OPTIMAL PLACEMENT OF PHASOR MEASUREMENT UNITS USING MULTI-CRITERIA ANALYSIS

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The Synchrophasor Technology is more precise than traditional measuring systems, and it must be installed in the power system to ensure wide-area monitoring. The disadvantages of a phasor measurement system include the high cost of the equipment and installation. PMUs are primarily utilized in power systems due to their high reporting rate, synchronized measurement, and capacity to observe adjacent buses. With optimal placement, PMUs allow for the acquisition of enough information to make the power system observable with a minimum total cost. The optimization problem that tries to determine the optimal placement of PMUs can be solved using both deterministic and metaheuristic algorithms.

Keywords: Synchrophasor Technology (ST), Phasor Measurement Unit (PMU), Wide Area Measurement System, Zero-injection bus, Optimal Placement Problem (OPP), System Observability Index (SORI), Integer linear programming, Harris Hawk Optimization, Grey Wolf Optimization, Particle Swarm Optimization.

1. Introduction

Climate change is caused by the increasing carbon footprint of humanity. In this context, the European Commission imposed measures to reduce the greenhouse gas emissions produced by conventional power plants that use fossil fuels to generate electricity. The European target is to achieve climate neutrality by 2050 [1]. Thus, the future power systems will include more renewable sources of energy, like wind and solar power plants. Replacing the conventional power plants with renewable energy sources will decrease the power system's inertia due to the low inertia of power electronics converters. Furthermore, these are stochastic generation sources, meaning that they are weather-dependent. This dependance increase the power system uncertainties which might cause power oscillations. Integrating many such power plants into the grid is a threat to the stability of the power systems. This

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will require to improve the actual monitoring and control systems which are needed to maintain the stability of the power systems [2]. Conventionally, the SCADA system is used for monitoring and control, but it only provides power flow measurement and the magnitude of voltage and current. The main SCADA's disadvantages are represented by the lack of time synchronization, the slow reporting rate, and the fact that phasor angle is not directly measured. The technological advancement of measurement technology and communications systems made it possible to solve these drawbacks and facilitate the transition to smart grids [3]. The concept of phasor monitoring was introduced in 1988, and this concept started commercial production in 1991 by Macrodyne Corporation. The phasor and measurement unit (PMU) is time-synchronized by the GPS receptor, and it can directly estimate the voltage and current phasor for each bus on the grid, the frequency, and the rate of frequency changes. The PMU reporting rate is higher than SCADA, and it can ensure up to 50/60 measurements per second. Based on these advantages, phasor measurement systems can ensure the observability of wide-area power systems, facilitating monitoring and controlling them in real-time and reducing the drawbacks and power oscillations that may be caused by stochastic power generation. The power system's observability can be reached if all buses on the grid are observable. Buses are considered observable if their voltage phasor can be obtained by direct measurement or calculated indirectly, known as pseudo-measurement [4]. Placing the PMU's equipment in the power system buses involves a high total cost, which is composed of the costs of the measurement units, the installation, the maintenance, the communication network, and the power consumption of the measurement system. Considering a purely technical approach, full measurement redundancy and observability can be achieved by placing the PMUs in each bus of the grid, but this approach isn't economically feasible because it involves a far too high total cost. Redundancy means that the voltage phasors for each bus can be estimated, directly or indirectly, from multiple measurement units. To minimize the cost of the entire measurement system, a redundancy compromise has to be made. Considering these assumptions, the optimal placement of the phasor measurement problem. which takes into consideration to ensure full observability and to maximize the measurement redundancy with a minimum total cost, can be formulated [3]. Generally, full observability can be reached by placing the measurement units in a minimum of 20–30% of the power system buses [5]. For PMU placement, most of the current research papers have focused on using metaheuristic optimization techniques, including differential evolution (DE) [6], particle swarm optimization (PSO) [7], ant colony optimization (ACO) [8], and genetic algorithms (GA) [9]. However, these metaheuristic approaches do not ensure a globally optimal solution. Although the optimal global solution of the PMU placement problem can be achieved using the integer linear programming approach (ILP) [5], it comes with a significant computational cost when considering large networks [8].

This paper is divided into the following sections: observability aspects, placement problem formulation, case studies, and conclusions. This paper presents a multi-criteria approach to optimize the placement of phasor measurement units to maximize the power system's observability, considering the influence of zero injection buses on the PMU placement problem as well as the system observability redundancy index (SORI) and variable cost of the measurement system on different buses.

