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THE ANALYSIS OF SOME ACCOMMODATION AND DIS-ACCOMMODATION (HYSTERESIS) PROCESSES IN SOME MAGNETIC MATERIALS AND SANDSTONES FROM THE PERSPECTIVE OF PHENOMENOLOGICAL UNIVERSALITY

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Based on recent approaches in terms of Universality classes, this paper re-examines the analysis of the growth-accommodation processes in [1] *regarding photo-relaxation processes of magnetic materials (low complexity). Also the analysis is extended to compression-decompression (hysteresis) processes for rich complexity materials (sandstones). The fitting accuracies of the growth-accommodation processes are compared for three models: original West, U1 (extended Gompertz) and U2 (extended West). The last two models show a considerable better accuracy of the accommodation-stagnation processes fitting.*

Keywords: Phenomenological Universality, Universality Classes**,** growth-accommodation, complexity.

1. Introduction

In the analysis [1] of photo-relaxation processes of some magnetic materials [2], [3] , for fitting of the accommodation processes, it was used the West model [4]:

$$y = y_0\left[1+b(1-\exp(-t/\tau))\right]^{1/b}. \qquad (1)$$

The achieved results, even if quite satisfactory, do not correspond to the complexity of the studied systems due to the low flexibility of the West equation (1), which operates only with two parameters.

This, and the desire to expand the application of the Phenomenological Universality (PUN) [5] to as many growth processes, urged us to study the compatibility of the models of the Universality classes (CUN): U1 (extended Gompertz) and U2 (extended West) with the experimental data. On this occasion, the compatibility study has been extended also to some medium complexity systems (rocks), for which the compression-decompression (hysteresis branches) curves were already experimentally studied.

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2. Phenomenological Approximation

To analyze the evolution of a physical parameter *y*(*t*), we considered as especially useful the Phenomenological Universality (PUN) approach, proposed, developed and used by the research group of Prof. P. P. Delsanto [6] - [9].

It is assumed that the evolution of the system can be described by a nonlinear first-order differential equation:

$$\dot{y} = v(y,t)\cdot y(t)\,, \tag{2}$$

where $\dot{y}$ is the absolute growing rate parameter y . This can be converted into

$$\dot{z} = \frac{\dot{y}}{y} = v(z,t)\,, \tag{3}$$

using the obvious substitution

$$z = \ln(y)\,. \tag{4}$$

In (3), $v(z,t) \equiv \dot{z} = \dfrac{dy}{y\cdot dt}$ is the relative growing rate of the parameter *y*.

The equations (2) and (3) look too general to be used and, therefore, it is necessary to introduce some assumptions on the function $v(z,t)$, independent of the actual studying problem.

We can assume that the "velocity" *v* depends only on *t*, and its first derivative – the "acceleration" - can be developed in power series of *v*:

$$a \equiv \dot{v} = \sum_{n=1}^{\infty} \kappa_n \cdot v^n(z) = \beta\cdot v + \gamma\cdot v^2 + \delta\cdot v^3 + ... \tag{5}$$

Depending on the actual situation, this series will be limited, of course, to a certain number N of terms, thereby generating a corresponding Universality Class (CUN).

A mechanical analogy is visible (which justifies the notations): *z* – the position, $v(z,t) = \dot{z}$ - the speed and $a = \dot{v} = \ddot{z}$ - the acceleration, the equation (5) being obviously a differential equation of motion in a viscous medium. The integration of equations (5) and (3) will provide the real expressions for the evolution (growing) in each Universality Class (CUN).

The first successive values of N (= 0, 1, 2) are leading to the:

a) U0 class: even if N = 0 is not part of the series development (5), this value is very important, accounting for a constant growing relative rate:

$$a = 0,\quad v = v_0 = \text{const, and } y(t) = y_0 \exp(v_0\cdot t). \tag{6}$$

This is the expression of an autocatalytic development (explosive growth or relaxation) of the parameter *y(t)*, where (here and beyond) $v_0 = v(0)$ is the initial value of relative growth rate.

b) U1 class: N = 1, $a = \beta\cdot v$ and:

$$y(t) = y_0 \exp\left(\frac{\nu_0}{\beta}\left(\exp(\beta t) - 1\right)\right), \tag{7}$$

which represents a similar expression to the Gompertz equation [10]:

$$y(t) = \exp\left[-m\left(c^t - 1\right)\right], \text{ with } c > 1.$$

c) U2 class: N = 2, $a = \beta \cdot \nu + \gamma \cdot \nu^2$ and

$$y(t) = y_o\left[1 + \frac{\nu_o \gamma}{\beta}\left(1 - \exp(\beta \cdot t)\right)\right]^{-\frac{1}{\gamma}} \tag{8}$$

corresponding to a generalization of the West law (1): [4], [11], [12].

