t)-v_j(t)\right\|=0\end{cases}\quad i=1,2,3,\cdots,N \tag{3}$$

3. Main Results

Let $x_i(t_k)$ and $v_i(t_k)$ be the position and velocity sampling state of the i_{th} UAV, and t_k be the sampling moment. The fixed directed graph topology of system (1) contains a directed spanning tree. Similar to the [18], let the consistent control strategy for position and velocity state information be as follows

$$u_i(\mathrm{t})=-k_0\sum_{j=N_i}w_{ij}\Big(\big(x_i(t_k)-x_j(t_k)\big)+k_0\big(v_i(t_k)-v_j(t_k)\big)\Big) \tag{4}$$

where $k_0>0$, N_i denotes the neighbours set of the i_{th} UAV.

Let

$$\begin{cases} e_{x_i}(t) = x_i(t) - x_1(t) \\ e_{v_i}(t) = v_i(t) - v_1(t) \end{cases} \quad i = 1, 2, \cdots, N \tag{5}$$

Then, substituting the equation (5) into the equations (1) and (4)

$$\begin{cases} \dot{e}_{x_i}(t) = e_{v_i}(t) + h_i(t) - h_1(t) \\ \dot{e}_{v_i}(t) = -k_0 \sum\limits_{j=N_i} w_{ij} \left(\left(e_{x_i}(t_k) - e_{x_j}(t_k) \right) + k_0 \left(e_{v_i}(t_k) - e_{v_j}(t_k) \right) \right) \\ -k_0 \sum\limits_{j=N_1} w_{1j} \left(e_{x_j}(t_k) + k_0 e_{v_j}(t_k) \right) + f_i(t) - f_1(t) \end{cases} \tag{6}$$

Let

$$\begin{cases} e_x(t) = \left(e_{x_2}{}^T(t), e_{x_3}{}^T(t), \cdots, e_{x_N}{}^T(t) \right)^T \\ e_v(t) = \left(e_{v_2}{}^T(t), e_{v_3}{}^T(t), \cdots, e_{v_N}{}^T(t) \right)^T \\ w_i = \left(w_{i1}, \cdots, w_{i(i-1)}, w_{i(i+1)}, \cdots, w_{iN} \right)^T \end{cases} \tag{7}$$

According to the [22]

$$L_{22} = \begin{pmatrix} d_2 & -w_{23} & \cdots & -w_{2N} \\ -w_{32} & d_3 & \cdots & -w_{3N} \\ \vdots & \vdots & & \vdots \\ -w_{N2} & -w_{N3} & \cdots & d_N \end{pmatrix} \tag{8}$$

$$H = L_{22} + 1_{N-1} w_1^T \tag{9}$$

Then, the equation (6) can be transformed into the equation (10) as follows

$$\begin{cases} \dot{e}_x = e_v + \left[h_2(t) - h_1(t), h_3(t) - h_1(t), \right. \\ \left. \cdots, h_N(t) - h_1(t) \right]^T \\ \dot{e}_v = -k_0 H \otimes I_n e_x(t_k) - k_0{}^2 H \otimes I_n e_v(t_k) \\ + \left[f_2(t) - f_1(t), f_3(t) - f_1(t), \cdots, f_N(t) - f_1(t) \right]^T \end{cases} \tag{10}$$

To simplify writing, $x(t)$ is written as x and $v(t)$ is written as v. Let

$$\begin{cases} \overline{e}_x(t) = e_x(t_k) - e_x(t) \\ \overline{e}_v(t) = e_v(t_k) - e_v(t) \end{cases} \tag{11}$$

Substituting the equation (11) into the equation (10)

$$\begin{cases} \dot{e}_x = e_v + \left[h_2(t) - h_1(t), h_3(t) - h_1(t), \right. \\ \left. \cdots, h_N(t) - h_1(t) \right]^T \\ \dot{e}_v = -k_0 H \otimes I_n \left(e_x(t) + \overline{e}_x(t) \right) - k_0{}^2 H \otimes I_n \left(e_v(t) + \overline{e}_v(t) \right) \\ + \left[f_2(t) - f_1(t), f_3(t) - f_1(t), \cdots, f_N(t) - f_1(t) \right]^T \end{cases} \tag{12}$$

