INRUSH CURRENT INVESTIGATION FOR SINGLE PHASE POWER TRANSFORMERS BY MEANS OF MAGNETIC MATERIAL CORE CHARACTERISTICS

Emil CAZACU[1], Lucian PETRESCU[2]

The paper proposes a quantitatively study of the inrush current that occurs when a no loaded single phase power transformers is energized. An analytical approach is suggested, in which, the main magnetic core material characteristics are considered. Thus, a comparison between the parameters of the inrush current for different core magnetic materials of a 6 kVA single phase transformer (general purpose low voltage) is carried out. The paper predetermined the peak value of inrush current and evaluates the harmonic content for its first cycle. Consequently, important features of the energizing current are revealed, which can be very useful in a proper design of the transformer protecting device or for the evaluation of these transient process consequences to the electric network.

Keywords: inrush current, single phase transformer, magnetization curves.

1. Introduction

The commutation of a non loaded power transformer to the supply voltage is essentially a transient phenomenon characterized by extreme values of the two main electromagnetic quantities of transformer: the core magnetic flux and the electric current through the winding. Consequently, an extreme high value of the current is initially surged at the switching instant. The amplitude of these inrush currents can reach up to 50 times the rated current of a transformer [1-4]. The current is also indicated as *inrush current* or *magnetization inrush current*. During the energizing process, due to this inrush current (IC) effects, both the transformer and the electric network where the device works, experience important electrodynamical and thermal stress that could affect their functionality. Additionally, these extreme values of the energizing current could be interpreted by the transformer protection equipment as a malfunction current (*e.g.* short circuit) and determine their abnormal operation generating numerous undesirable consequences. In order to avoid that, various method of discrimination the IC from other fault currents have been developed. They mainly rely on the particular features of the IC waveform – *i. e.* a reach second harmonic content of the first IC waveform cycle [5-7].

---

[1] University POLITEHNICA of Bucharest, Romania, e-mail: emil.cazacu@upb.ro
[2] University POLITEHNICA of Bucharest, Romania, e-mail: lucian.petrescu@upb.ro

Important power quality problems are initiated by IC (especially harmonic and unbalances problems) that propagate to the electric installation in the nearby of the switched transformer. Consequently, taking into account even only the above mentioned issues, the necessity of the IC accurate prediction becomes very clear. Thus, numerous both analytical and numerical methods of IC computation have been reported, along with procedures of IC suppression or limitation [8-11]. All these techniques try to consider as many parameters as possible on which the IC depends in order to provide an accurate evaluation for its main parameters: waveform, peak value and duration.

Synthesizing, the main factors that determine the inrush current characteristics could be divided in at least three categories:

- *the electric power source parameters*: the effective voltage values, frequency and, very important, the supply voltage initial phase at the instant of energizing;
- *the nearby network parameters* where the transformer operates (the corresponding connecting inductance and resistance of electric network along with its short circuit power at the common coupling point);
- *the magnetic core characteristics* of the transformer (magnetization curve or hysteresis loop of the core material and, accordingly, the core remnant magnetization and its polarity at the connection instant);

Recently, due to the important development in the magnetic materials field, new and performing magnetic materials (highly grain oriented or domain refined silicon steel) are used in nowadays transformer manufacturing technologies. These new materials significantly affect the transformer IC feature and should be properly considered [12, 13].

This paper is going to investigate the influences of the magnetic materials properties, as the magnetic core of a single phase power transformer over the IC characteristics. The analysis focuses on both the IC amplitude evaluation and the harmonic content of the IC first cycle. These determinations are achieved using an analytical computation procedure that is based on the solution of the transient state equation of the transformer and the core magnetic characterization. Thus, the aggregated equation system is solved by a very fast and convergent classical method. In spite of this simplicity the adopted method turns to be very flexible and able to give accurate solutions for a large class of low-voltage single phase power transformer with rated power up to 25 kVA. Although the proposed procedure is illustrated on the transformers with particular core geometry (UI shape), it could be easily adapted to any other core shape geometry. In order to validate the results, the peak values of IC for some transformers were compared to those indicated by some major transformer manufacturers.

