EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR UNCERTAIN NONLINEAR SWITCHED SYSTEMS WITH V-N JUMPS

Zhifu Jia[1] and Cunlin Li[2]

In this paper, an uncertain nonlinear switched system with V-n jumps, characterized by its sensitivity to subjective uncertainties, is modeled using uncertain differential equations with V-n jumps. To account for the discontinuous jump behavior in each subsystem, a V-n jumps process associated with an uncertain $\mathfrak{Z}$-n jumps variable, defined by a jump uncertainty distribution, is introduced. Under the assumptions of linear growth and Lipschitz conditions, an existence and uniqueness theorem for the solutions of uncertain nonlinear switched systems with V-n jumps is established and rigorously proven. An illustrative example is provided to demonstrate the effectiveness and applicability of the theoretical results.

Keywords: Uncertainty theory; uncertain nonlinear switched systems with V-n jumps; existence and uniqueness

MSC2020: 34M 99; 37N 99; 65L 99; 93E 99.

1. Introduction

Over the past two decades, the study of the properties of differential systems has advanced rapidly, resulting in significant achievements in areas such as the oscillatory behavior of differential equations [1, 2, 3], their asymptotic properties [4], and the stabilization of switched differential systems [5, 6, 7]. The development of stochastic analysis, particularly with the introduction of theories like Itô calculus [8], has greatly facilitated progress in the investigation of the stability of stochastic systems. As a specific class of stochastic systems, stochastic switching systems have drawn substantial attention from the academic community. Major developments in this field include the proposal of multiple Lyapunov function methods for stability analysis [9], the establishment of exponential m-stability [10], and the exploration of input-to-state stability via Lyapunov-based approaches for nonlinear systems [11]. Unlike stochastic switched systems, uncertain nonlinear switched systems are nonlinear switched systems that are influenced by subjective uncertainties, which can be modeled using uncertain differential equations characterized by belief degrees. This form of uncertainty, defined by belief degrees, is distinct from the uncertainty described by probability theory and is best framed within the context of uncertainty theory [12, 13]. In practical uncertain control systems, environmental factors such as component aging or damage may alter the system dynamics, leading to abrupt changes in the mathematical model. Furthermore, during actual operation, these systems are frequently subjected to various disturbances, resulting in uncertain noise and uncertain jumps with time-varying parameters. The uncertain switching jump system, which integrates the features of both uncertain switching systems and uncertain jump systems, encapsulates the complexities associated

[1] Dr., School of Mathematics and Physics, Suqian University, Suqian, 223800, China, e-mail: jzflzbx@163.com

[2] Prof.,(Ningxia Key Laboratory of intelligent information and Big Data Processing, Governance and Social Management Research Center of Northwest Ethnic Regions, North Minzu University, Yinchuan, China, e-mail: bitlcl@163.com(corresponding author)

with these combined uncertainties. Uncertainty theory has become widely applied across diverse fields, including uncertain variational inequalities [14, 15], the stability analysis of uncertain systems [16, 17], as well as in differential games and equilibrium strategy control problems for uncertain systems [18, 19].

To describe the intrinsic properties of uncertain differential systems, the concept of stability in measure was introduced in 2009. Later, Yao et al. [20] provided a sufficient criterion for evaluating the stability in measure for uncertain systems. Su et al. [21] expanded this research by introducing the concept of stability for multidimensional uncertain differential equations based on uncertain measure. Subsequent studies investigated stability in terms of the p-th moment [22], mean stability [23], and almost sure stability [24]. In 2022, Su et al. [25] explored three types of stability for uncertain nonlinear switched systems, while Jia and Li [26] extended this line of inquiry by introducing stability in the p-th moment specifically for uncertain nonlinear switched systems. These analyses of stability are fundamentally dependent on the existence of a unique solution for the uncertain system. Chen and Liu [27] offered a sufficient condition ensuring the existence of a unique solution for uncertain differential equations and derived an analytical solution for a class of linear uncertain differential equations. Deng et al. [28] demonstrated the existence and uniqueness theorem for uncertain differential equations involving jumps. Zhu [29] established an existence and uniqueness theorem for uncertain fractional systems, under the conditions of Lipschitz continuity and linear growth. Yang and Ni [30] examined a class of uncertain heat equations and proposed conditions to ensure the existence and uniqueness of solutions. Jia et al. [31] studied uncertain equations incorporating delay and V-jumps, while Su et al. [32] developed an existence and uniqueness theorem for uncertain nonlinear switched systems. This paper presents two key contributions. First, it investigates nonlinear switched systems with subjective uncertainties and jumps within a finite-time domain. Second, using the V-n jumps process [33], uncertainty theory, and Banach's fixed-point theorem, an existence and uniqueness theorem is proved for a class of uncertain nonlinear switched systems with V-n jumps, under conditions of linear growth and Lipschitz continuity. Compared to the results in Ref. [32], this work enhances the understanding and description of the properties of solutions for nonlinear switched systems influenced by subjective uncertainties and V-n jumps. Beyond theoretical advancements, uncertain differential equations have wide-ranging applications in dynamical systems, particularly in nonlinear switched systems, as discussed in this paper and in previous studies [34, 35].

The structure of the paper is organized as follows: Section 2 presents an overview of the Liu process, the V-n jumps process, and the concept of uncertain nonlinear switched systems with V-n jumps, along with two key foundational assumptions. Section 3 formulates the existence and uniqueness theorem for uncertain nonlinear switched systems with V-n jumps. In Section 4, an example is provided to demonstrate the validity and effectiveness of the proposed theoretical results. Lastly, Section 5 concludes the paper with a concise summary of the main findings.

2. Uncertain nonlinear switched systems with V-n jumps

Definition 2.1. [12, 13] *An uncertain process C_t as a Liu process based on the following conditions:*

(1) $C_0 = 0$ and almost all sample paths are Lipschitz continuous.