Define $e_{xv}(t)$ as the state error, i.e.

$$e_{xv}(t) = \left(\overline{e}_x{}^T(t), \overline{e}_v{}^T(t) \right)^T$$

Let

$$\eta(t) = \left(e_x{}^T(t), e_v{}^T(t) \right)^T$$

Then

$$\begin{aligned} \dot{\eta}(t) = {} & M \otimes I_n \eta(t) + J \otimes I_n e_{xv}(t) \\ & + \left[h_2(t) - h_1(t), \cdots, h_N(t) - h_1(t), \right. \\ & \left. f_2(t) - f_1(t), \cdots, f_N(t) - f_1(t) \right]^T \end{aligned} \tag{13}$$

Where

$$M = \begin{pmatrix} 0_{(N-1)\times(N-1)} & I_{N-1} \\ -k_0 H & -k_0^2 H \end{pmatrix},$$

$$J = \begin{pmatrix} 0_{(N-1)\times(N-1)} & 0_{(N-1)\times(N-1)} \\ -k_0 H & -k_0^2 H \end{pmatrix}.$$

Referring to [23], if the fixed topology graph of the system contains only one directed spanning tree, then the eigenvalues of the matrix H have positive real parts and there exists a positive definite matrix P_1 such that the following equation (14) holds

$$P_1 H + H^T P_1 = I_{N-1} \tag{14}$$

Considering that both the position and velocity states of multi-UAV system can be disturbed in practical application, and that multi-UAV system consistency control does not only occur during the initial time period. In the process of maintaining consistency, the consistency state may be corrupted, and then the update control input needs to be triggered again to bring the system back to the consistency state. Therefore, the event-triggered function shown in equation (15) is designed.

$$g_i\left(e_{xv}(t)\right)=\left\|e_{xv}(t)\right\|-e^{-\alpha\left(t-t_{k_0}\right)}\frac{\lambda_{\min}\left(P_3\right)-2\beta\lambda_{\max}\left(P_2\right)}{2k_0\sqrt{k_0^2+1}\left\|H\right\|\lambda_{\max}\left(P_2\right)}\left\|\eta(t)\right\| \tag{15}$$

Where

$$P_2=\begin{pmatrix}k_0 & 1\\ 1 & k_0\end{pmatrix}\otimes P_1,\ P_3=-\left(M^TP_2+P_2M\right).$$

$\lambda_{\min}\left(P_3\right)-2\beta\lambda_{\max}\left(P_2\right)>0$. $\alpha>0$ is a constant.

Then, the next trigger moment t_{k+1}^i is shown in equation (16).

$$t_{k+1}^i=\inf\left\{t>t_k^i\left|g_i\left(e_{xv}\right)>0\right.\right\} \tag{16}$$

The control input is updated when $g_i\left(e_{xv}\right)>0$. t_{k_0} is the initial moment or the initial moment when system consistency is broken. When the system reaches the consistent state, t_{k_0} is set to zero to wait for the next event to trigger an update.

Theorem 1 When the communication topology graph of system (1) contains a directed spanning tree and satisfies Assumption 1, then there exists $k_0>1$, $\beta_1<\frac{\lambda_{\min}\left(P_3\right)}{2\lambda_{\max}\left(P_2\right)}$, $\beta_2<\frac{\lambda_{\min}\left(P_3\right)}{2\lambda_{\max}\left(P_2\right)}$, such that the control strategy (4) and event-triggered mechanism (15) are feasible in system (1), i.e., the multi-UAV system is guaranteed to achieve consistent control. And the event-triggered interval $t_{k+1}-t_k\geq\tau$, i.e., Zeno behavior will not be triggered in the system. Where

$$\tau=\frac{1}{\left\|M\right\|-\left\|J\right\|}\ln\frac{f(\tau,0)+1}{f(\tau,0)\frac{\left\|J\right\|}{\left\|M\right\|}+1},$$

$$f(\tau,0)=e^{-\alpha\left(t_k-t_{k_0}\right)}\frac{\lambda_{\min}\left(P_3\right)-2\beta\lambda_{\max}\left(P_2\right)}{2k_0\sqrt{k_0^2+1}\left\|H\right\|\lambda_{\max}\left(P_2\right)}$$