2. The influence of the magnetic core characteristic on the transformer inrush current

The magnetic materials used for the transformer cores are usually described by the magnetization curves that exhibit a linear and saturation areas. Under the normal operation, the transformer works near the saturation knee of the characteristic in order to benefit of both great flux density value and large magnetic permeability. A half period after the device is switched on, the core magnetic flux $\Phi(t)$ could reach almost twice its rated value (if the device is energized when the voltage crosses zero line). Additionally, the remnant flux Φ_r could also add to this double rated value (when remnant flux density B_r has its positive maxima). This process can be mathematically described using the fundamental transformer equations:

$$u(t) = U\sqrt{2}\sin(\omega t + \varphi) \cong N_1 \frac{d\Phi}{dt},$$

$$\Phi(t) = \int u(t)\,dt = -\frac{U\sqrt{2}}{\omega N_1}\cos(\omega t + \varphi) + C,$$

$$\Phi\big|_{t=0} = \Phi_r = B_r A_c,$$

$$\Phi(t) = \Phi_m\big[\cos\varphi - \cos(\omega t + \varphi)\big] + \Phi_r, \qquad (1)$$

$$\Phi_m = \frac{U\sqrt{2}}{\omega N_1};$$

$$\Phi_M = \Phi(t)\big|^{\varphi=0;}_{t=\frac{T}{2}} = 2\Phi_{max} + \Phi_r,$$

where $u(t)$ is the supply voltage of effective value U, angular frequency ω (period $T = 2\pi/\omega$) and initial phase φ, A_c is the effective cross section of the core, and N_1 is the primary turns number.

For simplicity reasons and aiming to better explain inrush current generation phenomena, in (1) the winding electrical resistance is not taking into account. In the following sections this will be also considered, being evaluated according to the transformer main rated data.

Hence, at the first instance of the energizing process, due to the extreme value of the core magnetic flux, the operating point of the transformer is pushed in the "deep" saturation area where the device behave as a air core inductance (due to the very low magnetic permeability in that region). As a consequence, a high current is initially surged by the connected transformer. The magnetic core repeatedly gets in and out the saturation area, generating different peak values for each waveform cycle, so that the specific IC signature arises. The IC decays after a few cycles and its steady state value, also called the *transformer magnetization*

current, is rapidly achieved. This latter is commonly just one percent of the transformer rated current.

Figure 1 qualitatively explains the inrush current generation and the major influence of the magnetic core characteristic on the IC first cycle peak value and waveform. One can notice that an accurate description of the transformer magnetic core may lead to a precise IC predetermination. Hence, due to the flat nature of the magnetization curve in the saturation area, a small increase to the residual flux can significantly affect the IC first amplitude.
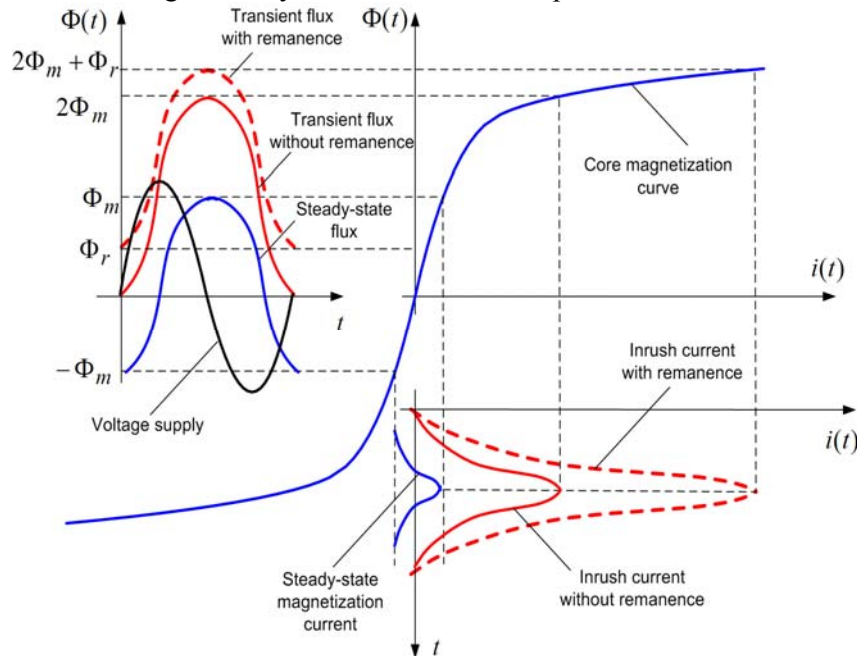