2. Observability

The power systems are operated and supervised by a control center. The system's operators need to have accurate information about what is happening in the entire power system in real time. To achieve this, the state estimator is used. The state estimator is a mathematical tool that determines the most likely state of the power system based on a set of measurements from the substations. In some cases, the state estimator can't compute and can't find the power system state because the measurement set may be incomplete. To compute the state estimation of the power system, the measurement set needs to ensure the system's observability. So, before computing the state estimation, it is necessary to verify the full observability of the system, which ensures that the state of all buses in the power system can be properly estimated [10].

The power system's observability can be estimated by different methods, which can be categorized as topological, numerical, or hybrid approaches. The topological methods are based on graph theory, and the numerical approaches consist of analyzing the null space of the Jacobian matrix and studying the gain matrix. The hybrid techniques use topological transformation to reduce the model of the network, and then a numerical algorithm is used to determine the observability score. In this paper, a topological perspective is used to ensure the full observability of the power system [10].

Topological observability rules

Assuming that all measurements from the entire power system are provided only by phasor measurement units, the following six observability rules can be formulated [10]:

Rule 1: The buses that are equipped with PMUs are referred to as directly observable because the voltage phasors of these buses are measured directly by PMUs. Considering that the equipment can also measure the current phasor, all incident lines are considered directly measurable.

Rule 2: If the current and voltage of one end of a line are measured by PMU, the voltage phasor of the other end of the line can be determined by the law of electrical conduction and the voltage phasor is referred to as indirectly observable.

Rule 3: Furthermore, for a line that has both voltage phasors known, directly or indirectly, the current phasor can also be estimated using the law of electrical conduction. The current obtained in this way is considered indirectly observable.

These first three rules can be applied to any bus in the power system and are considered general rules. The last three rules are more specific and applicable only to some buses because they take into consideration the effect of the zero-injection bus (ZIB). The ZIB is a node that doesn't have a load, or a generator connected to it, and the particularity of this type of bus is that the sum of all the current phasors connected to this node is equal to 0.

Rule 4: If all voltage phasors for neighbors of an observable ZIB are known except one, it can be calculated by Kirchhoff's current law on the ZIB node.

Rule 5: If the ZIB's voltage phasor is unknown but all its neighbors are observable, the ZIB node becomes observable, and its voltage phasor can be calculated by Kirchhoff's current law on the ZIB node.

Rule 6: An unobservable group of interconnected ZIBs becomes observable if all its neighbors are observable. In this case, the voltage phasors of ZIBs can be calculated by the nodal equation using the voltage phasor values of the neighbors of the entire group of ZIBs.

All these rules can be applied under the assumption that the line impedance is known.

Modeling of the ZIB

Considering the ZIBs effect in the network topology, we can significantly reduce the total number of PMUs that are needed to ensure full observability. For this reason, the following four rules can be formulated for merging the ZIBs that are used in the topology transformation of the network [10]:

Rule 1: The buses that are selected to be merged must not have already been merged in a previous iteration.

Rule 2: The ZIB will be merged with a radial bus connected to it if this exists. Radial buses, by definition, are those that are connected to the power system only through one line.

Rule 3: If there are many interconnected buses that are connected to the ZIB too, then the ZIB will be merged with the bus which has the greatest number of lines connected to it.

Rule 4: The ZIB will be merged with one of its neighbors that has the greatest number of buses connected to it.

3. Optimal PMU placement problem formulation

The aim of optimal PMU placement is to minimize the installation cost of the measurement units in the power systems to achieve full observability. Solving that problem will return the minimum number of PMU needed, the installation cost, and the bus location that needs to be equipped with measurement units. The cost of installing the measurement unit may be different for each bus, and usually it's dependent on the number of incident lines in each bus and implicitly the number of measurement unit channels needed. Beside this, measurement redundancy is also necessary because it is imperative to ensure full observability in the event of measurement unit or line unavailability. Traditionally, the mathematical model includes the objective function (1) and the constraints (2). It, also, considers that the installation cost is the same for each bus, and there isn't any redundancy measurement [10].

$$Minimize \sum_{i=1}^{N} X_i \tag{1}$$

$$Subject\ to \sum_{j=1}^{N} a_{i,j} X_j \geq b_i\ , \forall i\ \in \boldsymbol{N} \tag{2}$$