(2) C_t has stationary and independent increments

(3) Every increment $C_{r+t} - C_r$ is a normal uncertain variable with an expected value of 0 and variance t^2. The uncertainty distribution of this increment is given by:

$$\Phi(x) = \left(1 + \exp\left(-\frac{\pi x}{\sqrt{3}t}\right)\right)^{-1}, \quad x \in \mathbb{R}.$$

Definition 2.2. [33] *An uncertain variable* $\mathfrak{Z}(\mu_{i1}, \mu_{i2}, n, t)$ *is described as an uncertain* $\mathfrak{Z}$*-n jumps variable with parameters* μ_{i1} *and* μ_{i2} *under specific conditions. The definition of the uncertain variable outlines a piecewise function* $\Phi(x)$*, which characterizes the jump uncertainty distribution of this variable for* $t > 0$*. The function* $\Phi(x)$ *is defined as follows:*

$$\Phi(x) = \begin{cases} 0, & \text{if } x < 0, \\ \frac{(n+1)\mu_{11}}{t}x, & \text{if } 0 \le x < \frac{t}{n+1}, \\ \mu_{i2} + \frac{(n+1)(\mu_{(i+1)1} - \mu_{i2})}{t}\left(x - \frac{it}{n+1}\right), & \text{if } \frac{it}{n+1} \le x < \frac{(i+1)t}{n+1},\ i = 1, \dots, n, \\ 1, & \text{if } x \ge t. \end{cases}$$

Remark 2.1. *The function* $\Phi(x)$ *defines the jump uncertainty distribution for different intervals of* x*. Parameters* μ_{i1} *and* μ_{i2} *determine the characteristics of the jumps within the distribution. The conditions* $0 < \mu_{i1} < \mu_{i2} < \mu_{(i+1)1} < \mu_{(i+1)2} < \mu_{(n+1)1} = 1$ *for* $i = 1, 2, \dots, n-1$ *ensure that the jump points are ordered correctly.*

Definition 2.3. [33] *An uncertain process* V_t *is said to be an uncertain* V*-n jumps process with parameters* μ_{i1} *and* μ_{i2} $(0 < \mu_{i1} < \mu_{i2} < \mu_{(i+1)1} < \mu_{(i+1)2} < \mu_{(n+1)1} = 1, i = 1, 2, \cdots, n-1)$ *for* $t > 0$ *if (i)* $V_0 = 0$*; (ii)* V_t *has stationary and independent increments; (iii) every increment* $V_{r+t} - V_r$ *is an uncertain* $\mathfrak{Z}$*-n jumps variable* $\mathfrak{Z}(\mu_{i1}, \mu_{i2}, n, t)$*.*

Lemma 2.1. [32] *Suppose that* C_t *is a Liu process, and* $\mathbf{Z}_t$ *is an integrable* n*-dimensional uncertain process on* $[a, b]$ *with respect to* t*. Then the inequality*

$$\left\| \int_a^b \mathbf{Z}_t(\gamma) dC_t(\gamma) \right\|_1 \le K_\gamma \int_a^b \|\mathbf{Z}_t(\gamma)\|_1 dt$$

holds, where K_γ *is the Lipschitz constant of the sample path* $C_t(\gamma)$*.*

In fact, the uncertain nonlinear switched system with V-n jumps discussed in this section is a nonlinear switched system influenced by both a Liu process and a V-n jumps process. In the finite-time domain, the uncertain nonlinear switched system with V-n jumps, represented by a series of uncertain differential equations with V-n jumps, will be analyzed in the following sections.

$$\begin{cases} d\mathbf{Z}_t = \mathbf{f}_{i(k)}(t, \mathbf{Z}_t)dt + \mathbf{g}_{i(k)}(t, \mathbf{Z}_t)dC_t + \mathbf{h}_{i(k)}(t, \mathbf{Z}_t)dV_t, & t \in [0, T), \\ i(k) \in \{1, 2, \dots, M\}, \\ \mathbf{Z}_t|_{t=0} = \mathbf{Z}_0, & t = 0, \end{cases} \tag{1}$$

where $\mathbf{Z}_t \in \mathbb{R}^n$ represents the state vector of the system, coefficient functions $\mathbf{f}_{i(k)}(t, \mathbf{z}) : [0, T] \times \mathbb{R}^n \to \mathbb{R}^n$, $\mathbf{g}_{i(k)}(t, \mathbf{z}) : [0, T] \times \mathbb{R}^n \to \mathbb{R}^n$ and $\mathbf{h}_{i(k)}(t, \mathbf{z}) : [0, T] \times \mathbb{R}^n \to \mathbb{R}^n$ are all continuous for any $i(k) \in \{1, 2, \dots, M\}$, C_t and V_t respectively represent the noise and the jump of the subsystem, with C_t as a Liu process and V_t as a V-jump process defined on an uncertainty space.

Remark 2.2. *In the given system* (1)*, the uncertain nonlinear switched system with V-n jumps consists of multiple subsystems, each governed by a specific switching law* $i(k) \in \{1, 2, \dots, M\}$*. Each subsystem is described by an uncertain differential equation with V-n jumps. This jump process introduces discontinuities or sudden changes in the systems trajectory, which are characteristic of processes with V-n jumps.*

The 1-norm is used to measure the distance for a vector $\mathbf{Z} = (z_1, z_2, \dots, z_n)^T$ in this paper, specifically defined as:

$$\|\mathbf{Z}\|_1 = \sum_{i=1}^{n} |z_i|.$$

The switching law of uncertain switched system (1) defined on the interval $[0, T]$ is

$$\Lambda = \{(t_0, i(0)), (t_1, i(1)), \ldots, (t_n, i(N))\},$$

at the time instants $t_0 = 0 < t_1 < \cdots < t_N = T$, the tuple $(t_k, i(k))$ indicates that at time t_k, the system switches from sub-system $i(k-1)$ to sub-system $i(k)$. That is, sub-system $i(k)$ remains active during the time interval $[t_k, t_{k+1}]$ for each $k \in \{0, 1, \ldots, N\}$.

Next, two assumptions regarding the coefficient functions in system (1) are introduced to facilitate the concise analysis of the existence and uniqueness of its solutions. It is assumed that for each $i(k) \in \{1, 2, \ldots, M\}$, there exist corresponding positive constants such that

Assumption 1. The coefficient functions $\mathbf{f}_{i(k)}(t, \mathbf{z})$, $\mathbf{g}_{i(k)}(t, \mathbf{z})$ and $\mathbf{h}_{i(k)}(t, \mathbf{z})$ satisfy the linear growth condition

$$\|\mathbf{f}_{i(k)}(t, \mathbf{z})\|_1 + \|\mathbf{g}_{i(k)}(t, \mathbf{z})\|_1 + \|\mathbf{h}_{i(k)}(t, \mathbf{z})\|_1 \leq \mathfrak{Q}_{i(k)}(1 + \|\mathbf{z}\|_1)$$

for any $t \in [0, T]$, $\mathbf{z} \in \mathbb{R}^n$;

Assumption 2. The coefficient functions $\mathbf{f}_{i(k)}(t, \mathbf{z})$, $\mathbf{g}_{i(k)}(t, \mathbf{z})$ and $\mathbf{h}_{i(k)}(t, \mathbf{z})$ satisfy the Lipschitz condition

$$\begin{aligned}
&\|\mathbf{f}_{i(k)}(t, \mathbf{z}) - \mathbf{f}_{i(k)}(t, \tilde{\mathbf{z}})\|_1 + \|\mathbf{g}_{i(k)}(t, \mathbf{z}) - \mathbf{g}_{i(k)}(t, \tilde{\mathbf{z}})\|_1 + \|\mathbf{h}_{i(k)}(t, \mathbf{z}) - \mathbf{h}_{i(k)}(t, \tilde{\mathbf{z}})\|_1 \\
&\leq \mathfrak{L}_{i(k)}\|\mathbf{z} - \tilde{\mathbf{z}}\|_1, \forall t \in [0, T], \mathbf{z}, \tilde{\mathbf{z}} \in \mathbb{R}^n.
\end{aligned}$$