Fig.1 Inrush current generation phenomena and the major influence of the core magnetic characteristic.

3. The transformer modeling and the analytic inrush current solution

In order to obtain a deterministic characterization of the single phase transformer at the instant of switching, a simple circuit model was adopted. This one considers the winding resistance R_1 and the core nonlinear inductivity mainly described by the magnetic core *H-B* characteristic. Thus, one can analytically describe the inrush current $i(t)$ and magnetic flux $\Phi(t)$ time dependency:

$$\begin{cases} \dfrac{d\Phi(t)}{dt} = \dfrac{u(t) - R_1 i(t)}{N_1}, \quad u(t) = U\sqrt{2}\,\sin(\omega t + \varphi) \\[3mm] i(t) = \dfrac{H(t) \cdot l_m}{N_1}, \quad H(t) = f(B(t)), \\[3mm] \Phi(t) = B(t) \cdot A_c, \end{cases} \tag{2}$$

where $H(t)$ and $B(t)$ are the core magnetic field strength and flux density, respectively while l_m represent the mean magnetic path length of the transformer magnetic core.

Since the most common power class range for single phase power transformer (1.2 ÷25 kVA) exhibit similar geometrical core data with those presented in Appendix, our further computations will mainly refer to this UI core shape geometry. The transient state equations (2) require the magnetic core characteristic in terms of its *H-B* representation along with the detailed core geometrical description. That can be accomplished using different approaches. Some common adopted analytical relations are Ollendorff' [14] or Brauer' [15] functions:

$$\begin{cases} H = aB + b\,\mathrm{sh}(cB) & -\text{Ollendorff} \\[2mm] H = uB + vB\exp(wB^2) & -\text{Brauer} \end{cases} \tag{3}$$

where a, b, c, u, v, w constants could be determined for any type of silicon steel core by a simple fitting procedure (constraining the expression to pass through at least three of the steel given data points).

Regarding the selection of the analytical function it is useful to mention that also the *H-B* linear interpolation could be successfully chosen, since the first IC peak value derives from the heavy saturation aria of the magnetic core characterization.

In our further investigations, we select Brauer analytical function and we used Rosenbrock numerical method [16] for solving the non linear differential equations system (2). The initial parameter value is imposed by the remnant magnetization of the magnetic core B_{ro} before the energizing process initiation and it is being expressed in terms of initial magnetic flux $\Phi_0 = B_{ro}A_c$. In order to consider the worst case scenario for our computation method, we will always assume that at the connection moment the voltage crosses zero line and the core remnant magnetization has its positive maxima $B_{r0} = B_r$. Along with the IC waveform and amplitude prediction, the harmonic content of its first cycle should be also investigated.

That is mainly required by an accurate study over the effects of the transformer connection over the nearby network where the device operates

(expressed in terms of power quality indicators) [17, 18]. Additionally, many power transformers protection devices should be able to distinguish the IC from any fault current and, consequently, to avoid their false tripping. Thus, in order to accomplish this task, the *second harmonic restraint procedure* [19, 20] is commonly applied. This protection method bases on the evaluation of the transient current harmonic spectrum of the first cycle (during a period time T of the fundamental voltage supply waveform) and computes the current second harmonic level in respect to the fundamental. Typically, for IC, this level has a significant contribution to the waveform harmonic spectrum (more than 50 %).