Proof: The Lyapunov function is constructed as follows

$$V = \eta^T(t)(P_2 \otimes I_n)\eta(t) \tag{17}$$

P_1 is the positive definite matrix and satisfies the equation (14). Then it follows that P_2 is the positive definite matrix by the Lemma 1, and $k_0 > 1$.

Derivation of the equation (17) yields

$$\begin{aligned}\dot{V} &= \eta^T(t)\left(M^T P_2 + P_2 M\right) \otimes I_n \eta(t) + 2\eta^T(t) P_2 J \otimes I_n e_{xv} \\ &+ \left[h_2(t) - h_1(t), \cdots, h_N(t) - h_1(t),\right. \\ &\left. f_2(t) - f_1(t), \cdots, f_N(t) - f_1(t)\right](P_2 \otimes I_n)\eta(t) + \eta^T(t)(P_2 \otimes I_n) \\ &\left[h_2(t) - h_1(t), \cdots, h_N(t) - h_1(t), f_2(t) - f_1(t), \cdots, f_N(t) - f_1(t)\right]^T\end{aligned} \tag{18}$$

Let

$$Q = \left[h_2(t) - h_1(t), \cdots, h_N(t) - h_1(t), f_2(t) - f_1(t), \cdots, f_N(t) - f_1(t)\right]^T$$

Then the equation (18) can be transformed into

$$\begin{aligned}\dot{V} &= \eta^T(t) P_3 \otimes I_n \eta(t) + 2\eta^T(t) P_2 J \otimes I_n e_{xv} \\ &+ Q^T(P_2 \otimes I_n)\eta(t) + \eta^T(t)(P_2 \otimes I_n)Q\end{aligned} \tag{19}$$

Here

$$P_3 = \begin{pmatrix} k_0 I_{N-1} & k_0^2 I_{N-1} - k_0 P_1 \\ k_0^2 I_{N-1} - k_0 P_1 & k_0^3 I_{N-1} - 2P_1 \end{pmatrix}.$$

It follows from the Lemma 1 that P_3 must be a positive definite matrix when $k_0 > \sqrt{1 + \frac{\lambda_{\max}(P_1)}{2}}$. Then according to assumption 1

$$\|Q\| \le \left\|\left(\beta_1 e_x^{\ T}(t), \beta_2 e_v^{\ T}(t)\right)^T\right\| = \|\eta(t)\|$$

Because of $\beta_1 \le \beta,\ \beta_2 \le \beta$, and it can know that $\|Q\| \le \beta\|\eta\|$.

Further,

$$\dot{V} \le -\lambda_{min}(P_3)\|\eta(t)\|^2 + 2\lambda_{\max}(P_2)\|\eta(t)\|\|J\|\|e_{xv}\| + 2\|P_2\|\|Q\|\|\eta(t)\| \tag{20}$$