For the direct fitting of the growth curves, there were used UN functions of type ACS (accommodation - stagnation) from the classes U1 - (7), U2 - (8) and also West - (1).

3. Photo-induced relaxation of the relative magnetic permeability and coercive magnetic field

The curves of the photo-induced relaxation were fitted for the relative magnetic permeability and the coercive magnetic field strength of some spinelic ferrites, studied in the Physics Department of U.P.B. in 1978 – 1981 [13], [14].

For this purpose, there were used the equivalents of the equations (1), (7), (8), as it follows:

a) for the West equation (1):

$$y(t) = y_o\left[1 + \frac{1}{p}\left(1 - \exp\left(-t/\tau\right)\right)\right]^p, \tag{9}$$

b) for the U1 function (7):

$$y(t) = y_o \exp\left[\nu_o \tau\left(1 - \exp(-t/\tau)\right)\right], \tag{10}$$

c) for the U2 function (8):

$$y(t) = y_o\left[1 + \frac{\nu_o \tau}{p}\left(1 - \exp(-t/\tau)\right)\right]^p. \tag{11}$$

3.1. Results for the photo-induced relaxation

It was performed the fitting of the the experimental results obtained during the 1979-1981 interval for photo-induced relaxation of the relative magnetic permeability and magnetic coercive field on some magnetic materials, continuing so the calculations reported by paper [1].

The graphs presented by Figures 1 (for a stabilized laser irradiation), 2 (for a non-stabilized laser) and 3 (for the irradiation of a classical source), as well as the values of the standard errors indicated by Table 1, demonstrate that the studied processes of photo-relaxation of the relative magnetic permeability can be best fitted by the phenomenological universality functions of classes U1 and U2.

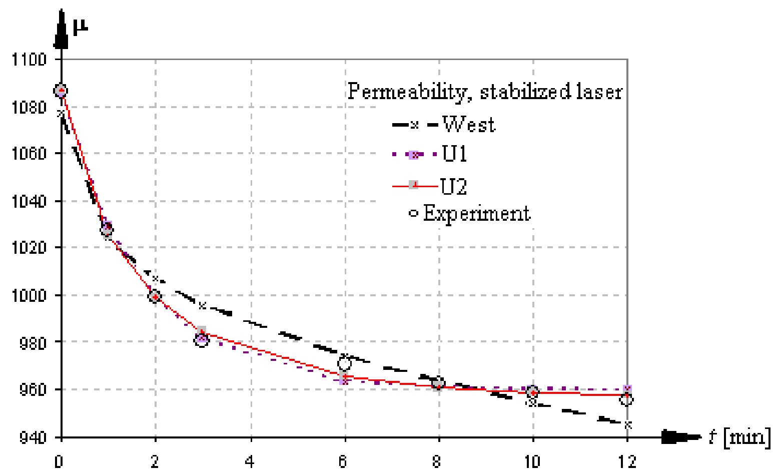

Fig. 1. Graphical fitting results of the photo-relaxation processes of the relative magnetic permeability under the influence of a stabilized laser radiation

