EFFECT OF A TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY ON A FIXED UNBOUNDED SOLID WITH A CYLINDRICAL CAVITY

Ashraf M. ZENKOUR[1]

This article investigates the thermoelastic interactions in an orthotropic unbounded solid containing a cylindrical cavity with variable thermal conductivity. A generalized solution is developed in the context of the one relaxation time thermoelasticity theory. The surface of the cylinder is constrained and subjected to a harmonically varying heat. The governing equations are treated to be timeless dependence by using the Laplace transform. Finally, the transformed equations are inverted by the numerical inversion of the Laplace transform. A numerical example has been calculated to illustrate the effects of the variability thermal conductivity parameter and the angular frequency of the thermal vibration on all fields.

Keywords: Thermoelasticity, Constrained cylindrical hole, Orthotropic medium, Variable thermal conductivity, Harmonically varying heat.

MSC2010: 74Dxx, 74Fxx, 74Kxx.

1. Introduction

The classical coupled theory of thermoelasticity is insufficient to deal with thermoelasticity problems. One part of its solution to the heat equation is extended to infinity. This matter contrary to the physical phenomenon since that a part of mechanical or thermal disturbance should include an infinite velocity of propagation. This paradox may be treated after using one of the generalized thermoelasticity theories [1–3]. Lord and Shulman [1] and Green and Lindsay [2] introduced new theories of generalized thermoelasticity that predict a finite speed for heat propagation. Tzou [3] formulated a new generalized thermoelasticity theory called dual-phase-lag (DPL) heat conduction model.

Most investigations in thermoelasticity are based on the assumption of the temperature-independent material properties [4–12]. The applicability of the solutions of such problems is limited to certain ranges of temperature. Generally, the thermal conductivity should be temperature-dependent at high temperature, which definitely alters the thermoelastic behaviors. The effect of temperature-dependent thermal conductivity is investigated by many authors [13–22]. Most of

[1] Prof., Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; Department of Mathematics, Faculty of Science, Kafrelsheikh University, Kafr El-Sheikh 33516, Egypt. e-mail: zenkour@kau.edu.sa, zenkour@sci.kfs.edu.eg

these authors are taken into account the variable thermal conductivity for problem concerned with generalized thermoelastic solids subjected to various types of heating sources.

The aim of the present paper is to investigate the thermoelastic interactions in an orthotropic unbounded body containing a cylindrical cavity with variable thermal conductivity. The surface of the present cylinder is constrained and subjected to a time-dependent thermal shock. The problem is solved in the context of generalized thermoelasticity with one relaxation time, developed by Lord and Shulman [1]. A direct approach of the Laplace transform is used to obtain the solution of the present problem in the Laplace domain. In addition, a numerical technique is employed to obtain the solution in the physical domain. The effect of the angular frequency of thermal vibration and the variability of thermal conductivity parameters is investigated graphically and discussed.

2. Formulation of the problem

The unbounded orthotropic body with cylindrical cavity and constrained surface is considered to be subjected to a harmonically varying heat. The cylindrical coordinates system (r, θ, z) with z-axis as the axial axis of the cylinder. The present problem is considered as a 1D problem due to symmetry and all the functions are depending on the radial distance r and the time t.

For axially symmetric problem, the radial, hoop, and axial displacement components are reduced to be

$$u_r = u(r,t), \quad u_\theta(r,t) = u_z(r,t) = 0, \tag{1}$$

with radial ε_r and hoop ε_θ strains given by

$$\varepsilon_r = \frac{\partial u(r,t)}{\partial r}, \quad \varepsilon_\theta = \frac{u(r,t)}{r}. \tag{2}$$

So, the stress-displacement relations may be written as

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \\ c_{13} & c_{23} \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial r} \\ \frac{u}{r} \end{Bmatrix} - \begin{Bmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{33} \end{Bmatrix} \Theta, \tag{3}$$

where σ_r, σ_θ, and σ_z are the radial, hoop, and axial stress components, respectively, c_{ij} are the isothermal elastic constants, β_{ij} are the thermal elastic coupling components, and $\Theta = T - T_0$ is the dynamical temperature increment of the resonator, in which T_0 is the environmental temperature. The dynamic equation of motion of the cylindrical cavity, without considering the body forces or the heat sources acting in the medium, is expressed as

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{4}$$

where ρ is the material density of the medium. From Eq. (3), the above equation of motion will be in the form

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-c_{22}\frac{u}{r^2}=\rho\frac{\partial^2 u}{\partial t^2}+\beta_{11}\frac{\partial \Theta}{\partial r}+(\beta_{11}-\beta_{22})\frac{\Theta}{r}. \qquad (5)$$

Here, the generalized heat conduction equation is given by [1]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rK_r\frac{\partial \Theta}{\partial r}\right)=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[\rho C_E\frac{\partial \Theta}{\partial t}+T_0\frac{\partial}{\partial t}\left(\beta_{11}\frac{\partial u}{\partial r}+\beta_{22}\frac{u}{r}\right)\right], \qquad (6)$$

where K_r is the thermal conductivity, C_E is the specific heat per unit mass at constant strain, and τ_0 is the thermal relaxation time parameter. The above heat equation is given according to the generalized dynamical theory of thermoelasticity of Lord and Shulman [1] that eliminate the paradox of the classical coupled theory of thermoelasticity.