The symbols $\mathfrak{Q}$ and $\mathfrak{L}$ are employed to denote the maximums of positive constants $\mathfrak{Q}_{i(k)}$ and $\mathfrak{L}_{i(k)}$ $(i(k) = 1, 2, \ldots, M)$, respectively, so the following equalities are established:

$$\mathfrak{Q} = \max_{i(k)}\{\mathfrak{Q}_{i(k)} |\ i(k) = 1, 2, \ldots, M\},\ \mathfrak{L} = \max_{i(k)}\{\mathfrak{L}_{i(k)} |\ i(k) = 1, 2, \ldots, M\}.$$

3. **Existence and uniqueness of the solutions**

In this section, the existence and uniqueness of solutions for the uncertain nonlinear switched system with V-n jumps (1) are investigated using uncertainty theory and the Banach fixed-point theorem. Let $C[0, T]$ represent the space of continuous $\mathbb{R}^n$-valued vector functions on $[0, T]$. Hence, it is straightforward to see that $C[0, T]$ forms a Banach space with the following norm:

$$\|\mathbf{Z}\|_{[0,T]} = \max_{t\in[0,T]} \|\mathbf{Z}_t\|_1.$$

The mapping $\chi(t)$ on $C[0, T]$ is now defined as follows: for a sample path $\mathbf{Z}(\gamma) \in C[0, T]$ with any given $\gamma \in \Gamma$, we denote that

$$\begin{aligned}
\chi(\mathbf{Z}_t(\gamma)) =&\mathbf{Z}_0 + \sum_{j=0}^{k-1} \int_{t_j}^{t_{j+1}} \mathbf{f}_{i(j)}(r, \mathbf{Z}_r(\gamma))dr + \int_{t_k}^{t} \mathbf{f}_{i(k)}(r, \mathbf{Z}_r(\gamma))dr \\
&+ \sum_{j=0}^{k-1} \int_{t_j}^{t_{j+1}} \mathbf{g}_{i(j)}(r, \mathbf{Z}_r(\gamma))dC_r(\gamma) + \int_{t_k}^{t} \mathbf{g}_{i(k)}(r, \mathbf{Z}_r(\gamma))dC_r(\gamma) \\
&+ \sum_{j=0}^{k-1} \int_{t_j}^{t_{j+1}} \mathbf{h}_{i(j)}(r, \mathbf{Z}_r(\gamma))dV_r(\gamma) + \int_{t_k}^{t} \mathbf{h}_{i(k)}(r, \mathbf{Z}_r(\gamma))dV_r(\gamma),
\end{aligned}$$

where $t \in [t_k, t_{k+1}] \subseteq [0, T]$ and $k = 0, 1, \ldots, N$.

To examine the existence and uniqueness of the solutions, the following two lemmas are first introduced and proven.

Lemma 3.1. *Suppose that V_t is an uncertain V-jump process, and $\mathbf{Z}_t$ is an integrable n-dimensional uncertain process on $[a,b]$ with respect to t. Then, for any sample γ, the inequality*

$$\left\|\int_a^b \mathbf{Z}_t(\gamma)dV_\gamma(t)\right\|_1 \le \int_a^b \|\mathbf{Z}_t(\gamma)\|_1\, dt$$

holds.

Proof Denote $\mathbf{Z}_t = (\mathbf{Z}_t^1, \mathbf{Z}_t^2, \ldots, \mathbf{Z}_t^n)^T$, where $\mathbf{Z}_t^i$ are integrable uncertain processes for $i = 1, 2, \ldots, n$. By applying Theorem 3.2 in Ref. [28], we obtain that

$$\left\|\int_a^b \mathbf{Z}_t(\gamma)dV_\gamma(t)\right\|_1 = \sum_{i=1}^n \left|\int_a^b \mathbf{Z}_t^i(\gamma)dV_\gamma(t)\right| \le \sum_{i=1}^n \int_a^b \left|\mathbf{Z}_t^i(\gamma)\right| dt = \int_a^b \|\mathbf{Z}_t(\gamma)\|_1\, dt.$$

This completes the proof.

Lemma 3.2. *If a vector function $\mathbf{Z}_t(\gamma) \in C[0,T]$ holds for any sample $\gamma \in \Gamma$, and coefficient functions $\mathbf{f}_{i(k)}(t,\mathbf{z})$, $\mathbf{g}_{i(k)}(t,\mathbf{z})$ and $\mathbf{h}_{i(k)}(t,\mathbf{z})$ satisfy the linear growth condition outlined in Assumption 1 for each $i(k) \in \{1, 2, \ldots, M\}$, then $\chi(\mathbf{Z}_t(\gamma)) \in C[0,T]$.*

Proof Let $r_1, r_2 \in [0,T]$ with $r_1 \le r_2$ and $|r_2 - r_1| < \min_{k=0,1,\ldots,N}\{t_{k+1} - t_k\}$. There are two cases in which the distance between $\chi(\mathbf{Z}_{r_2}(\gamma))$ and $\chi(\mathbf{Z}_{r_1}(\gamma))$ can be computed based on Lemma 3.1.

Firstly, r_1 and r_2 lie within the same interval, meaning $t_k \le r_1 < r_2 \le t_{k+1}$, and thus we have

$$\begin{aligned}
&\|\chi(\mathbf{Z}_{r_2}(\gamma)) - \chi(\mathbf{Z}_{r_1}(\gamma))\|_1 \\
=&\left\|\int_{r_1}^{r_2} \mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))dr + \int_{r_1}^{r_2} \mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))dC_r(\gamma) + \int_{r_1}^{r_2} \mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))dV_r(\gamma)\right\|_1 \\
\le&\int_{r_1}^{r_2} \|\mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))\|_1 dr + \|\int_{r_1}^{r_2} \mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))dC_r(\gamma)\|_1 + \|\int_{r_1}^{r_2} \mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))dV_r(\gamma)\|_1 \\
\le&\int_{r_1}^{r_2} \|\mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))\|_1 dr + K_\gamma \int_{r_1}^{r_2} \|\mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))\|_1 dr + \int_{r_1}^{r_2} \|\mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))\|_1 dr \\
\le&(2+K_\gamma)\int_{r_1}^{r_2} \mathfrak{Q}_{i(k)}(n + \|\mathbf{Z}_t(\gamma)\|_{[t_k,t_{k+1}]})dr \\
\le&\mathfrak{Q}(n + \|\mathbf{Z}_t(\gamma)\|_{[t_k,t_{k+1}]})(2+K_\gamma)(r_2 - r_1). \qquad (2)
\end{aligned}$$