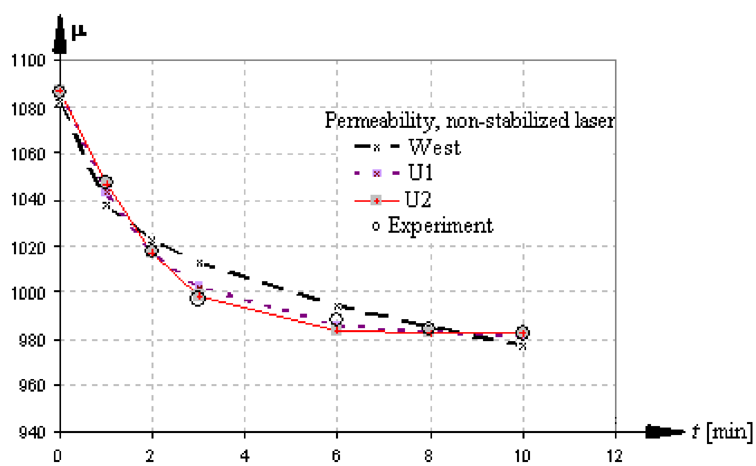

Fig. 2. Graphical fitting results of the photo-relaxation processes of the relative magnetic permeability under the influence of a non-stabilized laser radiation

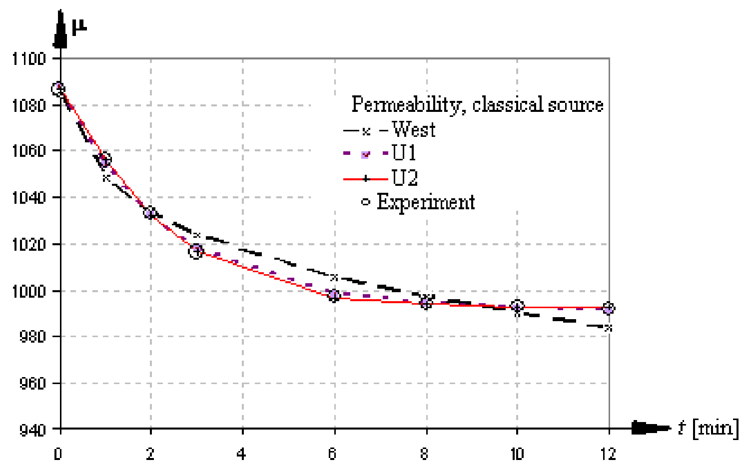

Fig. 3. Graphical fitting results of the photo-relaxation processes of the relative magnetic permeability under the influence of a classical source radiation

Table 1

Values of the fitting parameters for the photo-relaxation of the relative magnetic permeability

Source Type	Phenomenological Universality Model	Characteristic Parameters				Standard deviation (%)
		μ_0	τ [min]	ν_0 [min^{-1}]	p	
Classical	**WEST**	1085.56	-13.160	-	-0.024	0.62%
	U1	1087.61	2.431	-0.038	-	0.14%
	U2	1086.56	1.582	-0.031	+0.067	0.04%
Nonstabilized Laser	**WEST**	1082.15	-8.367	-	-0.022	0.85%
	U1	1087.84	1.930	-0.054	-	0.29%
	U2	1086.64	1.051	-0.040	+0.049	0.23%
Stabilized Laser	**WEST**	1076.94	-5.862	-	-0.023	0.89%
	U1	1085.65	1.764	-0.070	-	0.35%
	U2	1085.35	1.578	-0.066	+0.377	0.37%

The same conclusions (as those obtained from Figures 1-3 and Table 1) result from Figures 4 (for a stabilized laser irradiation), 5 (for a non-stabilized laser) and 6 (for the irradiation of a classical source), as well as from the values of the standard deviation presented by Table 2, for the photo-relaxation of the coercive magnetic field strength.

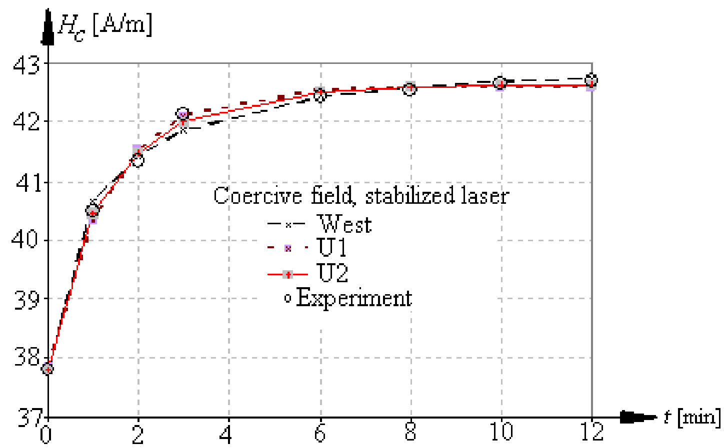

Fig. 4. Graphical fitting results of the photo-relaxation processes of
the coercive magnetic field strength under the influence of a stabilized laser radiation