3. Temperature-dependent thermal conductivity

Generally, the assumption that the solid body is thermosensitivity (the thermal properties of the material vary with temperature) leads to a nonlinear heat conduction problem. The exact solution of such problem can be found by assuming the thermal conductivity K_r and the specific heat C_E to be linearly-depending on the temperature [23], but thermal diffusivity is assumed be constant. That is

$$K_r=K_r(\Theta)=k_0(1+k_1\Theta), \qquad (7)$$

where k_0 is the thermal conductivity at ambient temperature T_0 and k_1 is the slope of the thermal conductivity-temperature curve divided by the intercept k_0. Now, we will consider the Kirchhoff transformation [23]

$$\psi=\frac{1}{k_0}\int_0^{\Theta}K_r(\Theta)\mathrm{d}\Theta, \qquad (8)$$

where ψ is a new function expressing the heat conduction. By substituting Eq. (7) in Eq. (8), one gets

$$\psi=\Theta\left(1+\tfrac{1}{2}k_1\Theta\right). \qquad (9)$$

From Eq. (9), it follows that

$$\nabla\psi=\frac{K_r(\Theta)}{k_0}\nabla\Theta, \quad \frac{\partial\psi}{\partial t}=\frac{K_r(\Theta)}{k_0}\frac{\partial\Theta}{\partial t}. \qquad (10)$$

After substituting Eq. (10) into Eq. (6), the new form of the general heat equation of solids with temperature-dependent thermal conductivity is obtained by

$$\nabla^2\psi=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[\frac{1}{k}\frac{\partial\psi}{\partial t}+\frac{T_0}{k_0}\frac{\partial}{\partial t}\left(\beta_{11}\frac{\partial u}{\partial r}+\beta_{22}\frac{u}{r}\right)\right], \qquad (11)$$

where $k=K_r/\rho C_E$ is thermal diffusivity and

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}. \qquad (12)$$

Then, the equation of motion, Eq. (5), will be in the form

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-c_{22}\frac{u}{r^2}=\rho\frac{\partial^2 u}{\partial t^2}+\frac{\beta_{11}}{1+2k_1\Theta}\frac{\partial\psi}{\partial r}$$
$$+\frac{\beta_{11}-\beta_{22}}{k_1 r}\left[\sqrt{1+2k_1\psi}-1\right], \qquad (13)$$

or in an expanding form

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-c_{22}\frac{u}{r^2}=\rho\frac{\partial^2 u}{\partial t^2}+\beta_{11}\frac{\partial\psi}{\partial r}[1-(2k_1\Theta)+(2k_1\Theta)^2$$
$$-\cdots]+\frac{\beta_{11}-\beta_{22}}{k_1 r}\left[1+\tfrac{1}{2}(2k_1\psi)-\tfrac{1}{8}(2k_1\psi)^2+\cdots-1\right]. \quad (14)$$

Now, it is assumed that the temperature change $\Theta=T-T_0$ accompanying the deformation is small and does not result in significant variations of the elastic and thermal coefficients. So, one can consider these coefficient be regarded as independent of T. In addition to the assumption $|\Theta/T_0|\ll 1$ one can assume that second powers and products of the components of strain may be neglected in comparison with the strains themselves. Thus, the usual linear theory of thermoelasticity is obtained by considering the case where only terms linear in strain and temperature change. Then, Eqs. (14) and (3) take the forms

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-c_{22}\frac{u}{r^2}=\rho\frac{\partial^2 u}{\partial t^2}+\beta_{11}\frac{\partial\psi}{\partial r}+(\beta_{11}-\beta_{22})\frac{\psi}{r}, \quad (15)$$

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\end{Bmatrix}=\begin{bmatrix}c_{11} & c_{12}\\ c_{12} & c_{22}\\ c_{13} & c_{23}\end{bmatrix}\begin{Bmatrix}\frac{\partial u}{\partial r}\\ \frac{u}{r}\end{Bmatrix}-\begin{Bmatrix}\beta_{11}\\ \beta_{22}\\ \beta_{33}\end{Bmatrix}\psi. \quad (16)$$

In what follows, the following dimensionless variables will be used

$$\{r',u',R'\}=\frac{c_0}{k}\{r,u,R\},\quad \{t',\tau_0'\}=\frac{c_0^2}{k}\{t,\tau_0\},$$
$$\psi'=\frac{\psi}{T_0},\quad \sigma_j'=\frac{\sigma_j}{c_{11}},\quad k_1'=T_0k_1,\quad c_0^2=\frac{c_{11}}{\rho},\quad (j=r,\theta,z). \quad (17)$$

Therefore, the governing equations take the following forms after dropping the primes for convenience,

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-c_2\frac{u}{r^2}=\frac{\partial^2 u}{\partial t^2}+\varepsilon_1\frac{\partial\psi}{\partial r}+\varepsilon_6\frac{\psi}{r}, \quad (18)$$

$$\nabla^2\psi=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[\frac{\partial\psi}{\partial t}+\frac{\partial}{\partial t}\left(\varepsilon_4\frac{\partial u}{\partial r}+\varepsilon_5\frac{u}{r}\right)\right], \quad (19)$$