Accordingly, a numerical harmonic analysis over the first cycle of IC waveform is also carried out. This quantitatively investigation relies on the Fourier series applied to the IC first cycle waveform. Thus, the spectrum of IC first cycle is evaluated, by means of each harmonic percentage level (h_k) with respect to the fundamental:

$$a_0 = \frac{1}{T}\int_0^T i(t)\,\mathrm{d}t; \quad a_k = \frac{2}{T}\int_0^T i(t)\cos(k\omega t)\,\mathrm{d}t,$$

$$b_k = \frac{2}{T}\int_0^T i(t)\sin(k\omega t)\,\mathrm{d}t, \qquad k = 1, 2, 3...$$

$$i(t) = \sum_{k=0}^n \left[a_k\cos(k\omega t) + b_k\sin(k\omega t)\right],$$

$$c_k = \sqrt{a_k^2 + b_k^2}, \quad h_k[\%] = \frac{c_k}{c_1}\cdot 100.$$

(4)

4. Simulation results –case studies

In order to validate the IC computation method presented in the previous section, a general purpose UI core shape single phase power transformer with a rated power of 6 kVA, was selected for our investigation (see Appendix).

Table 1

The magnetic materials used in our investigations

	Materials	Remnant Flux Density [T]
M1	*Silicon Steel M27*	1.150
M2	*Silicon Steel M43*	1.150
M3	*Silicon Steel M45*	1.150
M4	*Carpenter SiFe 1066C Anneal*	0.868
M5	*Hiperco-50*	1.170
M6	*Vanadium Permandur*	1.195
M7	*Low Carbon Steel 1020*	1.289
M1	*Silicon Steel M27*	1.150

Aiming to evaluate the effect of core magnetic material on IC feature, different magnetic materials were used for the transformer magnetic circuit. Since the magnetic materials characterization (extracted from [21]) are indicated by a finite number of pair points, the Brauer analytical approximation was applied for the necessary *H-B* dependency. Table I indicates all the investigated materials along with the corresponding remnant flux density, while Fig. 2 qualitatively represents the magnetization curve for some of these materials.
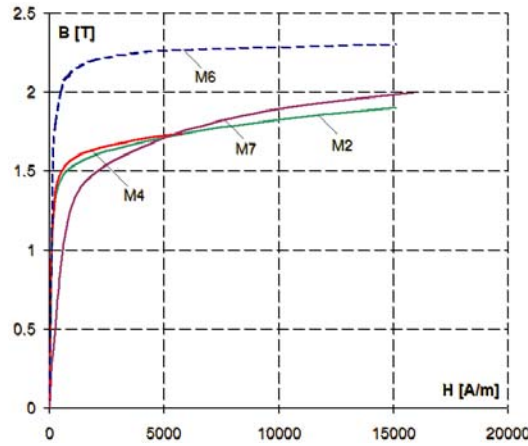
Fig.2 Magnetization curve for some of the materials used for the transformer magnetic circuit.

We will present the simulation results only for two materials used in the transformer core construction: *Carpenter SiFe 1066C Anneal* (M4) and *Low Carbon Steel 1020* (M7). Consequently, the inrush current and the transformer core magnetic flux time variation are presented in Fig. 3 (for M4) and in Fig. 4 (for M7), respectively.
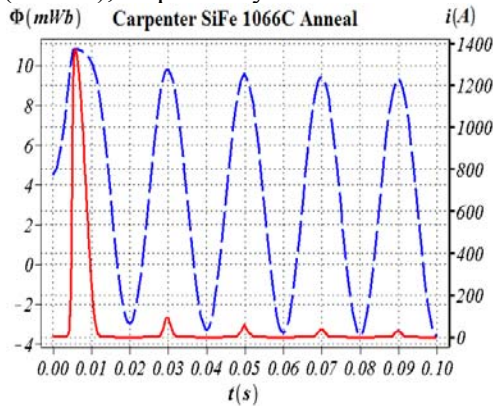