Secondly, when r_1 and r_2 belong to two different intervals, i.e., $t_{k-1} \le r_1 < t_k \le r_2 \le t_{k+1}$, we have

$$\begin{aligned}
&\|\chi(\mathbf{Z}_{r_2}(\gamma)) - \chi(\mathbf{Z}_{r_1}(\gamma))\|_1 \\
=&\Bigg\|\int_{r_1}^{t_k} \mathbf{f}_{i(k-1)}(r,\mathbf{Z}_r(\gamma))dr + \int_{t_k}^{r_2} \mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))dr \\
&+ \int_{r_1}^{t_k} \mathbf{g}_{i(k-1)}(r,\mathbf{Z}_r(\gamma))dC_r(\gamma) + \int_{t_k}^{r_2} \mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))dC_r(\gamma) \\
&+ \int_{r_1}^{t_k} \mathbf{h}_{i(k-1)}(r,\mathbf{Z}_r(\gamma))dV_r(\gamma) + \int_{t_k}^{r_2} \mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))dV_r(\gamma)\Bigg\|_1 \\
\le&\int_{r_1}^{t_k} \|\mathbf{f}_{i(k-1)}(r,\mathbf{Z}_r(\gamma))\|_1 dr + \|\int_{r_1}^{t_k} \mathbf{g}_{i(k-1)}(r,\mathbf{Z}_r(\gamma))dC_r(\gamma)\|_1
\end{aligned}$$

$$
\begin{aligned}
&+\|\int_{r_1}^{t_k}\mathbf{h}_{i(k-1)}(r,\mathbf{Z}_r(\gamma))dV_r(\gamma)\|_1+\int_{t_k}^{r_2}\|\mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))\|_1dr\\
&+\|\int_{t_k}^{r_2}\mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))dC_r(\gamma)\|_1+\|\int_{t_k}^{r_2}\mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))dV_r(\gamma)\|_1\\
\leq&\int_{r_1}^{t_k}\|\mathbf{f}_{i(k-1)}(r,\mathbf{Z}_r(\gamma))\|_1dr+K_\gamma\int_{r_1}^{t_k}\|\mathbf{g}_{i(k-1)}(r,\mathbf{Z}_r(\gamma))\|_1dr\\
&+\int_{r_1}^{t_k}\|\mathbf{h}_{i(k-1)}(r,\mathbf{Z}_r(\gamma))\|_1dr+\int_{t_k}^{r_2}\|\mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))\|_1dr\\
&+K_\gamma\int_{t_k}^{r_2}\|\mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))\|_1dr+\int_{t_k}^{r_2}\|\mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))\|_1dr\\
\leq&(2+K_\gamma)\int_{r_1}^{t_k}\mathfrak{Q}_{i(k-1)}(n+\|\mathbf{Z}_t(\gamma)\|_{[t_{k-1},t_k]})dr\\
&+(2+K_\gamma)\int_{t_k}^{r_2}\mathfrak{Q}_{i(k)}(n+\|\mathbf{Z}_t(\gamma)\|_{[t_k,t_{k+1}]})dr\\
\leq&\mathfrak{Q}_{i(k-1)}(n+\|\mathbf{Z}_t(\gamma)\|_{[t_{k-1},t_k]})(2+K_\gamma)(t_k-r_1)\\
&+\mathfrak{Q}_{i(k)}(n+\|\mathbf{Z}_t(\gamma)\|_{[t_k,t_{k+1}]})(2+K_\gamma)(r_2-t_k)\\
\leq&\mathfrak{Q}(n+\|\mathbf{Z}_t(\gamma)\|_{[t_{k-1},t_{k+1}]})(2+K_\gamma)(r_2-r_1).
\end{aligned}\tag{3}
$$

Combining the above (2) and (3), we obtain that:

$$\|\chi(\mathbf{Z}_{r_2}(\gamma))-\chi(\mathbf{Z}_{r_1}(\gamma))\|_1\leq\mathfrak{Q}(n+\|\mathbf{Z}_t(\gamma)\|_{[0,T]})(2+K_\gamma)(r_2-r_1),$$

which implies that

$$\|\chi(\mathbf{Z}_{r_2}(\gamma))-\chi(\mathbf{Z}_{r_1}(\gamma))\|_1\to 0,$$

as $r_2-r_1\to 0$. Hence, $\chi(\mathbf{Z}_t(\gamma))$ is continuous on $[0,T]$ for any $\gamma\in\Gamma$. This concludes the proof of the lemma.

Building on the results from Lemmas 3.1 and 3.2, the existence and uniqueness of solutions for the uncertain nonlinear switched system with V-n jumps (1) can be explored. Subsequently, an existence and uniqueness theorem will be established on a small interval $[t,t+c]$ by applying the two lemmas along with the Banach fixed-point theorem.

Theorem 3.1. *There exists a constant $c>0$ such that, for any $t\in[t_k,t_{k+1}]\subseteq[0,T]$, system (1) has a unique solution on the interval $[t,t+c]$ (with $t+c=t_{k+1}$ if $t+c>t_{k+1}$, for $k=0,1,\ldots,N$). This holds provided that the vector functions $\mathbf{f}_{i(k)}(t,\mathbf{z})$, $\mathbf{g}_{i(k)}(t,\mathbf{z})$, and $\mathbf{h}_{i(k)}(t,\mathbf{z})$ satisfy both the linear growth condition and the Lipschitz condition specified in Assumptions 1 and 2 for each $i(k)\in\{1,2,\ldots,M\}$.*

Proof Let $c>0$ such that $\varrho(\gamma)=\mathfrak{L}(2+K_\gamma)c\in(0,1)$.

For any given $t\in[0,T]$, $\tau\in[t,t+c]$ and $\gamma\in\Gamma$, define a mapping Ψ on the space $C[t,t+c]$ in the following:

$$
\begin{aligned}
\Psi(\mathbf{Z}_\tau(\gamma))=&\mathbf{Z}_t(\gamma)+\int_t^\tau\mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))dr+\int_t^\tau\mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))dC_r(\gamma)\\
&+\int_t^\tau\mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))dV_r(\gamma).
\end{aligned}\tag{4}
$$

By Lemma 3.2, it is able to derive that $\Psi(\mathbf{Z}_\tau(\gamma))\in C[t,t+c]$ for $\mathbf{Z}_\tau(\gamma)\in C[t,t+c]$.