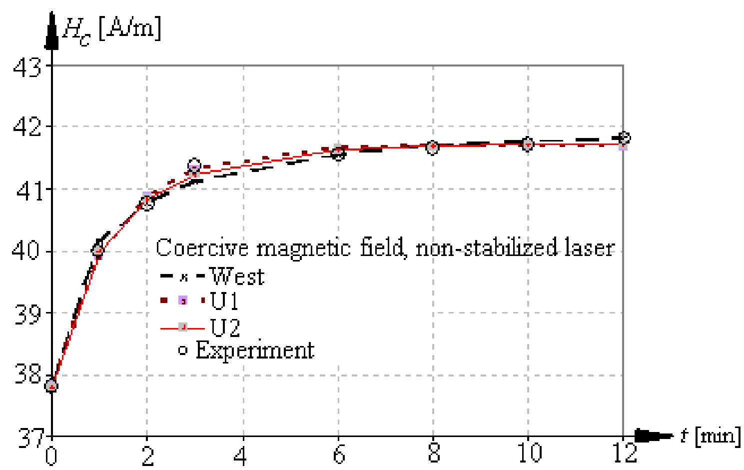

Fig. 5. Graphical fitting results of the photo-relaxation processes of
the coercive magnetic field strength under the influence of a non-stabilized laser radiation

The examination of figure 6 points out a somewhat similar behavior (photo-relaxation) at the photo-irradiation by means of a classical light source of the coercive magnetic field strength of some ferri-magnetic spinelic materials.

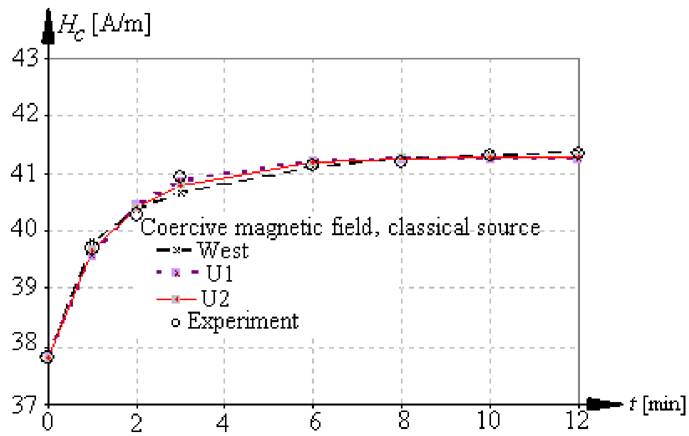

Fig. 6. Graphical fitting results of the photo-relaxation processes of the coercive magnetic field strength under the irradiation of a classical light source

Table 2

Values of the fitting parameters for the photo-relaxation of the coercive magnetic field strength

Source Type	Phenomenological Universality Model	Characteristic Parameters				Standard deviation (%)
		H_{co} [A/m]	τ [min]	ν_0 [min^{-1}]	p	
Classical	WEST	37.77	5.007	-	0.025	0.30%
	U1	37.83	1.351	0.064	-	0.23%
	U2	37.80	1.913	0.085	0.079	0.20%
Nonstabilized Laser	WEST	37.77	4.195	-	0.029	0.28%
	U1	37.82	1.253	0.078	-	0.21%
	U2	37.80	1.777	0.104	0.086	0.17%
Stabilized Laser	WEST	37.76	3.918	-	0.038	0.31%
	U1	37.84	1.291	0.092	-	0.28%
	U2	37.80	1.897	0.127	0.096	0.21%

The high accuracy of the obtained fittings is pointed out by the practical coincidence of all 9 values of the fitting plots crossings with the coercive magnetic field (strength) H_c (see the first numerical column of Table 2).

4. Analysis of the UN fitting for mechanical hysteresis processes for some types of sandstones

The used experimental data concerning the mechanical hysteresis processes of some sandstones are those established by the works [15] (Castlegate), [16] (Berea 1) and [17] (Berea 2).