Fig.3 Inrush current (solid curve) and magnetic flux (dashed curve) for a 6 kVA transformer with core magnetic material M4.
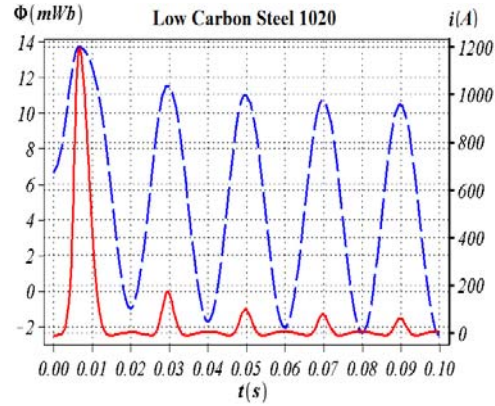
Fig.4 Inrush current (solid curve) and magnetic flux (dashed curve) for a 6 kVA transformer with core magnetic material M7.

It is useful to stress out that the computed IC amplitude are 1371 A and 1199 A respectively, while the transformer rated current is around 15 A and its steady-state magnetization current is only 0.22 A. These IC peak values results are in good agreement with those reported by some major transformer manufacturers [22].

The spectrum histograms for the transformer IC first cycle, for the above mentioned core materials, are shown in Fig. 5 and Fig. 6 respectively.
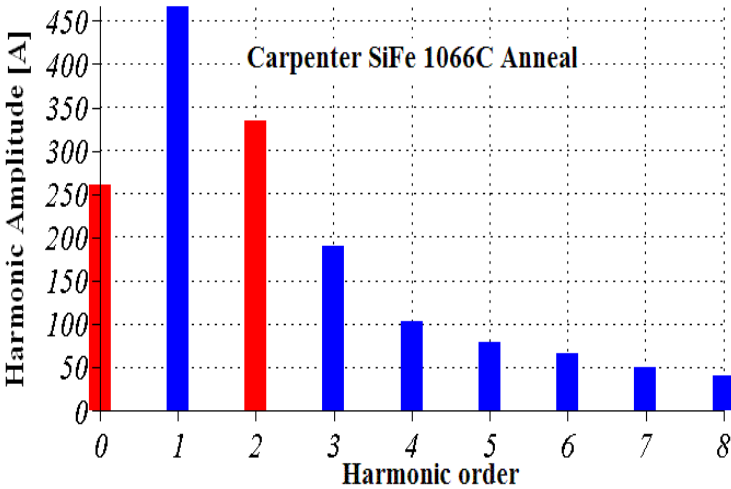

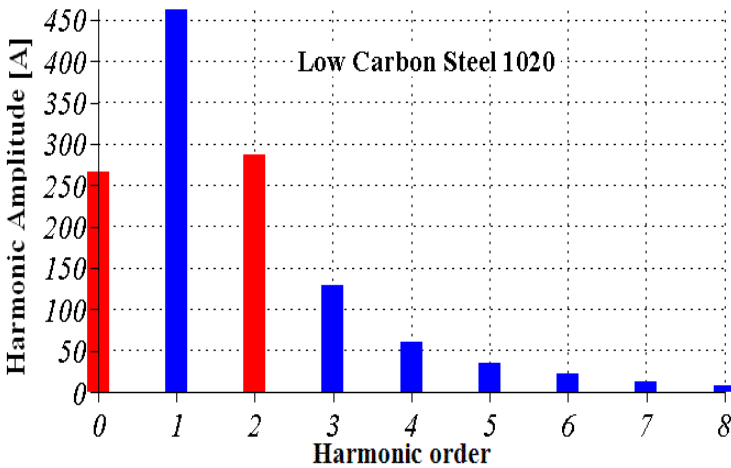Fig.5 Histogram spectrum of the IC first cycle for core material M4.