Meanwhile, because of $\|J\| = k_0\sqrt{1 + k_0^2}\,\|H\|$, then

$$
\begin{aligned}
\dot{V} \le & -\|\eta(t)\|\left(\lambda_{min}(P_3)\|\eta\| - 2\beta\|P_2\|\|\eta(t)\| - \right.\\
& \left. 2k_0\sqrt{1+k_0^2}\lambda_{\max}(P_2)\|H\|\|e_{xv}(t)\|\right) + 2\beta\|P_2\|\|\eta(t)\|^2 \\
\le & -\|\eta\|\left(\lambda_{min}(P_3)\|\eta(t)\| - 2\beta\|P_2\|\|\eta(t)\| - \right.\\
& \left. 2k_0\sqrt{1+k_0^2}\lambda_{\max}(P_2)\|H\|\|e_{xv}(t)\|\right) + 2\beta\lambda_{\max}(P_2)\|\eta(t)\|^2 \\
\le & -\|\eta(t)\|\left(\lambda_{min}(P_3)\|\eta(t)\| - 2\beta\lambda_{\max}(P_2)\|\eta(t)\| \right.\\
& \left. -2\beta\|P_2\|\|\eta(t)\|\right) - 2k_0\sqrt{1+k_0^2}\lambda_{\max}(P_2)\|H\|\|e_{xv}(t)\|\Big)
\end{aligned}
\tag{21}
$$

If system consistency holds, then the event trigger $g_i(e_{xv}) \le 0$ holds, and then further equation (22) can be obtained.

$$
\dot{V} \le -\left(1 - e^{-\alpha\left(t_k - t_{k_0}\right)}\right)\left(\lambda_{\min}(P_3) - 2\beta\lambda_{\max}(P_2)\right)\|\eta(t)\|^2 \tag{22}
$$

Since $\alpha > 0$ and $t_k > t_{k_0} \ge 0$, then $0 < e^{-\alpha\left(t - t_{k_0}\right)} < 1$, and when $\lambda_{\min}(P_3) - 2\beta\lambda_{\max}(P_2) > 0$

$$
\dot{V} \le -\left(1 - e^{-\alpha\left(t - t_{k_0}\right)}\right)\left(\lambda_{\min}(P_3) - 2\beta\lambda_{\max}(P_2)\right)\|\eta\|^2 < 0 \tag{23}
$$

It is known that $\dot{V} < 0$ when $\beta < \frac{\lambda_{\min}(P_3)}{2\lambda_{\max}(P_2)}$, *i.e.* $\beta_1 < \frac{\lambda_{\min}(P_3)}{2\lambda_{\max}(P_2)}$, $\beta_2 < \frac{\lambda_{\min}(P_3)}{2\lambda_{\max}(P_2)}$. According to Lyapunov stability theory, the system (1) can guarantee consistent control when $k_0 > \sqrt{1 + \frac{\lambda_{\max}(P_1)}{2}}$ under the control strategy (4) and the event-triggered mechanism (15). That is, the consistency proof is completed.

In order to show that there is no Zeno behaviour in the system, the following proof process is given.

For the derivation of $\frac{\|e_{xv}\|}{\|\eta\|}$, the following equation (24) can be obtained.

$$\frac{d\left\|e_{xv}\right\|}{dt\|\eta\|}=-\frac{e_{xv}\dot{\eta}}{\left\|e_{xv}\right\|\|\eta\|}-\frac{\eta^T\dot{\eta}\left\|e_{xv}\right\|}{\|\eta\|^3} \tag{24}$$

Then

$$\begin{aligned}&-\frac{e_{xv}\dot{\eta}}{\left\|e_{xv}\right\|\|\eta\|}-\frac{\eta^T\dot{\eta}\left\|e_{xv}\right\|}{\|\eta\|^3}\\&\leq\frac{\|\dot{\eta}\|}{\|\eta\|}+\frac{\|\dot{\eta}\|\left\|e_{xv}\right\|}{\|\eta\|^2}=\frac{\|\dot{\eta}\|}{\|\eta\|}\left(1+\frac{\left\|e_{xv}\right\|}{\|\eta\|}\right)\\&=\left(\|M\|+\|J\|\frac{\left\|e_{xv}\right\|}{\|\eta\|}\right)\left(1+\frac{\left\|e_{xv}\right\|}{\|\eta\|}\right)\end{aligned} \tag{25}$$

Then

$$\frac{d\left\|e_{xv}\right\|}{dt\|\eta\|}\leq\left(\|M\|+\|J\|\frac{\left\|e_{xv}\right\|}{\|\eta\|}\right)\left(1+\frac{\left\|e_{xv}\right\|}{\|\eta\|}\right) \tag{26}$$