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\end{Bmatrix}=\begin{bmatrix}1 & c_1\\ c_1 & c_2\\ c_3 & c_4\end{bmatrix}\begin{Bmatrix}\frac{\partial u}{\partial r}\\ \frac{u}{r}\end{Bmatrix}-\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{Bmatrix}\psi, \quad (20)$$

where

$$\{\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_6\}=\frac{T_0}{c_{11}}\{\beta_{11},\beta_{22},\beta_{33},\beta_{11}-\beta_{22}\},$$
$$\{\varepsilon_4,\varepsilon_5\}=\frac{1}{\rho C_E}\{\beta_{11},\beta_{22}\},\{c_1,c_2,c_3,c_4\}=\frac{1}{c_{11}}\{c_{12},c_{22},c_{13},c_{23}\}. \quad (21)$$

4. Initial and boundary conditions

Both the initial and boundary conditions of the problem should be considered. The initial conditions are assumed to be in the form

$$u(r,0)=\left.\frac{\partial u(r,t)}{\partial t}\right|_{t=0}=0,\quad \psi(r,0)=\left.\frac{\partial\psi(r,t)}{\partial t}\right|_{t=0}=0. \quad (22)$$

The following boundary conditions hold since the boundary of the cylinder is constrained and subjected to a to harmonically varying heat

- The surface of the cylinder $\boldsymbol{r} = \boldsymbol{R}$ is subjected to a harmonically varying heat

$$\Theta(R,t) = \Theta_0\, cos(\omega t)\,, \quad \omega > 0, \tag{23}$$

where ω is the angular frequency of the thermal vibration and Θ_0 is a constant. Using Eq. (9), then one gets

$$\psi(R,t) = \Theta_0\, cos(\omega t) + \tfrac{1}{2} k_1 [\Theta_0\, cos(\omega t)]^2. \tag{24}$$

It is to be noted that $\omega = 0$ for the thermal shock problem.

- The mechanical boundary condition is due to the displacement of the surface is constrained. That is

$$u(R,t) = 0. \tag{25}$$

5. Solution of the problem in the Laplace transform domain

Using the Laplace transform of Eqs. (18)-(20) and taking into account the initial conditions given in Eq. (21) and assuming that $\beta_{11} = \beta_{22}$ (i.e., $\varepsilon_4 = \varepsilon_5 = \varepsilon$) and $c_{11} = c_{22}$ (i.e., $c_2 = 1$), we obtain the following equations:

$$\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\bar{u}}{\mathrm{d}r} - \frac{\bar{u}}{r^2} - s^2\bar{u} = \varepsilon_1 \frac{\mathrm{d}\bar{\psi}}{\mathrm{d}r}, \tag{26}$$

$$\frac{\mathrm{d}^2\bar{\psi}}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\bar{\psi}}{\mathrm{d}r} = s(1+\tau_0 s)\left[\bar{\psi} + \varepsilon\left(\frac{\mathrm{d}\bar{u}}{\mathrm{d}r} + \frac{\bar{u}}{r}\right)\right], \tag{27}$$

$$\begin{Bmatrix} \bar{\sigma}_r \\ \bar{\sigma}_\theta \\ \bar{\sigma}_z \end{Bmatrix} = \begin{bmatrix} 1 & c_1 \\ c_1 & 1 \\ c_3 & c_4 \end{bmatrix} \begin{Bmatrix} \frac{\mathrm{d}\bar{u}}{\mathrm{d}r} \\ \frac{\bar{u}}{r} \end{Bmatrix} - \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_1 \\ \varepsilon_3 \end{Bmatrix} \bar{\psi}. \tag{28}$$

Here, any variable with an over bar denotes the Laplace transform of this variable and s denotes the transform parameter. Equations (26) and (27) can be written in the forms

$$(DD_1 - s^2)\bar{u} = \varepsilon_1 D\bar{\psi}, \tag{29}$$

$$\varepsilon q D_1 \bar{u} = (D_1 D - q)\bar{\psi}, \tag{30}$$

where

$$D = \frac{\mathrm{d}}{\mathrm{d}r}, \quad D_1 = \frac{\mathrm{d}}{\mathrm{d}r} + \frac{1}{r}, \quad q = s(1+\tau_0 s). \tag{31}$$

Now, let us define the radial displacement $\bar{u}$ in terms of the thermoelastic potential function $\bar{\varphi}$ by the relation

$$\bar{u} = \frac{\mathrm{d}\bar{\varphi}}{\mathrm{d}r}, \tag{32}$$

then, one can rewrite Eqs. (29) and (30) as

$$(D_1 D - s^2)\bar{\varphi} = \varepsilon_1 \bar{\psi}, \tag{33}$$

$$\varepsilon q D_1 D \bar{\varphi} = (D_1 D - q)\bar{\psi}. \tag{34}$$

Eliminating $\bar{\psi}$ from Eqs. (33) and (34), one gets

$$\{\nabla^4 - [s^2 + q(1+\varepsilon\varepsilon_1)]\nabla^2 + qs^2\}\bar{\varphi} = 0. \tag{35}$$