Fig.6 Histogram spectrum of the IC first cycle for core material M7.

As the histograms reveal, the IC first cycle spectrum is dominated by the zero (DC) and second harmonic components. Their percentage level relative to the

fundamental can be easily derived according to (4). Hence, for the *Carpenter SiFe 1066C Anneal* (M4) core magnetic material:

$$h_0[\%] = \frac{c_0}{c_1} \cdot 100 \cong 55\,\%, \quad h_2[\%] = \frac{c_2}{c_1} \cdot 100 \cong 71\%. \qquad (5)$$

Similarly, for the *Low Carbon Steel 1020* (M7), the zero and second harmonic percentage level in respect to the fundamental are:

$$h_0[\%] = \frac{c_0}{c_1} \cdot 100 \cong 57\,\%, \quad h_2[\%] = \frac{c_2}{c_1} \cdot 100 \cong 61\,\%. \qquad (6)$$

Equations (5) and (6) quantitatively underline one of the major features of the IC first cycle waveform: a rich even harmonic attendance into its harmonic spectrum. Most of the power transformer protecting equipment identifies the IC according to these very particular characteristics.

Similar simulations were carried out for all the magnetic core materials from Table I. The results are presented in Fig.7 and Fig 8 respectively: Fig. 7 illustrates the amplitude of transformer IC when different core magnetic materials are used while Fig. 8 shows the zero and second harmonic percentage level from the IC first cycle.
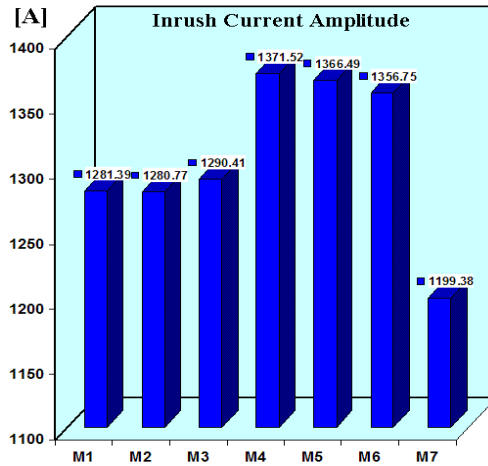

Fig.7 Inrush current amplitude for different transformer core materials
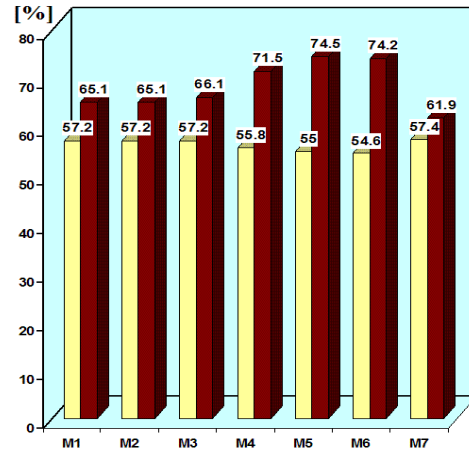
Fig. 8 The zero (light bars) and second (dark bars) harmonic percentage level of the IC first cycle for different core magnetic materials

6. Conclusions

Using a simple circuit model for the energizing process of a single phase transformer, a quantitative study over the inrush current was accomplished. Thus, solving the nonlinear equations system, which derived from the proposed model, a quite accurate predetermination of the inrush current amplitude and waveform

characteristics was predicted. The influence of the magnetic core characteristic was systematically analyzed for a large class of materials used as magnetic core of the single phase transformer of powers class around 6 kVA. Appling an analytical model of the *H-B* magnetic material description, our computation method manages to simulate both the linear and the heavy saturation region of the magnetization curve. The study reveals the importance of material characteristics for the accuracy of inrush current amplitude computation. Hence, more advanced is the magnetic material description, more precise are the predetermined IC features.