Let $h_0=\frac{\left\|e_{xv}\right\|}{\|\eta\|}$, then the equation (26) can be transformed into

$$\dot{h}_0\leq\left(\|M\|+\|J\|h_0\right)\left(1+h_0\right) \tag{27}$$

Where h_0 satisfies $h_0(t)\leq f\left(t,f_0\right)$ and $f\left(t,f_0\right)$ is the solution to equation (27).

$$\begin{cases}\dot{f}=\left(\|M\|+\|J\|f\right)(1+f)\\f\left(0,f_0\right)=f_0\end{cases} \tag{28}$$

If system consistency holds, then the event trigger $g_i\left(e_{xv}\right)\leq 0$ holds, i.e.

$$\left\|e_{xv}(t)\right\|-e^{-\alpha\left(t_k-t_{k_0}\right)}\frac{\lambda_{\min}\left(P_3\right)-2\beta\lambda_{\max}\left(P_2\right)}{2k_0\sqrt{k_0^2+1}\|H\|\lambda_{\max}\left(P_2\right)}\|\eta(t)\|\leq 0 \tag{29}$$

Then, the solution of the equation (28) can be satisfied by

$$f(\tau,0)=e^{-\alpha\left(t-t_{k_0}\right)}\frac{\lambda_{\min}\left(P_3\right)-2\beta\lambda_{\max}\left(P_2\right)}{2k_0\sqrt{k_0^2+1}\|H\|\lambda_{\max}\left(P_2\right)} \tag{30}$$

$$\tau = \frac{1}{\|M\| - \|J\|} \ln \frac{f(\tau,0)+1}{f(\tau,0)\frac{\|J\|}{\|M\|}+1} \tag{31}$$

From the above equations τ can be analysed. If $\|M\| \geq \|J\|$, $\tau > 0$ can be obtained, and if $\|M\| < \|J\|$, $\tau > 0$ can also be obtained. Then $\tau > 0$ can be known.

From the above proof, it follows that $t_{k+1} - t_k \geq \tau > 0$, i.e., the system has no Zeno behavior.

The proof is completed.

4. Simulation Research

To verify the effectiveness of the control algorithm proposed in this paper, the following simulation application study is done.

Taking a 4 UAV system as an example, let $x_i(t)$, $v_i(t)$ be the position state information and velocity state information of the ith UAV respectively, whose initial values are randomly generated from [-100,100] and [-10,10] as follows

$$x(t) = [x_1(t), x_2(t), x_3(t), x_4(t)]^T$$
$$= \begin{bmatrix} -71 & 91 & 16.5 \\ 50.72 & 3.6 & 90.81 \\ 8.7 & -70 & 69 \\ 63.4 & 85 & -51 \end{bmatrix}$$

$$v(t) = [v_1(t), v_2(t), v_3(t), v_4(t)]^T$$
$$= \begin{bmatrix} -0.3 & -2.8 & -5 \\ -4 & 4 & 8.6 \\ -6.6 & -4.43 & -5.5 \\ -4.6 & -7.2 & -5.01 \end{bmatrix}$$

A topology for the four UAVs is chosen as shown in Fig. 1.

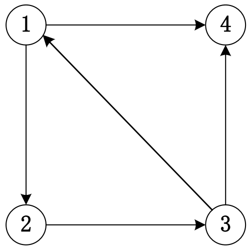

Fig. 1 Topology diagram

The Laplace matrix L can be given as

$$L=\begin{bmatrix}1 & -1 & 0 & -1\\ 0 & 1 & -1 & 0\\ -1 & 0 & 1 & -1\\ 0 & 0 & 0 & 2\end{bmatrix}$$

Thus, H can be obtained as follows

$$H=\begin{bmatrix}2 & -1 & 1\\ 1 & 1 & 0\\ 1 & 0 & 3\end{bmatrix}$$

The matrix P_1 was chosen as follows

$$P_1=\begin{bmatrix}0.3211 & \text{-}0.0409 & \text{-}0.1013\\ \text{-}0.0409 & 0.4591 & \text{-}0.0151\\ \text{-}0.1013 & \text{-}0.0151 & 0.2004\end{bmatrix}$$