For this purpose were used the equivalents of the equations (1), (7), (8), as follows:

a) for the West equation (1):

$$\varepsilon(\sigma) = \varepsilon_o \left[1 + \frac{1}{p}\left(1 - \exp\left(-\sigma / \sigma_{rel}\right)\right)\right]^p \tag{12}$$

b) for the U1 function (7):

$$\varepsilon(\sigma) = \varepsilon_o \exp\left[\nu_o \sigma_{rel}\left(1 - \exp\left(-\sigma / \sigma_{rel}\right)\right)\right], \tag{13}$$

c) for the U2 function (8):

$$\varepsilon(\sigma) = \varepsilon_o \left[1 + \frac{\nu_o \sigma_{rel}}{p}\left(1 - \exp\left(-\sigma / \sigma_{rel}\right)\right)\right]^p . \tag{14}$$

The mathematical processing of all numerical data (referring to some magnetic and mechanical properties, respectively) was performed using the application "Solver" to program Excel, with the conjugate gradient method. It was minimized the absolute standard deviation of the analytical results relative to the experimental data.

4.1. Results from the analysis of mechanical hysteretic process

The analyses of the plots from Figures 7-9 and that of the fitting errors in Table 3 point out that while the West model offers the lowest line (only a mediocre agreement) with the experimental results [15] - [18], the best agreement is provided by the U2 model. This is the sole model that faithfully reproduces the entire hysteresis process (see Figures 7 -9).

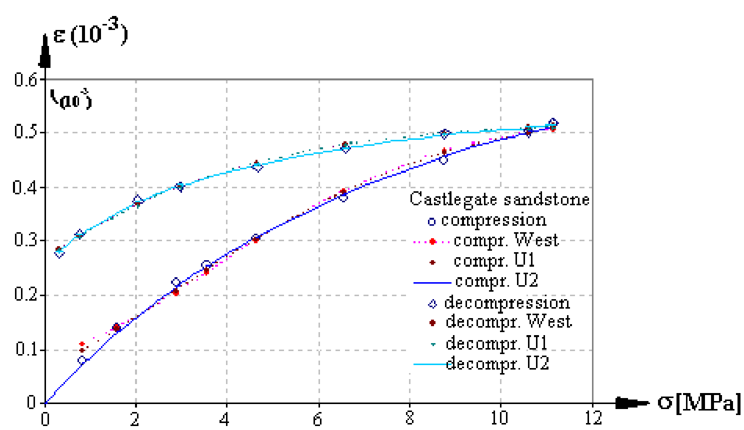

Fig. 7. Graphical plots (strain ε – stress σ) of the fitting of the compression-decompression elastic hysteresis processes of the Castlegate sandstones [15], by means of different phenomenological universality models (West, U1, U2)

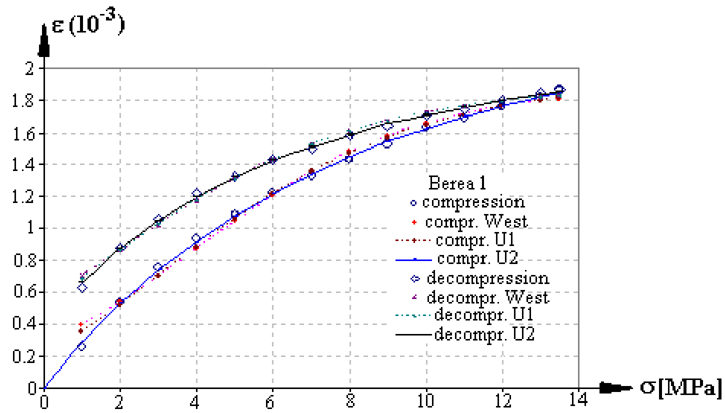

Fig. 8. Graphical plots (strain ε – stress σ) of the fitting of
the compression-decompression elastic hysteresis processes of the Berea 1 sandstones [16],
by means of different phenomenological universality models (West, U1, U2)