The above equation leads to the modified Bessel equation for $\bar{\varphi}$ of zero order

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}-m_1^2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}-m_2^2\right)\bar{\varphi}=0, \tag{36}$$

where m_1^2 and m_2^2 are the roots

$$m_1^2=\tfrac{1}{2}\left(2A+\sqrt{A^2-4B}\right),\quad m_2^2=\tfrac{1}{2}\left(2A-\sqrt{A^2-4B}\right), \tag{37}$$

in which

$$A=s^2+q(1+\varepsilon\varepsilon_1),\quad B=qs^2. \tag{38}$$

The solution of Eq. (36) under the regularity conditions that u, Θ, $\psi\to 0$ as $r\to\infty$ can be written as

$$\bar{\varphi}=\sum_{i=1}^{2}A_iK_0(m_ir), \tag{39}$$

where $K_0(\cdot)$ is the modified Bessel's function of the first kind of order zero and A_i, $i=1,2$ are two parameters depending on s of the Laplace transform. Using Eqs. (33) and (39), we obtain

$$\bar{\psi}=\frac{1}{\varepsilon_1}\sum_{i=1}^{2}(m_i^2-s^2)A_iK_0(m_ir). \tag{40}$$

Substituting from Eq. (41) into the Laplace transform of Eq. (32), we obtain

$$\bar{u}=-\sum_{i=1}^{2}A_iK_1(m_ir). \tag{41}$$

where $K_1(\cdot)$ is the modified Bessel function of the first kind of order one. So, the stresses can be written as

$$\bar{\sigma}_r=\sum_{i=1}^{2}\left[s^2K_0(m_ir)+\frac{m_i(1-c_1)}{r}K_1(m_ir)\right]A_i, \tag{42}$$

$$\bar{\sigma}_\theta=\sum_{i=1}^{2}\left\{[s^2+m_i^2(c_1-1)]K_0(m_ir)+\frac{m_i(c_1-1)}{r}K_1(m_ir)\right\}A_i, \tag{43}$$

$$\bar{\sigma}_z=\sum_{i=1}^{2}\left\{\left[\frac{m_i^2c_3}{2}-\frac{\varepsilon_3}{\varepsilon_1}(m_i^2-s^2)\right]K_0(m_ir)-\frac{m_ic_4}{r}K_1(m_ir)+\frac{m_i^2c_3}{2}K_2(m_ir)\right\}A_i. \tag{44}$$

where $K_2(\cdot)$ is the modified Bessel function of the first kind of order two. In addition, the boundary conditions given in Eqs. (24) and (25), after using Laplace transform, take the forms

$$\bar{\psi}(R,s)=\Theta_0\left[\frac{s}{s^2+\omega^2}+\frac{k_1(s^2+2\omega^2)}{2s(s^2+4\omega^2)}\right]=\bar{G}(s), \tag{45}$$

$$\bar{u}(R,s)=0. \tag{46}$$

Substituting Eqs. (40) and (41) into the above boundary conditions, one obtains two equations in the unknown parameters A_i, as

$$\sum_{i=1}^{2}(m_i^2-s^2)A_iK_0(m_iR)=\varepsilon_1\bar{G}(s), \tag{47}$$

$$\sum_{i=1}^{2}m_iA_iK_1(m_iR)=0. \tag{48}$$

The solution of the problem will be completed in the Laplace transform domain after obtaining the two constants A_1 and A_2. Solving the above two equations, one gets

$$\begin{aligned}A_1&=\frac{\varepsilon_1\bar{G}(s)m_2K_1(m_2R)}{m_2(m_1^2-s^2)K_0(m_1R)K_1(m_2R)-m_1(m_2^2-s^2)K_0(m_2R)K_1(m_1R)},\\ A_2&=\frac{\varepsilon_1\bar{G}(s)m_1K_1(m_1R)}{m_1(m_2^2-s^2)K_0(m_2R)K_1(m_1R)-m_2(m_1^2-s^2)K_0(m_1R)K_1(m_2R)}.\end{aligned} \tag{49}$$

Hence, one can easily obtain the displacement and stresses as well as other physical quantities of the medium. The temperature $\bar{\Theta}$ can be obtained by solving Eq. (9) after applying the Laplace transform as

$$\bar{\Theta}(r,s) = \frac{-1+\sqrt{1+2k_1\bar{\psi}}}{k_1}. \tag{50}$$

6. Numerical results and discussion

In this section, the temperature Θ, radial displacement u, and stresses σ_r, σ_θ and σ_z distributions will be obtained inside the medium in their inverted forms. To invert the Laplace transform in Eqs. (39)-(44), a numerical inversion method based on a Fourier series expansion [24] shouuld be adopted. Any function in Laplace domain can be inverted in this method to the time domain as

$$f(t) = \frac{e^{ct}}{t}\left\{\frac{1}{2}\bar{f}(c) + \mathrm{Re}\left[\sum_{n=1}^{N}(-1)^n \bar{f}\left(c+\frac{in\pi}{t}\right)\right]\right\}. \tag{53}$$

In most numerical experiments, the reliable value of c should satisfies the relation $ct \approx 4.7$ [25]. So, the numerical calculations are faster convergence for the same value of c.