Then it can be found

$$\lambda_{\max}\left(P_1\right)=0.471,$$

$$k_0>\sqrt{1+\frac{\lambda_{\max}\left(P_1\right)}{2}}\approx 1.1115\,.$$

Then

$$\beta<\frac{\lambda_{\min}\left(P_3\right)}{2\lambda_{\max}\left(P_2\right)}=\frac{2.1831}{2\times 0.9946}=1.0975$$

Case 1. The parameters $k_0 = 1.42$, $\alpha = 0.35$ and $\beta_1 = 1, \beta_2 = 0.5$ are selected and the disturbance functions are chosen as $g_i(t) = 0$ and $f_i(t) = 0$. Through simulation, the UAV state curves and the event triggering time point can be obtained in the Fig. 2, Fig. 3 and Fig. 4.

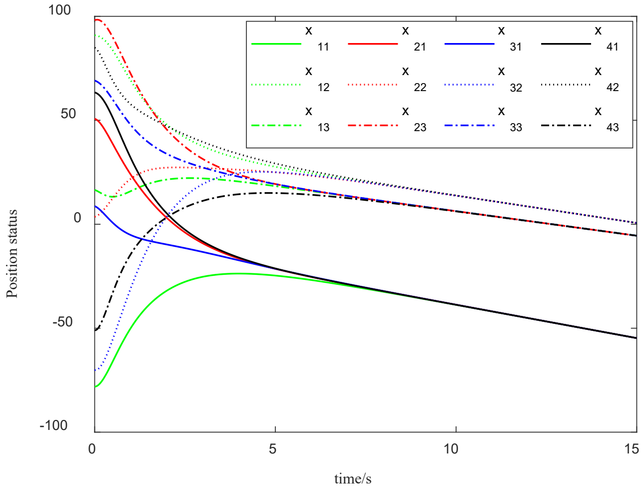

Fig. 2 Position state curves in case 1

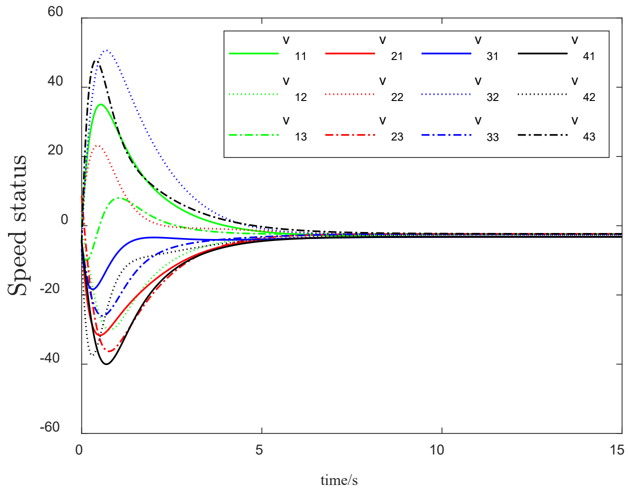

Fig.3 Velocity state curves in case 1

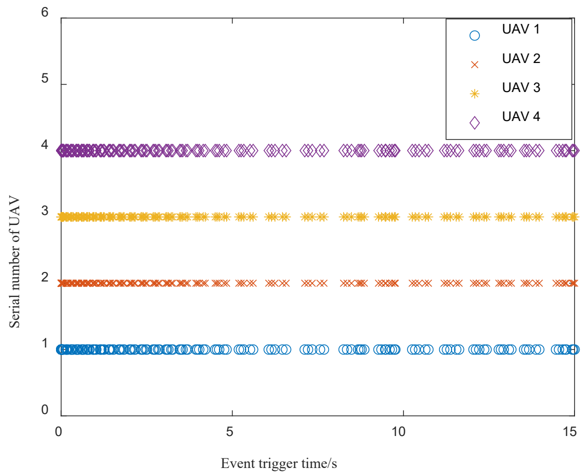

Fig. 4 Event triggering time point in case 1

As can be seen in Fig.2 and Fig.3, the state of the UAV is converged after approximately 7s. During the 15s simulation time, 165 communication times are triggered between each UAV respectively in Fig.4.