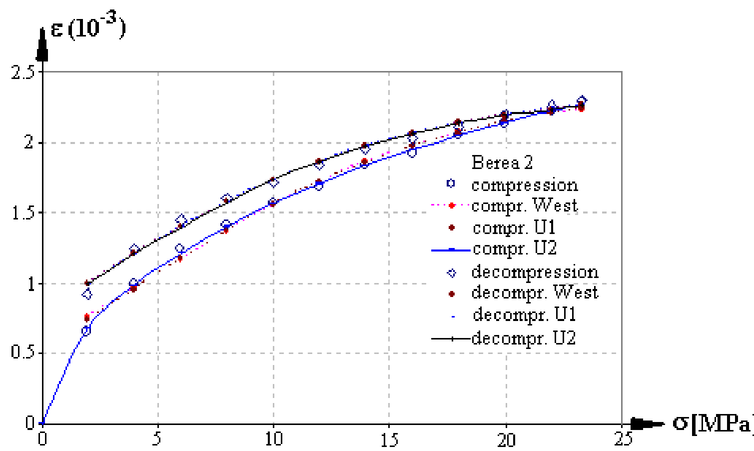

Fig. 9. Graphical plots (strain ε – stress σ) of the fitting of
the compression-decompression elastic hysteresis processes of the Berea 2 sandstones [17],
by means of different phenomenological universality models (West, U1, U2)

The obtained numerical values of the parameters of the studied phenomenological universalities (West, U1, U2) describing the mechanical hysteresis processes of the 3 examined sandstones were synthesized in frame of Table 3.

Table 3

Values of the fitting parameters of the 3 studied phenomenological universality models (West, U1, U2) describing the mechanical hysteresis processes for some sandstones

Sandstone Type [Reference]	Phenomenological Universality Model	Elastic hysteresis branch	Characteristic Parameters				Standard deviation (%)
			$\varepsilon_o(10^{-3})$	$\sigma_{rel.}$[MPa]	ν_o[MPa^{-1}]	p	
Castlegate [15]	WEST	compression	0.0791	2.60	-	-1.2961	14.0%
		decompression	0.2641	4.10	-	1.0100	1.4%
	U1	compression	0.0625	3.68	0.598	-	8.9%
		decompression	0.2685	3.27	0.203	-	1.6%
	U2	compression	0.0020	12.56	0.8267	90.776	2.4%
		decompression	0.02489	9.42	0.3077	0.453	0.8%
Berea 1 [16]	WEST	compression	0.2869	2.89	-	-1.3148	15.3%
		decompression	0.5422	3.59	-	-2.6576	3.6%
	U1	compression	0.2142	3.88	0.569	-	10.4%
		decompression	0.5108	3.99	0.332	-	3.0%
	U2	compression	0.0037	8.13	96.195	0.9561	3.4%
		decompression	0.3701	7.59	0.939	0.7298	1.5%
Berea 2 [17]	WEST	compression	0.5780	6.77	-	-1.9026	5.8%
		decompression	0.7902	7.70	-	-5.0864	3.6%
	U1	compression	0.5278	8.15	0.1881	-	4.6%
		decompression	0.7714	8.13	0.1406	-	2.9%
	U2	compression	0.0463	31.79	32.4749	0.5684	1.7%
		decompression	0.2671	31.61	3.9200	0.4278	0.8%

5. Conclusions

The processes taking place in the complex systems can be modeled with high precision by means of the CUN functions. Low Complexity Systems (magnetic materials) are described with practically the same accuracy by means of both classes of functions UN. Medium and high complexity systems (rocks) are well described only by U2 function. Based on the results presented in [18] and [19], we propose to be compared the description accuracies of two models of hysteresis loops: a) that [19] by means of some hyperbolic functions and b) the one [18] by means of the CUN functions.

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R E F E R E N CES

[1] *D. A. Iordache*, "Accommodation and dis-accommodation processes in physics and biophysics", in U.P.B. Sci. Bulletin, Series A, **71**(2) 47-58(2009).

[2] *L. Daniello, D. Iordache, I. M. Popescu, C. Cristescu* "The photo-induced decrease of permeability of some manganese ferrimagnetic materials" (in Romanian), Bull. Polytechnic Institute Bucharest, Chem.-Metallurgy S., **41**(2) 9-12(1979).