The necessity of this study is required by the new trends in power transformer construction technologies too. Hence, the modern transformers took advantage of the important progress in magnetic material development. The new core materials (with more consistent grain orientation) exhibit a higher operating induction and have a higher value of saturation flux density along with a lower remnant magnetization. Additionally, they also provide a larger linear region on the *H-B* magnetization curve. Furthermore, in order to provide a continuous path for the magnetic flux and to decrease the core reluctance, these new laminations are also overlapped. The transformer core cross-sectional area is also increased for achieving a limited operating flux density. All these new core characteristics notably influence the main transformer IC parameters (amplitude, first cycle harmonic spectrum and duration) and should be accordingly considered for a precise quantitatively evaluation of this important transient phenomena.

A p p e n d i x

The geometry of the investigated power class transformers (UI core shape) is presented in Fig. 9 and the main technical data of a unit of 6 kVA rated power with Silicon Steel M43 laminations are shown in Table II.
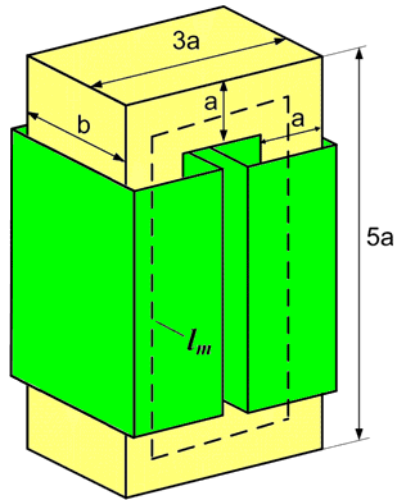
Fig.9 Magnetic core shape of the transformers and their geometrical dimensions

Table 2

Data sheet of the investigated single phase power transformer

$S_n = 6$ kVA	Rated power
$U_{1n} = 0.40$ kV	Primary rated voltage
$U_{2n} = 0.23$ kV	Secondary rated voltage
$P_0 = 78$ W	No load power losses
$P_{sc} = 180$ W	Short circuit power losses
$i_0 = 1.5$ %	Magnetization current
$u_{sc} = 3$ %	Short circuit voltage
$\Delta u_n = 3$ %	Rated voltage regulation
$N_1 = 279$	Number of turns in primary
$N_2 = 160$	Number of turns in secondary
$R_1 = 0.400$ Ω	Primary copper loss resistance
$R_2 = 0.1323$ Ω	Secondary copper loss resistance
$B_n = 1.25$ T	Operating magnetic core induction
$B_r = 1.15$ T	Core remnant magnetization
$A_c = ab = 0.005152$ m^2	Magnetic core area
$l_m = 12a = 0.672$ m	Magnetic path mean length
$a = 0.056$ m	UI Core main reference dimension
$b = 0.092$ m	Core thickness

R E F E R E N C E S

[1]   *R. A. Turner, K. S. Smith*, Transformer inrush currents, IEEE Industry Applications Magazine, vol. 16, no. 5, pp. 14-19, 2010.

[2]   *S. P. Patel*, Fundamentals of transformer inrush, The 64th Annual Conf. for Protective Relay Engineers, pp. 290 -300, Texas, USA 11 - 14 Apr. 2011.

[3]   *P. C. Y. Ling, A. Basak*, Investigation on magnetizing inrush current in single-phase transformer, IEEE Trans. Magn., vol. 24, no. 6, pp. 3217–3222, Nov. 1988.

[4]   *J. Faiz, S. Saffari*, Inrush Current Modeling in a Single-Phase Transformer*, IEEE Trans. on Magn., vol. 46, no. 2, pp. 578 - 581, Feb. 2010.*

[5]   *T. Chiulan, B. Pantelimon*, Experimental study on the large power transformer unit temperature variation, U.P.B. Sci. Bull., Series C, Vol. 71, Iss. 4, 2009, pp. 229-238

[6]   *C. L. Cheng, C. E. Lin, C. L. Huang, J. C. Yeh*, A simple model for transformer inrush current calculation and harmonic analysis, Electric Power Systems Research, vol. 23, no. 3, pp. 153-163, 1992.