For any $\tau\in[t,t+c]$, according to the Lipschitz condition in Assumption 3.2 we have

$$\|\Psi(\mathbf{Z}_\tau(\gamma))-\Psi(\tilde{\mathbf{Z}}_\tau(\gamma))\|_1$$

$$
\begin{aligned}
=&\left\|\int_t^\tau \left[\mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{f}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\right]dr\right.\\
&+\int_t^\tau \left[\mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{g}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\right]dC_r(\gamma)\\
&\left.+\int_t^\tau \left[\mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{h}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\right]dV_r(\gamma)\right\|_1\\
\le&\left\|\int_t^\tau \left[\mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{f}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\right]dr\right\|_1\\
&+\left\|\int_t^\tau \left[\mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{g}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\right]dC_r(\gamma)\right\|_1\\
&+\left\|\int_t^\tau \left[\mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{h}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\right]dV_r(\gamma)\right\|_1\\
\le&\int_t^\tau \left\|\mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{f}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\right\|_1 dr\\
&+K_\gamma\int_t^\tau \left\|\mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{g}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\right\|_1 dr\\
&+\int_t^\tau \left\|\mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{h}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\right\|_1 dr\\
\le&(2+K_\gamma)\int_t^\tau \Big[\|\mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{f}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\|_1\\
&+\|\mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{g}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\|_1\\
&+\|\mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))-\mathbf{h}_{i(k)}(r,\tilde{\mathbf{Z}}_r(\gamma))\|_1\Big]dr\\
\le&(2+K_\gamma)\mathfrak{L}_{i(k)}\int_t^\tau \left\|\mathbf{Z}_r(\gamma)-\tilde{\mathbf{Z}}_r(\gamma)\right\|_1 dr\\
\le&\mathfrak{L}(2+K_\gamma)\int_t^\tau \max_{t\le r\le t+c}\left\|\mathbf{Z}_r(\gamma)-\tilde{\mathbf{Z}}_r(\gamma)\right\|_1 dr\\
\le&\mathfrak{L}(2+K_\gamma)c\left\|\mathbf{Z}_r(\gamma)-\tilde{\mathbf{Z}}_r(\gamma)\right\|_{[t,t+c]},
\end{aligned}
$$

which means that

$$
\|\Psi(\mathbf{Z}_\tau(\gamma))-\Psi(\tilde{\mathbf{Z}}_\tau(\gamma))\|_{[t,t+c]}\le\varrho(\gamma)\|\mathbf{Z}_\tau(\gamma)-\tilde{\mathbf{Z}}_\tau(\gamma)\|_{[t,t+c]}, \tag{5}
$$

where $0<\varrho(\gamma)<1$.

From the above (5), it can be concluded that Ψ is a contraction mapping on $C[t,t+c]$. Therefore, by applying the well-known Banach fixed-point theorem, there exists a unique fixed point $\mathbf{Z}_\tau(\gamma)\in C[t,t+c]$ that satisfies (4).

This unique fixed point $\mathbf{Z}_\tau(\gamma)$, a continuous function of τ defined on $[t,t+c]$, can be treated as a sample path; and it is the unique solution of the following ordinary differential equation:

$$
\begin{aligned}
\mathbf{Z}_\tau(\gamma)=&\mathbf{Z}_t(\gamma)+\int_t^\tau \mathbf{f}_{i(k)}(r,\mathbf{Z}_r(\gamma))dr+\int_t^\tau \mathbf{g}_{i(k)}(r,\mathbf{Z}_r(\gamma))dC_r(\gamma)\\
&+\int_t^\tau \mathbf{h}_{i(k)}(r,\mathbf{Z}_r(\gamma))dV_r(\gamma),\quad \tau\in[t,t+c].
\end{aligned}
$$

In other words, for each sample point $\gamma\in\Gamma$, there exists a unique function $\mathbf{Z}_\tau(\gamma)$ that satisfies the given ordinary differential equation. Considering all sample points in Γ, $\mathbf{Z}_r$ becomes a mapping from the sample set Γ to the set of corresponding sample paths. Consequently,

$\mathbf{Z}_\tau$ is identified as an uncertain process, and for each $\tau \in [t, t+c]$, $\mathbf{Z}_\tau$ is measurable with respect to γ in Γ.

Therefore, the uncertain process $\mathbf{Z}_t$ is the unique solution of the uncertain differential equation

$$\mathbf{Z}_\tau = \mathbf{Z}_t + \int_t^\tau \mathbf{f}_{i(k)}(r, \mathbf{Z}_r)dr + \int_t^\tau \mathbf{g}_{i(k)}(r, \mathbf{Z}_r)dC_r + \int_t^\tau \mathbf{h}_{i(k)}(r, \mathbf{Z}_r)dV_r, \quad \tau \in [t, t+c].$$

In conclusion, system (1) has a unique solution $\mathbf{Z}_t$ on the small interval $[t, t+c]$. This completes the proof.

In the following, the existence and uniqueness of solutions for uncertain nonlinear switched system with V-n jumps (1) will be extended to the interval $[0, T]$ on the basis of Theorem 3.1.

Theorem 3.2. *For $0 < T < +\infty$, the uncertain nonlinear switched system with V-n jumps* (1) *possesses a unique solution over the interval $[0, T]$, provided that the coefficient functions $\mathbf{f}_{i(k)}(t, \mathbf{z})$, $\mathbf{g}_{i(k)}(t, \mathbf{z})$, and $\mathbf{h}_{i(k)}(t, \mathbf{z})$ meet the linear growth and Lipschitz conditions outlined in Assumptions 1 and 2 for each $i(k) \in \{1, 2, \ldots, M\}$.*

Proof For each $k \in \{0, 1, \ldots, N\}$, denote that

$$[t_k, t_k + c], [t_k + c, t_k + 2c], \ldots, [t_k + (l_k - 1)c, t_k + l_k c], [t_k + l_k c, t_{k+1}]$$

are the subsets of $[t_k, t_{k+1}]$ with

$$t_k + l_k c < t_{k+1} \leq t_k + (l_k + 1)c.$$

For any $\gamma \in \Gamma$, it follows from Theorem 3.1 that uncertain nonlinear switched system with V-n jumps (1) has a unique solution $\mathbf{Z}_t^{k,j}$ on the small interval $[t_k + jc, t_k + (j+1)c]$ for $j = 0, 1, \ldots, l_k$ and setting $t_k + (l_k + 1)c = t_{k+1}$.