Numerical evaluations are made by choosing an orthotropic material such as the *cobalt*. The properties of such material are thus given in SI units [26] as

$$\begin{gathered} c_{11} = c_{22} = 3.071\times10^{11}\ (\mathrm{N/m}), \\ c_{12} = 1.650\times10^{11}\ (\mathrm{N/m}), \\ \beta_{11} = \beta_{22} = 7.04\times10^{6}\ (\mathrm{N/m^2K}), \\ \beta_{33} = 6.90\times10^{6}\ (\mathrm{N/m^2K}), \\ C_E = 427\ (\mathrm{J/kg\,K}),\ \ K_r = 69\ (\mathrm{W/m\,K\,s}), \\ \rho = 8836\ (\mathrm{kg/m^3}). \end{gathered} \tag{54}$$

The cylindrical cavity of radius $R = 1$ with its center at the origin is considered. The results are illustrated graphically in Figures 1–5 for different values of R, ($R \geq 1$). It is assumed in all cases studied, except otherwise stated, that $T_0 = 298$ K, $k_1 = -0.5$, $\omega = 5$, and $t = 0.07$. The variations of the field quantities along the radial direction are plotted in Figures 1–5 for various parameters: (a) thermal conductivity k_1, (b) angular frequency ω, and (c) time t. Figures 1a, 2a, 3a, 4a and 5a represent the first case in which three different values the thermal conductivity parameter k_1 are considered to discuss the effect of thermal conductivity. It is assumed that $k_1 = -1$ and -0.5 for temperature-dependent thermal conductivity while $k_1 = 0$ otherwise. The second case of results is illustrated in Figures 1b, 2b, 3b, 4b and 5b for different values of the angular frequency parameter of thermal vibration ω. For thermal shock problem, we put $\omega = 0$ and for harmonically heat ω is set to be either 5 or 10. In the last case of results, Figures 1c, 2c, 3c, 4c and 5c display the values of the considered physical variables in the direction of wave propagation for different values of dimensionless time t which is taken to be 0.05, 0.07, and 0.1.

Fig. 1 shows that the temperature Θ is increasing as k_1 increases and as both ω and t decrease. The distribution of temperature may be found as a wave type of heat propagation in the medium. The heat wave front moves forward with a finite speed in the medium with the passage of time. The temperature Θ maybe have a local maximum values at the position $r = 1.19$.

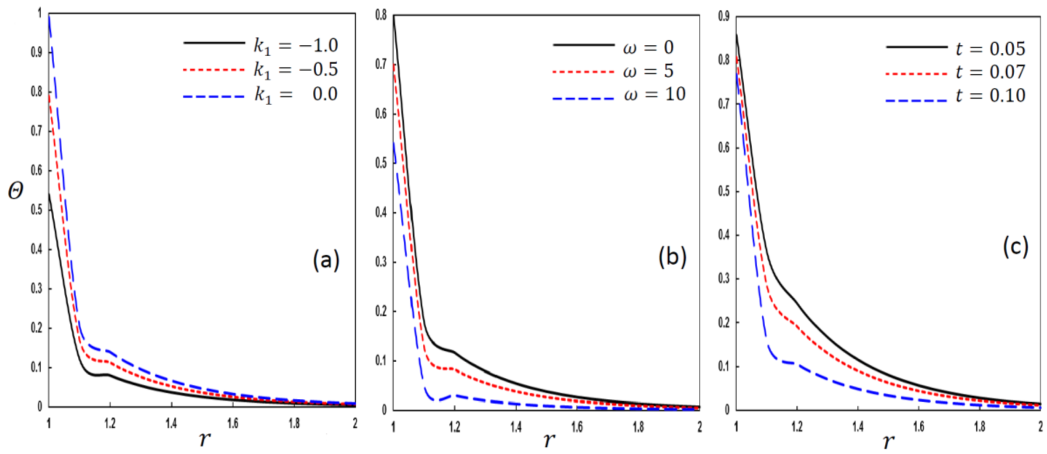

Fig. 1. Variation of temperature Θ along the radial direction for various parameters: (a) thermal conductivity k_1, (b) angular frequency ω, and (c) time t.