The total number of buses is $\boldsymbol{N}$, and the term $\boldsymbol{a_{i,j}}$ from the connectivity matrix is obtained as follows:

$$a_{i,j} = \begin{cases} 1\ , & if\ the\ bus\ i\ is\ conected\ to\ bus\ j \\ 1\ , & if\ i = j \\ 0\ , & otherwise \end{cases} \tag{3}$$

The term X_i is represented by binary decision values that indicate if in the bus i is placed a measurement unit or not.

$$X_i = \begin{cases} 1\ , & if\ a\ PMU\ is\ placed\ at\ bus\ i \\ 0\ , & otherwise \end{cases} \tag{4}$$

The b_i variable represents the minimum number of measurement sources that are imposed to determine the quantities for bus *i*. This value is a natural number greater than or equal to 1. In the simplified model, all b_i values are equal to 1 for all buses, which means all buses are observable, directly, or indirectly, from one measurement source. For some buses, the b_i variable can be chosen to be equal to 2, which means these are observable from two sources, so in the case of one measurement unit unavailability or one line fault, these buses will still be observable.

To improve the security of the power system, redundancy analysis is mandatory. If a large-scale redundancy is necessary, the b_i parameter can be set to be equal to 2 for each bus from the power system. In this case, full observability will be maintained even if the unavailability of any measurement unit or any line occurs. The main drawback of solutions with large-scale redundancy is the high implementation cost of these measurement systems because many measurement units must be placed in the network. To solve this drawback, the System Observability Redundancy Index (SORI) can be used.

The System Observability Redundancy Index (SORI) represents the total number of buses that the phasor measurement system covers, directly or indirectly [7]. Because the quantity of each bus can be estimated by multiple measurement units, the SORI value is most often higher than the total number of buses in the system. It is also a good indicator of the phasor measurement system's reliability and measurement redundancy.

The optimal PMU placement process can provide more solutions with the same cost of the measurement system's implementation or with the same minimal number of PMU [11]. In this case, it is necessary to evaluate it and choose the best solution based on multiple criteria that should take into consideration the system's security, reliability, redundancy, and the power system's critical buses. SORI can evaluate these optimal solutions and provide the best solution, but it doesn't guarantee the solution's uniqueness. The higher the value of SORI, the better the measurement system's redundancy. The SORI can be determined using the following equation:

$$SORI = \sum_{i=1}^{N}\left(\sum_{j=1}^{N} a_{i,j}\right)X_i \tag{5}$$

Based on simplified approaches, a new multicriterial mathematical model for optimal PMU placement is proposed in this paper. The new mathematical model includes the objective function (6) and the constraints (7). The new objective function includes the variable S_i, which corresponds to the SORI maximizing criteria, as well as the variable C_i, which minimizes the cost of installing the measurement units for each bus, which varies depending on the number of adjacent lines and the channel number needed for bus *i*. A_i represents the number of observable buses, directly or indirectly, by placing the PMU on the *i* bus, and its value is obtained by summing the elements of column *i* from the *A* connectivity matrix. The *R* parameter is chosen to impose a redundancy for each bus that depends on the number of adjacent buses on each bus. Therefore, for buses that have a connection number bigger than *R*, the b_i variable will be set to 2 in the mathematical model, and if they have a lower connection number than *R*, the b_i variable will be set to 1 for these buses.

$$Minimize \sum_{i=1}^{N} C_i X_i + S_i X_i \tag{6}$$

$$Subject\ to \sum_{j=1}^{N} a_{i,j} X_j \geq b_i\ , \forall i\ \in \boldsymbol{N} \tag{7}$$

$$b_i = \begin{cases} 1\ , & A_i < R \\ 2\ , & A_i \geq R \end{cases} \tag{8}$$

$$A_i = \sum_{j=1}^{N} a_{i,j} - 1, \forall i\ \in \boldsymbol{N} \tag{9}$$

$$S_i = \frac{1}{A_i}, \forall i\ \in \boldsymbol{N} \tag{10}$$

$$C_i = (1 + 0.35 * (A_i - 1))\ , \forall i\ \in \boldsymbol{N} \tag{11}$$

Each single-phase measurement is achieved using a single channel. Typically, a PMU has a total of around 10 channels: 3+1 voltage channels, 3+1 current channels, and 2 up to 4 binary signals. As per the literature, once the PMU's infrastructure is in place, the additional installation of an PMU increases the initial costs by approximately 35%. In the new mathematical model, this fact is considered by the C_i term, which is determined by eq (11).