Therefore, system (1) has a unique solution $\mathbf{Z}_t^k$ on the interval $[t_k, t_{k+1}]$ for every $k \in \{0, 1, \ldots, N\}$ by defining

$$\mathbf{Z}_t^k(\gamma) = \begin{cases} \mathbf{Z}_t^{k,0}(\gamma), & t \in [t_k, t_k + c], \\ \mathbf{Z}_t^{k,1}(\gamma), & t \in [t_k + c, t_k + 2c], \\ \vdots \\ \mathbf{Z}_t^{k,l_k-1}(\gamma), & t \in [t_k + (l_k - 1)c, t_k + l_k c], \\ \mathbf{Z}_t^{k,l_k}(\gamma), & t \in [t_k + l_k c, t_{k+1}]. \end{cases}$$

Then a multi-dimensional uncertain process $\mathbf{Z}_t$ on the interval $[0, T]$ is defined as follows:

$$\mathbf{Z}_t(\gamma) = \begin{cases} \mathbf{Z}_t^0(\gamma), & t \in [t_0, t_1] = [0, t_1], \\ \mathbf{Z}_t^1(\gamma), & t \in [t_1, t_2], \\ \vdots \\ \mathbf{Z}_t^{N-1}(\gamma), & t \in [t_{N-1}, t_N], \\ \mathbf{Z}_t^N(\gamma), & t \in [t_N, t_{N+1}] = [t_N, T], \end{cases}$$

for any $\gamma \in \Gamma$, which is the unique solution of uncertain nonlinear switched system with V-n jumps (1) on the interval [0, T]. This completes the proof.

Remark 3.1. *In Theorems 3.1 and 3.2, the coefficient functions in each sub-system are assumed to satisfy the linear growth and Lipschitz conditions. Therefore, regardless of the switching law, system (1) is composed of these sub-systems, ensuring a unique solution. In other words, for any given switching law, the system has a corresponding unique solution.*

4. **An example**

Example 4.1. *To verify the correctness of Theorem 3.2, an example of an uncertain nonlinear switched system with V-n jumps will be presented over a finite-time horizon.*

$$\begin{cases} d\mathbf{Z}_t = \mathbf{f}_{i(k)}(t,\mathbf{Z}_t)dt + \mathbf{g}_{i(k)}(t,\mathbf{Z}_t)dC_t + \mathbf{h}_{i(k)}(t,\mathbf{Z}_t)dV_t, \quad t \in [0,T], \\ i(k) \in \{1,2,3,4\}, \\ \mathbf{Z}_0 = (z_1(0), z_2(0))^T, \end{cases} \tag{6}$$

where $\mathbf{Z}_t = (z_1(t), z_2(t))^T \in \mathbb{R}^2$ *is the state vector of the system, and*

$$\begin{aligned} f_1(t,z) &= e^{-\frac{t}{4}} \cdot z, & g_1(t,z) &= \frac{3}{2+t^2} \cdot \exp(-|z|), & h_1(t,z) &= \frac{3}{2+t^2} \cdot \exp(-|z|), \\ f_2(t,z) &= e^{-\frac{t}{6}} \cdot z, & g_2(t,z) &= \frac{3}{1+t^2} \cdot \exp(-|z|), & h_2(t,z) &= \frac{3}{1+t^2} \cdot \exp(-|z|), \\ f_3(t,z) &= e^{-\frac{t}{3}} \cdot z, & g_3(t,z) &= \frac{1}{1+t^2} \cdot \exp(-|z|), & h_3(t,z) &= \frac{1}{1+t^2} \cdot \exp(-|z|), \\ f_4(t,z) &= e^{-\frac{t}{5}} \cdot z, & g_4(t,z) &= \frac{2}{1+t^2} \cdot \exp(-|z|), & h_4(t,z) &= \frac{2}{1+t^2} \cdot \exp(-|z|), \end{aligned}$$

where the notation $|z|$ *represents the vector* $(|z_1|, |z_2|)^T$ *for* $z = (z_1, z_2)^T$.

If the switching law of system (6) *defined on the interval* $[0,T]$ *is listed as follows:*

$$\Lambda = ((t_0,2),(t_1,4),(t_2,1),(t_3,3),(t_4,2),(t_5,3),(t_6,1),(t_7,2)),$$

where $T = 99$, *and* $t_k (k = 0,1,\ldots,7)$ *are the given switching times, i.e.,*

$$t_0 = 0, \quad t_1 = 8, \quad t_2 = 19, \quad t_3 = 29, \quad t_4 = 47, \quad t_5 = 56, \quad t_6 = 69, \quad t_7 = 79.$$

Then, we can obtain the following uncertain differential equations with V-n jumps

$$\begin{cases} dz_1(t) = e^{-\frac{t}{4}} \cdot z_1(t)dt + \frac{3}{2+t^2} \cdot \exp(-|z_1(t)|)dC_t + \frac{3}{2+t^2} \cdot \exp(-|z_1(t)|)dV_t, \\ dz_2(t) = e^{-\frac{t}{4}} \cdot z_2(t)dt + \frac{3}{2+t^2} \cdot \exp(-|z_2(t)|)dC_t + \frac{3}{2+t^2} \cdot \exp(-|z_2(t)|)dV_t, \\ \quad t \in [19,29] \cup [69,79]; \end{cases}$$

$$\begin{cases} dz_1(t) = e^{-\frac{t}{6}} \cdot z_1(t)dt + \frac{3}{1+t^2} \cdot \exp(-|z_1(t)|)dC_t + \frac{3}{1+t^2} \cdot \exp(-|z_1(t)|)dV_t, \\ dz_2(t) = e^{-\frac{t}{6}} \cdot z_2(t)dt + \frac{3}{1+t^2} \cdot \exp(-|z_2(t)|)dC_t + \frac{3}{1+t^2} \cdot \exp(-|z_2(t)|)dV_t, \\ \quad t \in [0,8] \cup [47,56] \cup [79,99]; \end{cases}$$

$$\begin{cases} dz_1(t) = e^{-\frac{t}{3}} \cdot z_1(t)dt + \frac{1}{1+t^2} \cdot \exp(-|z_1(t)|)dC_t + \frac{1}{1+t^2} \cdot \exp(-|z_1(t)|)dV_t, \\ dz_2(t) = e^{-\frac{t}{3}} \cdot z_2(t)dt + \frac{1}{1+t^2} \cdot \exp(-|z_2(t)|)dC_t + \frac{1}{1+t^2} \cdot \exp(-|z_2(t)|)dV_t, \\ \quad t \in [29,47] \cup [56,69]; \end{cases}$$

$$\begin{cases} dz_1(t) = e^{-\frac{t}{5}} \cdot z_1(t)dt + \frac{2}{1+t^2} \cdot \exp(-|z_1(t)|)dC_t + \frac{2}{1+t^2} \cdot \exp(-|z_1(t)|)dV_t, \\ dz_2(t) = e^{-\frac{t}{5}} \cdot z_2(t)dt + \frac{2}{1+t^2} \cdot \exp(-|z_2(t)|)dC_t + \frac{2}{1+t^2} \cdot \exp(-|z_2(t)|)dV_t, \\ \quad t \in [8,19]. \end{cases}$$

Based on the uncertainty distributions of ΔC_t *and* ΔV_t, *the sample points* $\tilde{c}_t$ *and* $\tilde{v}_t$ *are obtained from their respective inverse uncertainty distributions as* $\tilde{c}_t = -\frac{\sqrt{3}\Delta t}{\pi} \ln\left(\frac{1-\alpha}{\alpha}\right)$, $0 < \alpha < 1$, *and* $\tilde{v}_t = \begin{cases} \frac{\Delta t}{(n+1)\mu_{11}}\alpha, & 0 \le \alpha < \frac{\mu_{11}}{n+1}, \\ \frac{\Delta t}{n+1}\left(i + \frac{\alpha - \mu_{i2}}{\mu_{(i+1)1} - \mu_{i2}}\right), & \mu_{i2} \le \alpha < \mu_{(i+1)1},\ i = 1,\ldots,n, \\ \Delta t, & \alpha = 1. \end{cases}$

When $n = 20$ of V-n jumps, we conducted the following simulation as Figure 1, (a) illustrates coefficient dynamics of the sample points $\tilde{c}_t$ and $\tilde{v}_t$ under different belief degree α; (b) illustrates the solutions of above uncertain differential equations with V-n jumps under four different belief degree α.