Case 2. Other parameters are the same as case1. The disturbance are chosen as $g_i(t) = 0.1\sin(x_i(t))$, and $f_i(t) = 0.05\sin(v_i(t))$. Through simulation, the UAV state process curves and the event triggering time point can be obtained in the Fig.5, Fig.6 and Fig.7.

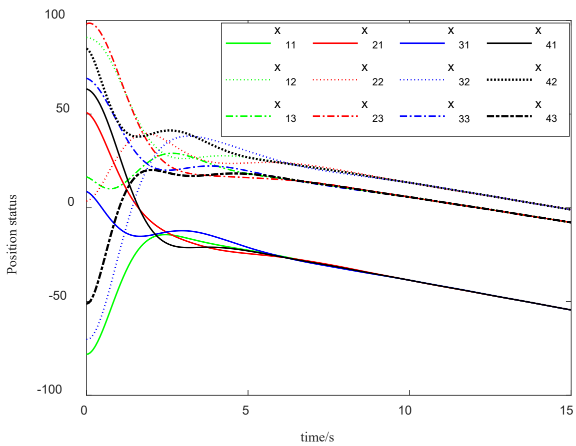

Fig.5 Position state curves in case 2

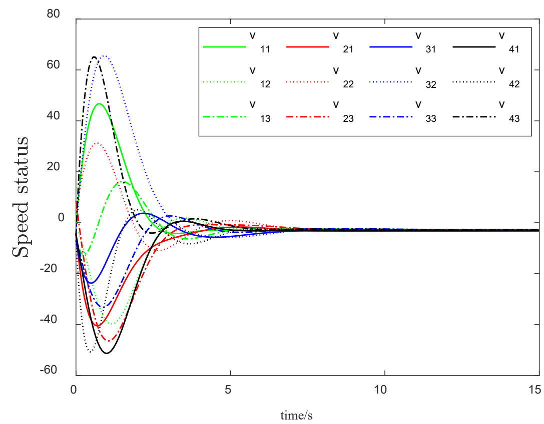

Fig.6 Velocity state curves in case 2

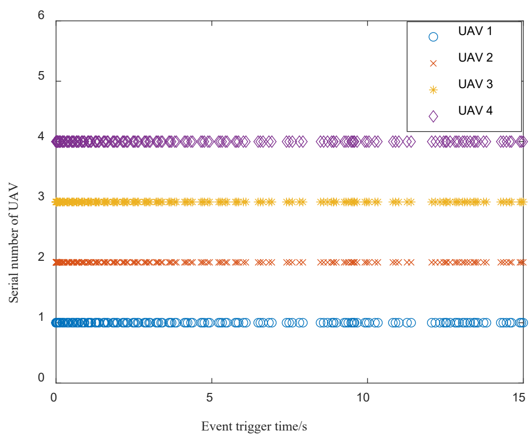

Fig.7 Event triggering time point in case 2

It can be seen in Fig.5 and Fig.6, the state of the UAV is converged after approximately 8s. During the 15s simulation time, 179 communication times are triggered between each UAV respectively in Fig.7.

Case 3. Now, the simulation comparison study with the approach in (Huang Hongwei et al.,2017) is performed. In the (Huang Hongwei et al.,2017), The trigger mechanism and control input functions are as follows.

$$\begin{cases} u_i(\mathrm{t}) = -k^2 \sum\limits_{j=N_i} a_{ij} \left(\left(x_i\left(t_k^i\right) - x_j\left(t_k^i\right) \right) + k^3 \left(v_i\left(t_k^i\right) - v_j\left(t_k^i\right) \right) \right) \\ g_i\left(e_{xv}\right) = \left\| e_{xv} \right\| - \sigma \dfrac{\lambda_{\min}\left(P_3\right)}{2k^2 \sqrt{k+1} \left\| H \right\| \lambda_{\max}\left(P_2\right)} \left\| \eta \right\| \end{cases}$$

The coefficient of the event-triggered mechanism is $\sigma = 0.35$ and the control is $k = 1.42$. The initial values of the states are chosen in the same way as in case 1 and case 2.