[3] *L. Daniello, D. Iordache, I. M. Popescu, C. Cristescu, Gh. Stanciu, E. Oltean* "Behavior at low temperatures and under laser photo-irradiation of some spinelic ferrimagnetic materials", Mém. Sci. Sections Acad. Roum., 4^{th} series, **4**(1) 159-163(1981)

[4] *G. B. West, J. H. Brown,* Life's Universal Scaling Laws, Physics Today, **9**, 36-42(2004).

[5] *P. Castorina, P. P. Delsanto, C. Guiot* "A classification scheme for phenomenological universalities in growth problems in physics and other sciences", Phys. Rev. Letters, **96**, 188701(2006).

[6] *P. P. Delsanto, A. S. Gliozzi, C. L. E. Bruno, N. Pugno, A. Carpinteri,* Scaling laws and fractality in the framework of a Phenomenological Approach, Chaos, Solitons and Fractals, **41**, 2782-2786(2009).

[7] *P. P. Delsanto, C. Guiot, A. S. Gliozzi,* Scaling, growth and cyclicity in biology: a new computational approach, Theoretical Biology and Medical Modeling, **5**, 5, (2008).

[8] *P. P. Delsanto,A. S. Gliozzi, D. A. Iordache, C. Guiot,* Phenomenological Universalities approach to the analysis of perinatal growth data, U. P. B. Sci. Bull., **70**(4) 3-10 (2009).

[9] *A. Gliozzi, C. Guiot, P. P. Delsanto, D. A. Iordache,* A novel approach to the Analysis of Human Growth, *Theor Biol and Med. Mod.,* 2012, **9**:17

[10] *B. Gompertz*, On the nature of the function expressive of the law of human mortality and on a new mode of determining life contingencies, R. Soc. London, **23**, 513, (1825).

[11] *G. B. West, J. B. Brown, B. J. Enquist*, A general model for ontogenetic growth, Nature, **413**, 628-631(2001).

[12] *G. B. West, J. H. Brown, B. J. Enquist,* The fourth dimension of life: Fractal geometry and allometric scaling of organisms, Science, **284**, 1677-1679(2000).

[13] *L. Daniello, D. Iordache, I. M. Popescu, C. Cristescu* "The photo-induced decrease of permeability of some manganese ferri-magnetic materials" (in Romanian), Bull. Polytechnic Institute Bucharest, Chem.-Metallurgy S., **41**(2) 9-12(1979).

[14] *L. Daniello, D. Iordache, I. M. Popescu, C. Cristescu, Gh. Stanciu, E. Oltean* "Behavior at low temperatures and under laser photo-irradiation of some spinelic ferrimagnetic materials", Mém. Sci. Sections Acad. Roum., 4^{th} series, **4**(1) 159-163(1981).

[15] *T. J. Plona, J. M. Cook* "Effects of stress cycles on static and dynamic Young's moduli in Castlegate sandstone", in Rock Mechanics: Proceedings of the 35^{th} US Symposium, edited by J. J. K. Daemen and R. A. Schultz, p. 155, A.A. Balkema, Rotterdam, Netherlands, 1995.

[16] *G. N. Boitnott* "Fundamental observations concerning hysteresis in the deformation intact and joined rock, with applications in the near source region", Proc. Numerical Modeling for Under-ground Nuclear Test Monitoring Symposium, Los Alamos Natl. Lab. Rep., LA-UR93-3839, 121(1993).

[17] *L. B. Hilbert Jr., T. K. Hwong, N. G. W. Cook, K. T. Nihei, L. R. Myer* "Effects of strain amplitude on the static and dynamic nonlinear deformation of Berea sandstone", in Rock Mechanics Models and Measurements Challenges from Industry, edited by P.P. Nelson and S. E. Laubach, p. 497, A.A. Balkema, Rotterdam, Netherlands, 1994.

[18] *I. Apostol,* Contributions to the study of some physical, technical, and biological, respectively, complex systems, PhD Dissertation, University "Politehnica" of Bucharest, September 2012.
[19] *I. Nova, V. Havlicek, I. Zemanek,* Dynamic hysteresis loops modeling by means of extended hyperbolic model, EMSA 2012.