(a) (b)

FIGURE 1. (a) gives the coefficient dynamics of the sample points $\tilde{c}_t$ and $\tilde{v}_t$ under different belief degree α, (b) gives solutions of uncertain differential equations with jumps under $\alpha = 0.2, 0.4, 0.6, 0.8$.

For any $t \in [0, T]$, $\mathbf{z} \in \mathbb{R}^2$, it is not difficult to obtain that

$$\|f_1(t,\mathbf{z})\|_1 + \|g_1(t,\mathbf{z})\|_1 + \|h_1(t,\mathbf{z})\|_1 \leq \left(e^{-\frac{t}{4}} + \frac{6}{2+t^2}\right) \cdot (2 + \|\mathbf{z}\|_1) \leq 4(2 + \|\mathbf{z}\|_1),$$

$$\|f_2(t,\mathbf{z})\|_1 + \|g_2(t,\mathbf{z})\|_1 + \|h_2(t,\mathbf{z})\|_1 \leq \left(e^{-\frac{t}{6}} + \frac{6}{1+t^2}\right) \cdot (2 + \|\mathbf{z}\|_1) \leq 7(2 + \|\mathbf{z}\|_1),$$

$$\|f_3(t,\mathbf{z})\|_1 + \|g_3(t,\mathbf{z})\|_1 + \|h_3(t,\mathbf{z})\|_1 \leq \left(e^{-\frac{t}{3}} + \frac{2}{1+t^2}\right) \cdot (2 + \|\mathbf{z}\|_1) \leq 3(2 + \|\mathbf{z}\|_1),$$

$$\|f_4(t,\mathbf{z})\|_1 + \|g_4(t,\mathbf{z})\|_1 + \|h_4(t,\mathbf{z})\|_1 \leq \left(e^{-\frac{t}{5}} + \frac{4}{1+t^2}\right) \cdot (2 + \|\mathbf{z}\|_1) \leq 5(2 + \|\mathbf{z}\|_1),$$

which means that, for each $i(k) \in \{1, 2, 3, 4\}$, the coefficient functions $\mathbf{f}_{i(k)}(t,\mathbf{z})$, $\mathbf{g}_{i(k)}(t,\mathbf{z})$ and $\mathbf{h}_{i(k)}(t,\mathbf{z})$ satisfy the linear growth condition in Assumption 1.

For any $t \in [0, T]$, $\mathbf{z}, \tilde{\mathbf{z}} \in \mathbb{R}^2$, it is able to derive that

$$\begin{aligned} &\|f_1(t,\mathbf{z}) - f_1(t,\tilde{\mathbf{z}})\|_1 + \|g_1(t,\mathbf{z}) - g_1(t,\tilde{\mathbf{z}})\|_1 + \|h_1(t,\mathbf{z}) - h_1(t,\tilde{\mathbf{z}})\|_1 \\ &\quad \leq e^{-\frac{t}{4}} \cdot \|\mathbf{z} - \tilde{\mathbf{z}}\|_1 + \frac{6}{2+t^2} \cdot \|\mathbf{z} - \tilde{\mathbf{z}}\|_1 \leq 4\|\mathbf{z} - \tilde{\mathbf{z}}\|_1, \end{aligned}$$

$$\begin{aligned} &\|f_2(t,\mathbf{z}) - f_2(t,\tilde{\mathbf{z}})\|_1 + \|g_2(t,\mathbf{z}) - g_2(t,\tilde{\mathbf{z}})\|_1 + \|h_2(t,\mathbf{z}) - h_2(t,\tilde{\mathbf{z}})\|_1 \\ &\quad \leq e^{-\frac{t}{6}} \cdot \|\mathbf{z} - \tilde{\mathbf{z}}\|_1 + \frac{6}{1+t^2} \cdot \|\mathbf{z} - \tilde{\mathbf{z}}\|_1 \leq 7\|\mathbf{z} - \tilde{\mathbf{z}}\|_1, \end{aligned}$$

$$\begin{aligned} &\|f_3(t,\mathbf{z}) - f_3(t,\tilde{\mathbf{z}})\|_1 + \|g_3(t,\mathbf{z}) - g_3(t,\tilde{\mathbf{z}})\|_1 + \|h_3(t,\mathbf{z}) - h_3(t,\tilde{\mathbf{z}})\|_1 \\ &\quad \leq e^{-\frac{t}{3}} \cdot \|\mathbf{z} - \tilde{\mathbf{z}}\|_1 + \frac{2}{1+t^2} \cdot \|\mathbf{z} - \tilde{\mathbf{z}}\|_1 \leq 3\|\mathbf{z} - \tilde{\mathbf{z}}\|_1, \end{aligned}$$

$$\|f_4(t,\mathbf{z}) - f_4(t,\tilde{\mathbf{z}})\|_1 + \|g_4(t,\mathbf{z}) - g_4(t,\tilde{\mathbf{z}})\|_1 + \|h_4(t,\mathbf{z}) - h_4(t,\tilde{\mathbf{z}})\|_1$$

$$\leq e^{-\frac{t}{5}} \cdot \|\mathbf{z} - \tilde{\mathbf{z}}\|_1 + \frac{4}{1+t^2} \cdot \|\mathbf{z} - \tilde{\mathbf{z}}\|_1 \leq 5\|\mathbf{z} - \tilde{\mathbf{z}}\|_1.$$

That is to say, for every $i(k) \in \{1, 2, 3, 4\}$*, the coefficient functions* $\mathbf{f}_{i(k)}(t, \mathbf{z})$*,* $\mathbf{g}_{i(k)}(t, \mathbf{z})$ *and* $\mathbf{h}_{i(k)}(t, \mathbf{z})$ *satisfy the Lipschitz condition in Assumption 2.*

In short, by using Theorem 3.2, it is able to conclude that system (6) *has a unique solution on the interval* $[0, T]$*, the (b) of Figure 1 also illustrates the existence and uniqueness of the solution.*

5. Conclusions

In this paper, a class of uncertain nonlinear switched systems characterized by V-n jumps was explored, modeled by a set of uncertain differential equations that incorporated V-n jumps. Utilizing V-n jumps process and the Banach fixed point theorem, an existence and uniqueness theorem for the solutions was established and proved on a small interval $[t, t + c]$, assuming that the coefficient functions in each subsystem satisfied the Lipschitz condition and the linear growth condition. This result was then extended to the broader interval $[0, T]$. Finally, an example was provided to validate the the existence and uniqueness.