Using this approach, determining the minimum cost and the minimum number of PMUs placed aren't the only targets for the optimal PMU placement problem anymore. This perspective increases the redundancy of the measurement system at the expense of determining the minimum total cost and finding an optimal middle point that satisfactorily meets both adversarial criteria.
Different methods can be used to solve the problem of optimal PMU placement in power systems.

These techniques are classified as deterministic, heuristic, or meta-heuristic algorithms. The deterministic optimization methods guarantee finding the optimal solution but usually involve high computational effort and high calculation times. To solve this drawback, metaheuristic and heuristic methods can be used, which determine a satisfying solution faster but don't guarantee that it is the global best solution. Therefore, the right approach needs to be chosen depending on the specificity of the optimization problem. Integer programming, binary search, exhaustive search, and greedy algorithms are the most popular deterministic algorithms. The most popular metaheuristics include simulated annealing, genetic algorithms, tabu searches, differential evolution, binary particle swarm optimization, and others. In this paper, to solve the optimization problem, the deterministic algorithm Integer Linear Programming (ILP) and the metaheuristic algorithms Harris Hawk Optimization (HHO), Grey Wolf Optimization (GWO), and Particle Swarm Optimization (PSO) are used.

4. Case study

To solve the optimal PMU placement problem, deterministic and metaheuristic algorithms are used for the IEEE 57 and IEEE 118 test systems. For both power systems, the proposed mathematical model and the classical mathematical model were solved using integer linear programming (ILP), particle swarm optimization (PSO), Harris Hawk optimization (HHO), and grey wolf optimization (GWO) techniques.

Table 1

Optimal PMU placement without considering ZIBs in the IEEE 57 Power System

		Classic O.F.				Proposed O.F.			
		ILP	PSO	HHO	GWO	ILP	PSO	HHO	GWO
R=1	No PMU	33	33	33	33	33	33	33	33
	Cost	55.4	55.05	55.4	55.05	55.4	55.4	55.4	55.4
	SORI	130	129	130	129	130	130	126	130
R=2	No PMU	32	32	32	32	32	32	32	32
	Cost	54.4	54.05	54.05	53.7	54.4	54.4	54.05	54.05
	SORI	128	127	127	126	128	128	128	127
R=3	No PMU	21	21	21	21	21	21	21	21
	Cost	37.8	38.15	37.45	37.8	38.1	38.5	38.5	38.5
	SORI	90	91	89	90	91	92	92	92
R=4	No PMU	19	19	19	19	19	19	19	19
	Cost	34.05	35.1	34.75	34.05	35.1	35.45	35.1	34.75
	SORI	84	84	83	81	84	85	84	83
R=5	No PMU	17	17	17	17	17	17	17	17
	Cost	28.9	29.25	29.25	28.9	28.9	29.25	29.25	29.25
	SORI	68	69	69	68	68	69	69	69
R=6	No PMU	17	17	17	17	17	17	17	17
	Cost	28.2	28.9	29.25	28.55	29.95	29.95	29.6	29.95
	SORI	67	68	69	67	71	71	70	71

The primary aim of the optimization placement problem can be to minimize the total cost of the monitoring system or to ensure good measurement redundancy with a reasonable total cost. To highlight the impact of choosing the *R* parameter on the SORI and the total cost of the measurement system, the *R* parameter takes values between 1 and the maximum number of connections that one bus can have in the entire system. For example, the maximum number of connections that one bus has in the IEEE 57 power system is 6 and in the IEEE 118 power system is 9.