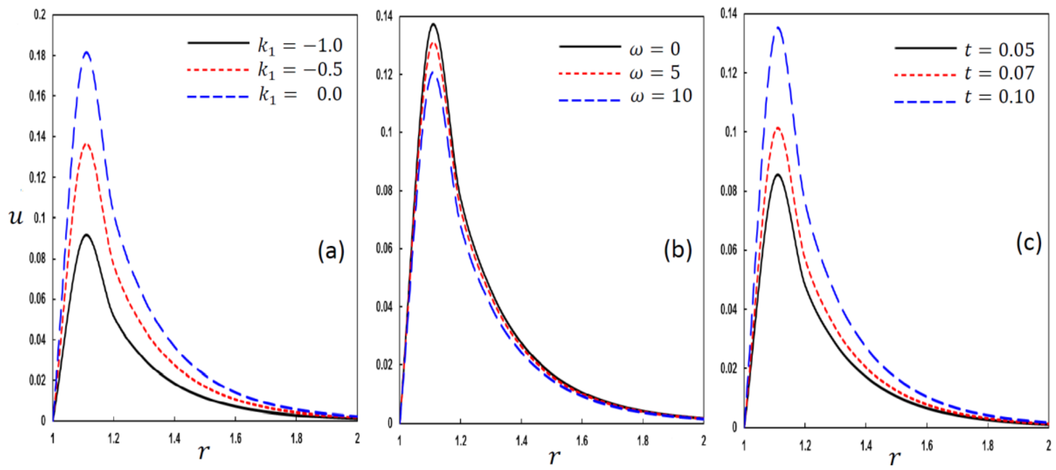

Fig. 2. Variation of radial displacement u along the radial direction for various parameters: (a) thermal conductivity k_1, (b) angular frequency ω, and (c) time t.

Fig. 2 shows that the radial displacement u is increasing as k_1 and t increase and as ω decreases. In Figure 2, u is no longer increasing along the radial direction and has its maximum value at the location $r = 1.11$ and this irrespective to the values of k_1, ω and t.

Fig. 3 shows that the radial stress σ_r is increasing as k_1 decreases and as ω and t increase. The radial stress is increasing through the radial direction according to all cases. It starts with negative values at $r = 1$ and it is continuously increasing to diminish to zero value.

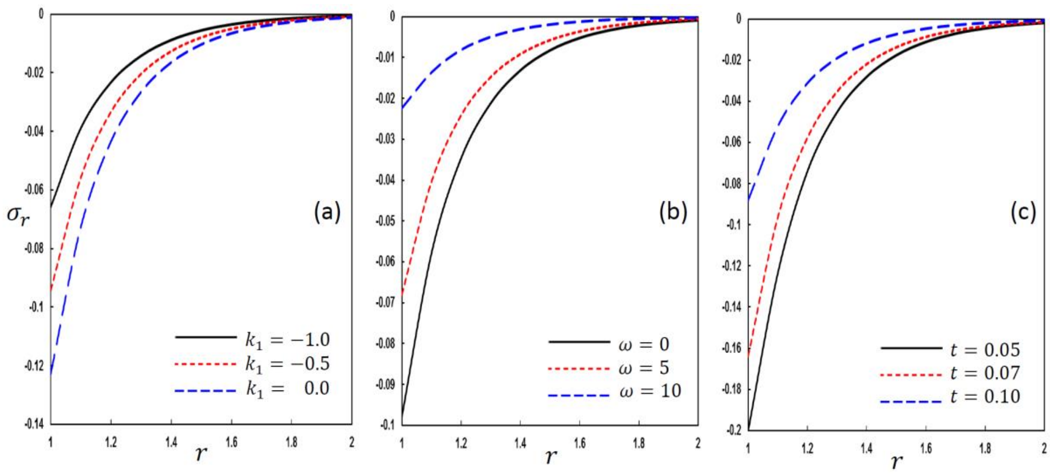

Fig. 3. Variation of radial stress σ_r along the radial direction for various parameters: (a) thermal conductivity k_1, (b) angular frequency ω, and (c) time t.

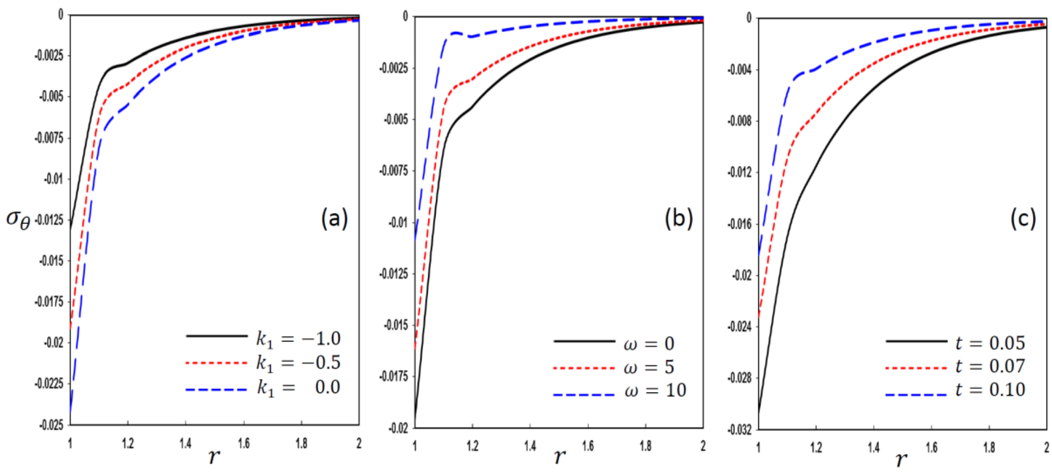

Fig. 4. Variation of hoop stress σ_θ along the radial direction for various parameters: (a) thermal conductivity k_1, (b) angular frequency ω, and (c) time t.