Acknowledgements

The authors are supported partially by Suqian Sci & Tech Program (Grant No. K202332) and Startup Foundation for Newly Recruited Employees of Suqian University(Grant No. 2024XRC004).

REFERENCES

[1] S. Santra, O. Bazighifan, M. Postolache, New conditions for the oscillation of second-order differential equations with sublinear neutral terms. Mathematics, **9(11)**(2021), 1-9.

[2] T. Nofal, O. Bazighifan, K.M. Kheder, M. Postolache, More effective conditions for oscillatory properties of differential equations. Symmetry-Basel, **13(2)**(2021),1-11.

[3] O. Bazighifan, M. Postolache, Improved conditions for oscillation of functional nonlinear differential equations. Mathematics, **8(4)**(2020),1-10.

[4] O. Bazighifan, M. Postolache, Multiple techniques for studying asymptotic properties of a class of differential equations with variable coefficients. Symmetry-Basel, **12(7)**(2020), 1-11.

[5] D. Liberzon, A. S. Morse, Basic problem in stability and design of switched systems. IEEE Control Systems Magazine, **19** (2001), 59-70.

[6] J. L. Aguilar, R. A. Garca, Some results on the stabilization of switched systems. Automatica, **49(2)** (2013), 441-447.

[7] M. S. Branicky, Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. IEEE Transactions on Automatic Control, **43(4)** (1998), 475-482.

[8] K. Itô, On stochastic differential equations, Memoirs of The American Mathematical Society, **4** (1951), 1-51.

[9] D. Chatterjee, and D. Liberzon, On stability of stochastic switched systems. In Proceedings of IEEE conference on decision and control, **4** (2005), 4125-4127.

[10] V. Filipovic, Exponential stability of stochastic switched systems. Transactions of the Institute of Measurement and Control, **31(2)** (2009), 205-212.

[11] M. S. Alwan, and X. Liu, Input-to-state stability for stochastic switched systems. In Nonlinear physical science, Theory of hybrid systems: Deterministic and stochastic. Singapore: Springer, (2018), 117-134.

[12] B. Liu, Uncertainty Theory, 2nd ed., Springer-Verlag,Berlin, 2007.

[13] B. Liu, Uncertainty theory, 4th ed., Springer, Berlin, 2015.

[14] C. Li, Z. Jia, M. Postolache, New convergence methods for nonlinear uncertain variational inequality problems. J. Nonlinear Convex Anal, **19**(2018), 2153-2164.

[15] C. Li, M. Postolache, Z. Jia, Weighted method for uncertain nonlinear variational inequality problems. Mathematics, **7** (2019), 974-996.

[16] Z. Jia, X. Liu, Y. Zhang, New stability theorem for uncertain pantograph differential equations. Journal of Intelligent and Fuzzy Systems, **40(5)** (2021), 9403-9411.

[17] Z. Jia, X. Liu, New sufficient conditions for p-th moment stability of uncertain delay differential equation. Journal of Intelligent and Fuzzy Systems, **43(1)** (2022), 451-461.

[18] Z. Jia, X. Liu, Uncertain stochastic hybrid differential game system with V-n jumps:Saddle point equilibrium, strategies and application to advertising duopoly game. Chaos Soliton and Fractals, **171** (2023), 113490.

[19] Z. Jia, X. Liu, Optimistic value models of saddle point equilibrium control problems for uncertain jump systems. Optimization, (2024), 1-35.

[20] K. Yao, J. Gao, Y. Gao, Some stability theorems of uncertain differential equation. Fuzzy Optimization and Decision Making, **12(1)** (2013), 3-13.

[21] T. Su, H. Wu, J. Zhou, Stability of multi-dimensional uncertain differential equation. Soft Computing, **20(12)** (2016), 4991-4998.

[22] G. Shi, X. Li, L. Jia, Stability in p-th moment of multi-dimensional uncertain differential equation. Journal of Intelligent and Fuzzy Systems, **38** (2020), 5267-5277.

[23] Y. Sheng, G. Shi, Stability in mean of multi-dimensional uncertain differential equation. Applied Mathematics and Computation, **353** (2019), 171-188.

[24] R. Gao, Almost Sure Stability for multi-dimensional uncertain differential equations. Mathematics, **10(19)** (2022), 1-10.

[25] Y. Shu, B. Li, Stability analysis for uncertain nonlinear switched systems with infinite-time domain. Fuzzy Optimization and Decision Making, **21** (2022), 405-428.

[26] Z. Jia, C. Li, Stability in p-th moment for uncertain nonlinear switched systems with infinite-time domain. University Politehnica of Bucharest Scientific Bulletin-Series A-Applied Mathematics and Physics, **86(2)** (2024), 97-108.

[27] X. Chen, B. Liu, Existence and uniqueness theorem for uncertain differential equations. Fuzzy Optimization and Decision Making, **2(9)** (2010), 69-81.

[28] L. Deng, Y. Zhu, Existence and uniqueness theorem of solution for uncertain differential equations with jump. ICIC Express Letters, **6** (2012), 2693-2698.

[29] Y. Zhu, Existence and uniqueness of the solution to uncertain fractional differential equation. Journal of Uncertainty Analysis and Applications, **3(1)** (2015), 1-11.

[30] X. Yang, and Y. Ni,. Existence and uniqueness theorem for uncertain heat equation. Journal of Ambient Intelligence and Humanized Computing, **8(1)** (2017), 717-725.

[31] Z. Jia, X. Liu, C. Li, Existence, uniqueness, and stability of uncertain delay differential equations with V-jump, Advances in Difference Equations, **440(1)** (2020), 1-21.

[32] Y. Shu, B. Li, Existence and uniqueness of solutions for uncertain nonlinear switched systems. Automatica, **149** (2023), 110803.

[33] L. Deng, Y. Zhu, An uncertain optimal control model with n jumps and application. Computer Science and Information Systems, **9(4)** (2012), 1453-1468.

[34] Y. Shu, T. Jin, Stability in measure and asymptotic stability of uncertain nonlinear switched systems with a practical application. International Journal of Control, **96(11)** (2023), 2917-2927.

[35] Y. Shu, B. Li, Stability and attractivity analysis of uncertain switched systems under optimistic value criterion. SIAM Journal on Control and Optimization, **60(4)** (2022), 2420-2439.