Table 2

Optimal PMU placement considering ZIBs in the IEEE 57 Power System

		Classic O.F.				Proposed O.F.			
		ILP	PSO	HHO	GWO	ILP	PSO	HHO	GWO
R=1	No PMU	25	25	25	25	25	25	25	25
	Cost	43.55	43.2	42.85	42.85	43.55	43.55	43.2	43.55
	SORI	103	102	101	101	103	103	102	103
R=2	No PMU	23	23	23	23	23	23	23	23
	Cost	41.2	40.85	41.2	40.85	41.2	41.2	41.2	41.2
	SORI	98	97	98	97	98	98	98	98
R=3	No PMU	17	17	17	17	17	17	17	17
	Cost	31.7	31.7	31.35	30.6	32.05	32.05	32.05	31.3
	SORI	76	76	75	73	77	77	77	75
R=4	No PMU	15	15	15	15	15	15	15	15
	Cost	27.25	27.6	27.95	28.3	29	27.95	28.65	29
	SORI	65	66	67	68	70	67	69	70
R=5	No PMU	13	13	13	13	13	13	13	13
	Cost	24.2	24.9	24.2	24.55	25.25	25.25	24.55	25.25
	SORI	58	60	58	59	61	61	59	61
R=6	No PMU	12	12	12	12	12	12	12	12
	Cost	23.2	22.85	22.85	22.85	23.2	23.2	23.2	23.2
	SORI	56	55	55	55	56	56	56	56

So, the natural *R* parameter varies between 1 and 6 for the IEEE 57 power system and between 1 and 9 for the IEEE 118 power system, as shown in tables (1-4). The cost of placing one PMU that can measure the bus voltage, and the current flow for one branch is considered 1, and it is increased by 0.35 for every branch observed.

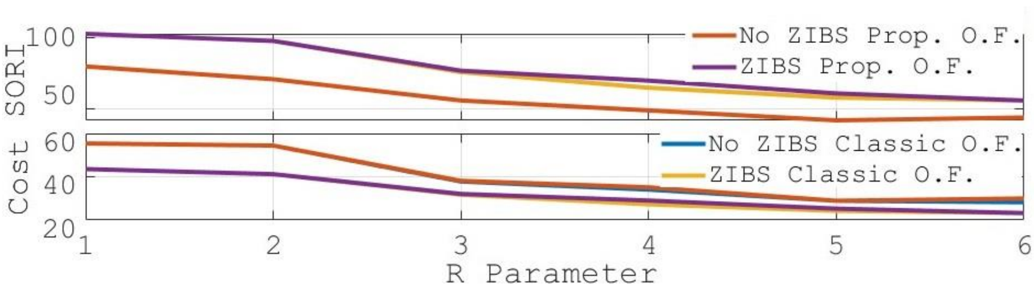

Fig. 1. Cost and SORI variation with R parameter in the IEEE 57 Power System

All four of these optimization techniques will provide almost the same results with minimal differences, given the stochastic nature of the metaheuristic algorithms, and in some cases, they will provide a better solution with a higher SORI. By using the proposed multicriterial approach, the system's observability can be improved with a minimum cost increase. More than this, considering ZIB's influence, the system's observability will be improved, and the minimum total cost of the system can be reached.

Table 3

Optimal PMU placement without considering ZIBs in the IEEE 118 Power System

		Classic O.F.				Proposed O.F.			
		ILP	PSO	HHO	GWO	ILP	PSO	HHO	GWO
R=1	No PMU	68	68	68	68	68	68	68	68
	Cost	126.45	127.15	125.4	127.75	128.55	128.55	128.2	127.85
	SORI	303	305	300	301	309	309	308	307
R=2	No PMU	62	62	62	62	62	6	62	62
	Cost	121.85	121.15	120.1	120.8	123.25	123.25	122.9	122.5
	SORI	295	293	290	292	299	299	298	297
R=3	No PMU	40	40	40	40	40	40	40	40
	Cost	82.3	87.25	85.85	86.2	89	89	87.6	88.6
	SORI	201	215	211	212	220	220	216	219
R=4	No PMU	37	37	37	37	37	37	37	37
	Cost	78.3	81.1	78.3	78.65	83.9	83.9	83.2	83.55
	SORI	192	200	192	193	208	208	206	207
R=5	No PMU	35	35	35	35	35	35	35	35
	Cost	68.6	72.45	71.05	72.1	77.35	77.35	77	76.65
	SORI	164	177	173	176	191	191	190	189
R=6	No PMU	32	32	32	32	32	32	32	32
	Cost	65.25	65.25	61.4	64.2	66.65	66.65	66.65	66.65
	SORI	159	159	148	156	163	163	163	163
R=7	No PMU	32	32	32	32	32	32	32	32
	Cost	61.4	65.95	62.25	63.5	67	67	62.8	66.3
	SORI	148	161	159	154	164	164	152	162
R=8	No PMU	32	32	32	32	32	32	32	32
	Cost	61.4	65.95	62.25	63.5	67	67	62.8	66.3
	SORI	148	161	159	154	164	164	152	162
R=9	No PMU	32	32	32	32	32	32	32	32
	Cost	61.4	65.95	62.25	63.5	67	67	62.8	66.3
	SORI	148	161	159	154	164	164	152	162