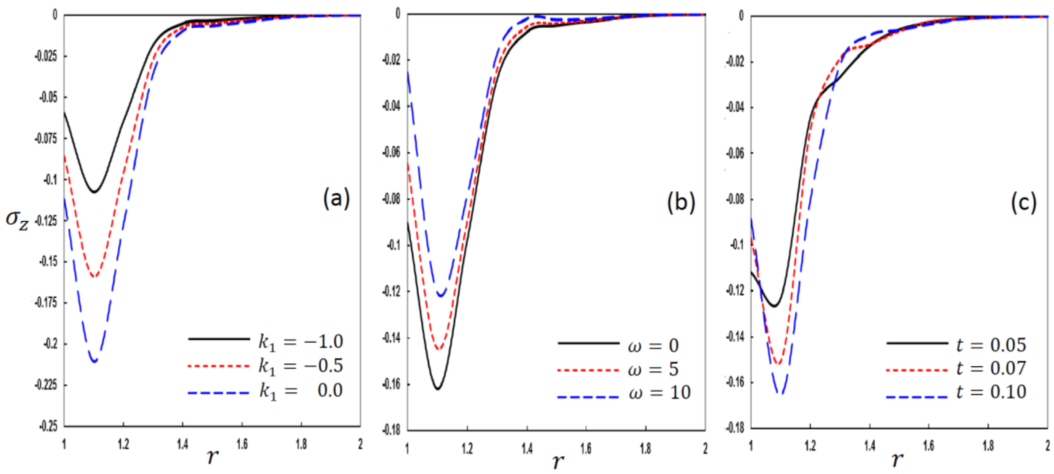

Fig. 5. Variation of axial stress σ_z along the radial direction for various parameters: (a) thermal conductivity k_1, (b) angular frequency ω, and (c) time t.

Fig. 4 shows that the hoop stress σ_θ is increasing as k_1 decreases, and as ω and t increase. The hoop stress is increasing through the radial direction. Also, it starts with negative values at $r = 1$ and it is increasing to diminish to zero value. It maybe have a local maximum value at $r = 1.125$ for some values of k_1, ω and t.

Finally, Figs. 5a and 5b show that the axial stress σ_z is increasing as k_1 decreases ω increases. The axial stress σ_z is no longer decreasing along the radial direction and has its minimum value at the location $r = 1.11$ and this irrespective to the values of k_1 and ω. In Figure 5c, the minimum values the axial stress σ_z are occurring at different positions in the neighborhoods of $r = 1.1$ and this depending on the value of t.

In general, it is to be noted that the variability thermal conductivity parameter k_1 has a significant effect on all the fields which add an importance to our consideration about the thermal conductivity to be variable. The behavior of the three cases of the angular frequency parameter ω is generally quite similar and ω has a significant effect on all fields. The influence of time parameter is very pronounced on all the studied field variables. It is to be noticed that all the variables behave the same manner due to the change in the values of time parameter with some difference in their magnitudes.

7. Conclusions

In this work, we construct the equations of generalized thermoelasticity for a homogeneous orthotropic infinite unbounded body containing a cylindrical cavity with a variable thermal conductivity based on the Lord-Shulman's model.

The outer surface is taken to be fixed and subjected to a time-dependent temperature. The problem has been solved numerically using the Laplace transform technique. Numerical results for radial displacement, temperature, and thermal stresses are illustrated graphically. Comparisons are made between the results predicted by the theory of generalized thermoelasticity with one relaxation time. It is concluded, from the numerical results, that the variability thermal conductivity parameter has significant effects on the speed of the wave propagation of all the studied fields. The thermoelastic temperature, displacement, and stresses have strong dependencies on the angular frequency parameter. The heat propagates as a wave with finite velocity instead of infinite velocity in medium since the generalized thermoelasticity theory with one relaxation time is used. The theory of coupled thermoelasticity can extracted from our model as special case. Finally, the results presented here should prove useful for researchers in scientific and engineering, as well as for those working on the development of mechanics of solids.

R E F E R E N C E S

[1]. *H. W. Lord, Y. Shulman*, "A generalized dynamical theory of thermoelasticity", J. Mech. Phys. Solids, **vol. 15**, no. 5, 1967, pp. 299-309.

[2]. *A. E. Green, K. A. Lindsay*, "Thermoelasticity", J. Elast., **vol. 2**, no. 1, 1972, pp. 1-7.

[3]. *D. Y. Tzou*, "A unified field approach for heat conduction from macro- to micro-scales", J. Heat Transfer, **vol. 117**, no. 1, 1995, pp. 8-16.

[4]. *M. Aouadi*, "Generalized thermo-piezoelectric problems with temperature-dependent properties", Int. J. Solids Struct., **vol. 43**, no. 21, 2006, pp. 6347-6358.

[5]. *S. Mukhopadhyay, R. Kumar*, "Solution of a problem of generalized thermoelasticity of an annular cylinder with variable material properties by finite difference method", Comput. Meth. Sci. Tech., **vol. 15**, no. 2, 2009, pp. 169-176.