[7]   *M. Jamali, M. Mirzaie, S. A. Gholamian*, Calculation and Analysis of Transformer Inrush Current Based on Parameters of Transformer and Operating Conditions, Electronics and Electrical Engineering, vol. 109, no. 3 , pp. 17-20, 2011.

[8]   *Y. Wang, S.G. Abdulsalam, W. Xu*, Analytical formula to estimate the maximum inrush current, IEEE Trans. Power Del., vol. 23, no. 2, pp. 1266-1268, Nov. 2008.

[9]   *M. G. Vanti, S. L. Bertoli, S. H. L. Cabra, A. G. Geren, Jr., P. Kuo-Peng*, Semianalytic Solution for a Simple Model of Inrush Currents in Transformers, IEEE Trans. Magn., vol. 44, no. 6, pp. 1270–1273, June 2008.

[10]   *S.D.L. Talu, M.D.L. Talu*, Dimensional optimization of frontal radiators of cooling system for power transformer 630 kVA 20/0.4 kV in terms of maximum heat transfer, U.P.B. Sci. Bull., Series C, Vol. 72, Iss. 4, 2010, pp. 249-260.

[11]   *D. I. Taylor, J. D. Law, B. K. Johnson, N. Fischer*, Single-Phase Transformer Inrush Current Reduction Using Prefluxing, IEEE Trans. Power Del., vol. 27, no. 1, pp. 245- 252, Jan. 2012.

[12]   *N. Chiesa, and H. K. Høidalen*, Novel Approach for Reducing Transformer Inrush Currents: Laboratory Measurements, Analytical Interpretation and Simulation Studies, IEEE Trans. Power Del., vol. 25, no. 4, pp. 2609- 2616, Oct. 2010.

[13]   *D. Zhang, Y. Liu, and S. Huang*, Differential Evolution Based Parameter Identification of Static and Dynamic J-A Models and Its Application to Inrush Current Study in Power Converters, IEEE Trans. on Magn., vol. 48, no. 11, pp. 3482-3485, Nov. 2012.

[14]   *F. Ollendorff*, Zur qualitativen Theorie gesättigter Eisendrosseln, Archiv für Elektrotechnik, Band XXI, pp. 9-24, 1928.

[15]   *J. R. Brauer,* Simple equations for the magnetization and reluctivity curves of steel, IEEE Trans. Magn., vol. MAG-11, pp.81, 1975.

[16]   *Shampine, L.F., Corless, R.M.* Initial Value Problems for ODEs in Problem Solving Environments, J. Comp. Appl. Math. Vol. 125(1-2). (2000): 31-40.

[17]   *K. Yabe*, Power differential method for discrimination between fault and magnetizing inrush current in transformers, IEEE Trans. Power Del., vol. 12, no. 3, pp. 1109- 1118, Jul. 1997.

[18]   *R. Hunt, J. Schaefer, B. Bentert*, Practical Experience in Setting Transformer Differential Inrush Restraint, The 61st Annual Conf. for Protective Relay Engineers, pp. 118- 141, Texas, USA 1-3 April 2008.

[19]   *J. Faiz, S. Lotfi-Fard*, A novel wavelet-based algorithm for discrimination of internal faults from magnetizing inrush currents in power transformers, IEEE Trans. Power Del., vol. 21, no. 3, pp. 1989-1996, Oct. 2006.

[20]   *E. Cazacu, V. Ioniţă, L. Petrescu*, Transformer inrush current predetermination for distorted waveform voltage supply, *Rev. Roum. Sci. Techn.– Electrotechn. et Energ.*, **58**, *3*, Bucarest, 2013, pp. 243-251.

[21]   *D. C. Meeker*, Finite Element Method Magnetics, Version 4.0.1 (03Dec2006 Build), http://www.femm.info.

[22]   EATON, Distribution Dry-Type Transformers—Low Voltage, Sheet No. 19001, 2012.