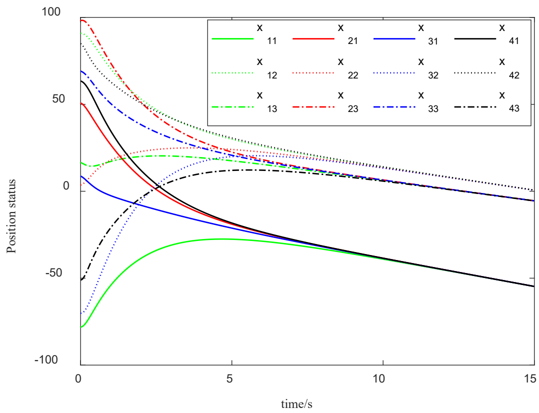

Fig. 8 Position state curves in case 3

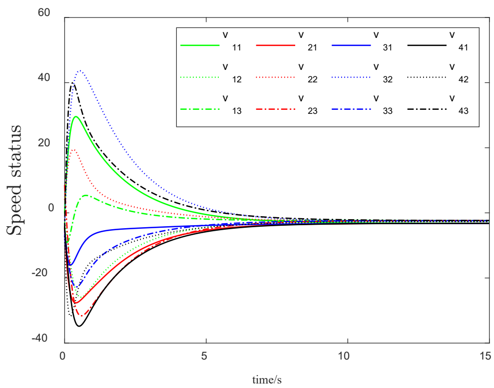

Fig. 9 Velocity state curves in case 3

It can be seen in Fig.8 and Fig.9, the state of the UAV is converged after approximately 10s. During the 15s simulation time, 226 communication times are triggered between each UAV respectively in Fig.10.

In summary, through the above experimental comparison, it can be seen that under the event-triggered mechanism and control algorithm proposed in this paper, the position and speed state curves of multiple UAVs with disturbance can quickly reach consistency. The number of triggers for the event-triggered mechanism in this paper is less in the same time. In addition, comparing case 1 and case 2, the number of event triggers only increases a little after the position and velocity state disturbances have been added to the system model, and the time to reach consistency is similar in both cases. It can be seen that the control strategy under the event-triggered mechanism in this paper has good anti-disturbance capability and the control algorithm is more advantageous.

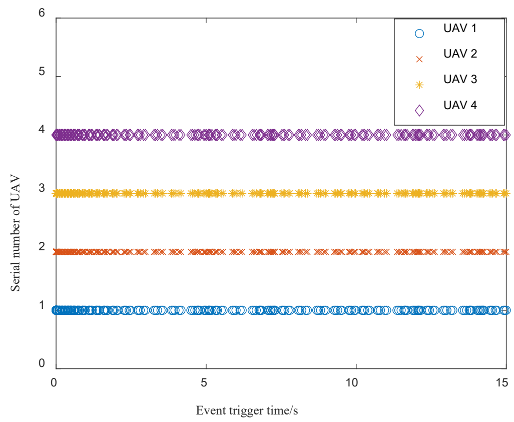

Fig. 10 Event triggering time point in case 3

5. Conclusions

The problem of the consistency control for second-order multi-UAV with disturbances in both position and velocity states is investigated in this paper. In order to further reduce the number of system triggers, the event-triggered mechanism whose threshold value depends on the combination of the real-time state error in the multi-UAV systems and the decay time function is proposed. Consistency control is proved by Lyapunov theory and it is shown that the system does not occur Zeno behavior. Finally, the validity of this control strategy is verified by simulation.

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