[6]. *M. N. Allam, K. A. Elsibai and A. E. Abouelregal*, "Magneto-thermoelasticity for an infinite body with a spherical cavity and variable material properties without energy dissipation", Int. J. Solids Struct., **vol. 47**, no. 20, 2010, pp. 2631-2638.

[7]. *A. M. Zenkour, D. S. Mashat and A. E. Abouelregal*, "Generalized thermodiffusion for an unbounded body with a spherical cavity subjected to periodic loading", J. Mech. Sci. Tech., **vol. 26**, no. 3, 2012, pp. 749-757.

[8]. *A. M. Zenkour, D. S. Mashat and A. E. Abouelregal*, "The effect of dual-phase-lag model on reflection of thermoelastic waves in a solid half space with variable material properties", Acta Mech. Solida Sinca, **vol. 26**, no. 6, 2013, pp. 659-670.

[9]. *A. M. Zenkour, A. E. Abouelregal*, "State-space approach for an infinite medium with a spherical cavity based upon two-temperature generalized thermoelasticity theory and fractional heat conduction", Z. angew. Math. Phys. ZAMP, **vol. 65**, no. 1, 2014, pp. 149-164.

[10]. *A. Sur, M. Kanoria*, "Fractional order generalized thermoelastic functionally graded solid with variable material properties", J. Solid Mech., **vol. 6**, no. 1, 2014, pp. 54-69.

[11]. *A. M. Zenkour, A. E. Abouelregal*, "Effect of temperature dependency on constrained orthotropic unbounded body with a cylindrical cavity due to pulse heat flux", J. Therm. Sci. Tech., **vol. 10**, no. 1, 2015, pp. 1-12.

[12]. *A. M. Zenkour*, “Three-dimensional thermal shock plate problem within the framework of different thermoelasticity theories”, Compos. Struct., **132**(2015), 1029-1042.
[13]. *A. M. Zenkour, I. A. Abbas*, “A generalized thermoelasticity problem of an annular cylinder with temperature-dependent density and material properties”, Int. J. Mech. Sci., **vol. 84**, 2014, pp. 54-60.
[14]. *A. M. Zenkour, I. A. Abbas*, “Nonlinear transient thermal stress analysis of temperature-dependent hollow cylinders using a finite element model”, Int. J. Struct. Stab. Dynam., **vol. 14**, no. 6, 2014, 1450025 (17 pages).
[15]. *A. M. Zenkour, A. E. Abouelregal, K. A. Alnefaie, X. Zhang and E.C. Aifantis*, “Nonlocal thermoelasticity theory for thermal-shock nanobeams with temperature-dependent thermal conductivity”, J. Therm. Stresses, **vol. 38**, no. 8, 2015, pp. 1049-1067.
[16]. *A. M. Zenkour, A.E. Abouelregal*, “Nonlocal thermoelastic nanobeam subjected to a sinusoidal pulse heating and temperature-dependent physical properties”, Microsys. Techno., **vol. 21**, no. 8, 2015, pp. 1767-1776.
[17]. *H. M. Youssef, I. A. Abbas*, “Thermal shock problem of generalized thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity”, Comput. Meth. Sci. Tech., **vol. 13**, 2007, pp. 95-100.
[18] *H. M. Youssef, A. A. El-Bary*, “Two-temperature generalized thermoelasticity with variable thermal conductivity”, J. Therm. Stresses, **vol. 33**, no. 3, 2010, pp. 187-201.
[19]. *S. M. Abo-Dahab, I. A. Abbas*, “LS model on thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity”, Appl. Math. Model., **vol. 35**, no. 8, 2011, pp. 3759-3768.
[20]. *H. H. Sherief, A. M. Abd El-Latief*, “Effect of variable thermal conductivity on a half-space under the fractional order theory of thermoelasticity”, Int. J. Mech. Sci., **vol. 74**, 2013, pp. 185-189.
[21]. *A. M. Zenkour, A. E. Abouelregal*, “Nonlocal thermoelastic vibrations for variable thermal conductivity nanobeams due to harmonically varying heat”, J. Vibroeng., **vol. 16**, no. 8, 2014, pp. 3665-3678.
[22]. *A. M. Zenkour, A. E. Abouelregal*, “Non-simple magnetothermoelastic solid cylinder with variable thermal conductivity due to harmonically varying heat”, Earthq. Struct., **vol. 10**, no. 3, 2016, pp. 681-697.
[23]. *N. Noda*, Thermal Stresses in Materials with Temperature-dependent Properties, Thermal Stresses I, R.B. Hetnarski (Editor), North-Holland, Amsterdam, 1986.
[24]. *G. Honig, U. Hirdes*, “A method for the numerical inversion of Laplace transform”, J. Comp. Appl. Math., vol. **10**, no. 1, 1984, pp. 113-132.
[25]. *D. Y. Tzou*, Macro to Micro-scale Heat Transfer: The Lagging Behavior, Taylor and Francis, Washington DC, 1997.
[26]. *J. C. Misra, N. C. Chattopadhyay and S. C. Samanta*, “Study of the thermoelastic interactions in an elastic half space subjected to a ramp-type heating–a state-space approach”, Int. J. Eng. Sci., **vol. 34**, no. 5, 1996, pp. 579-596.