To achieve full observability, it is necessary to install PMU devices in about 30% to 58% of the IEEE 57 (27% to 58% for IEEE118) system buses without considering ZIBs influence, and about 21% to 44% (23% to 53% for IEEE 118) in the case of considering ZIBs influence depending on the measurement system redundancy.

Table 4

Optimal PMU placement considering ZIBs in the IEEE 118 Power System

		Classic O.F.				Proposed O.F.			
		ILP	PSO	HHO	GWO	ILP	PSO	HHO	GWO
R=1	No PMU	63	63	63	63	63	63	63	63
	Cost	119	119.7	117.95	116.9	120.4	117.6	117.6	117.6
	SORI	268	288	283	280	290	282	282	282
R=2	No PMU	56	56	56	56	56	56	56	56

	Cost	112.35	112	111.3	111.65	113.4	112.7	112.7	112.7
	SORI	273	272	270	271	276	274	274	274
R=3	No PMU	36	36	36	36	36	36	36	36
	Cost	80.45	80.45	80.1	79.75	82.55	81.15	80.1	80.4
	SORI	199	199	198	197	205	201	198	199
R=4	No PMU	33	33	33	33	33	33	33	33
	Cost	70.1	74.3	72.2	70.45	77.1	74.65	75.35	73.25
	SORI	172	184	178	173	192	185	187	181
R=5	No PMU	31	31	31	31	31	31	31	31
	Cost	66.7	67.5	65.3	64.6	70.9	67.4	66.35	674
	SORI	168	165	160	158	176	166	163	166
R=6	No PMU	27	27	27	27	27	27	27	27
	Cost	57.8	58.5	57.45	58.15	59.55	58.85	58.85	59.2
	SORI	142	144	141	143	147	145	145	146
R=7	No PMU	27	27	27	27	27	27	27	27
	Cost	57.8	59.2	57.45	57.8	59.55	59.2	59.2	58.5
	SORI	142	146	141	142	147	146	146	144
R=8	No PMU	27	27	27	27	27	27	27	27
	Cost	57.8	59.2	57.45	57.8	59.55	59.2	59.2	58.5
	SORI	142	146	141	142	147	146	146	144
R=9	No PMU	27	27	27	27	27	27	27	27
	Cost	57.8	59.2	57.45	57.8	59.55	59.2	59.2	58.5
	SORI	142	146	141	142	147	146	146	144

By increasing the value of R, redundancy will decrease, and the total cost of the measurement system and the SORI value will also decrease, as shown in figures 1 and 2.

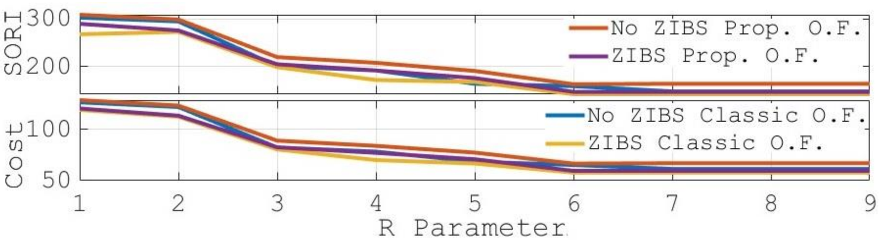

Fig. 2. Cost and SORI variation with R parameter in the IEEE 118 Power System

6. Conclusions

The results for different scenarios highlight the necessity of analyzing every power system using a multicriterial approach that takes into consideration the different specifications and needs that every power system has. The results show that better costs and better SORIs can be obtained for the same number of buses where PMUs are placed. For example, based on this approach, solutions with a minimum number of PMUs were found but differently placed, which improved the system redundancy with a minimal total cost increase. This aspect is very important because it proves that it is necessary to improve the classical mathematical model to take into consideration the redundancy and the total implementation cost to achieve the best solution for the placement problem. Thus, it is not necessary to post-evaluate the obtained solutions with the same number of PMUs.

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