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SUB-DAILY TIME SHIFT DETECTION BETWEEN DAILY SAMPLED TIME SERIES

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The paper presents a method to evaluating the average time shift between two time series by exploiting both components of the complex-valued cross coherence function: magnitude and relative phase. What is new is that it could detect temporal shifts smaller than the sampling interval of the series, which is one day in most cases of financial series. In addition, due to the properties of the coherence function, the method works on non-stationary series.

Keywords: sub-daily time shift, daily sampled time series, relative phase, magnitude coherence index, time synchronization.

1. Introduction

Here the possibility to detecting relative time shifts between two time series taken over the same calendar interval is demonstrated. Differing from the usual case of looking for the peak in the cross correlation function by shifting one series with integer multiples of sampling rate, the method is designed to detecting time shifts smaller than the sampling rate of the series. This might be a useful tool to revealing small differences among propagation times from the same driving source toward two correlated processes evidenced by distinct time series and eventually to get information in order to predict the behavior of the delayed one in the case of financial series [1-3], meteorological or seismic phenomena [4].

Generally, the complex-valued coherence function is computed using the estimator of the smoothed periodograms of appropriate time length. The magnitude coherence function (MCF) is largely used as adequate measure to disentangle the medium or short run correlations among distinct processes [5,6]. Here MCF value is rather an enabler for the relevance of the results, while the relative phase function (RPF) of the complex coherence function is the one that is exploited to extract information on sub-daily time delay/advance of one series with respect to its pair. This is of ultimate importance to predicting the immediate trend of a specific financial series when the behavior of the corresponding one in the pair is known. The present study is in line with the work focused onto fingerprints of transition and crisis in Romania [7-11].

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The organization of the work is as follows: section 2 explains the principles of the method, section 3 brings information about data, section 4 presents examples of how method works and its limitations. Finally, section 5 summarizes the conclusions.

2. Method

2.1 Theory

By denoting *x*(*n*) and *y*(*n*) two interfering waves and *X*(*f*) and *Y*(*f*) the corresponding Fourier images, the power at frequency *f* is *P*(*f*):

$$P_{xy}(f) \propto P_x(f) + P_y(f) + < 2\,\mathrm{Re}\{X(f)\cdot Y^*(f)\} >_\tau , \qquad (1)$$

where the window length τ depends on the spectral resolution of the detector, and the asterisk denotes the complex conjugate. The averaging time depends on the characteristic time of the detector. The ratio of the estimated cross-spectrum $< X(f)\cdot Y^*(f) >_\tau$ to the product of the square roots of the auto-spectra measures the normalized complex-valued coherence $\gamma_{xy}(f)$ [12]:

$$\gamma_{xy}(f) = \frac{< X(f)\cdot Y^*(f) >_\tau}{\sqrt{< X(f)\cdot X^*(f) >_\tau}\cdot\sqrt{< Y(f)\cdot Y^*(f) >_\tau}} = \left|\gamma_{xy}(f)\right|\cdot e^{\mathbf{i}\alpha_{xy}(f)} . \qquad (2)$$

In Eq.(2) MCF and RPF are $\left|\gamma_{xy}(f)\right| \le 1$ and $\alpha_{xy}(f)$ respectively. More precisely, the relative phase is:

$$\alpha_{xy}(f) = \mathrm{atan}\frac{\mathrm{Im}\{\gamma_{xy}(f)\}}{\mathrm{Re}\{\gamma_{xy}(f)\}} . \qquad (3)$$

According to the time delay theorem [13] one has $y(t-\delta t) \leftrightarrow Y(f)\cdot e^{\mathbf{i}(2\pi f\cdot\delta t)}$, where δt is a constant time shift. If such a constant time shift does really exist between two series *x* and *y* (zero included), the processes they are describing are timely synchronized [14]. It is easy to prove that

$$\gamma_{xy}(f,\delta t) = \frac{< X(f)\cdot Y^*(f,\delta t) >_\tau}{\sqrt{P_x(f)}\sqrt{P_y(f)}} = \gamma_{xy}(f)\cdot e^{-\mathbf{i}(2\pi f\cdot\delta t)} , \qquad (4)$$

where $\gamma_{xy}(f,\delta t)$ is the normalized complex-valued coherence of the pair consisting in the genuine and the shifted series.

Therefore RPF of any pair of series could be checked for the existence of statistically significant slope in coordinates (f, α_{xy}):

$$\alpha_{xy}(f) \cong \hat{a} + \hat{b}\cdot f , \qquad (5)$$

where $\hat{a}$ and $\hat{b}$ are the least squared error estimates of the regression. The estimated slope $\hat{b}$ should be a measure of the time shift if properly calibrated:

$$\hat{b} \propto -2\pi \cdot \delta t \,. \tag{5'}$$

The estimator “$< >_{\tau}$” is computed according to the Welch method [15] using the sliding window technique over the full length of the series [16]. The overlapping factor was less than 0.75 in order to allow for an anti-symmetric calibration. Since the waves are replaced with daily sampled financial time series and the interference is a computational one, an acceptable compromise between having enough frequency resolution on one side, and sufficient frequency points in the range of the shorter cycles on the other side is to set the window width to τ=24 days. According to Shannon-Nyquist criterion, for daily sampled time series the extreme values of the detectable cycles are given in Table 1.

Table 1

Time-frequency limits

Window width	Sampling time	Freq. resolution	Shortest cycle	Longest cycle
24 days	1 day	(1/24) day^{-1}	2 days	24 days

The method is implemented with the capabilities of Mathematica software [17] using the modified formulae [18]. Since the real frequency f (in day^{-1}) is related to the values of detectable cycles T_f (in days) by the formula $f = (T_f)^{-1}$, note that the dimensionless numeric frequency k is related to the real frequency by the relation:

$$k = \tau \cdot f \,, \quad f = 1, \ldots, f_{\max}. \tag{6}$$

Therefore Eq.(5’) comes to

$$\hat{b} = -2\pi \cdot \frac{\delta t}{\tau}. \tag{7}$$

Positive slopes mean advance of the second series with respect to the first series in the pair, while negative slopes mean delay. Eq.(7) relates the estimated slope to the normalized time:

$$\delta t = -\frac{2\pi}{\tau} \cdot \hat{b} \,. \tag{7'}$$

Despite of the time step is of one day, it is worth noting that the formula allows to computing time shifts smaller than the quanta of one day. In fact the minimum time delay that could be measured is limited by the accuracy to which $\hat{b}$ is determined. The time shift and the corresponding slope could be smaller than the sampling interval provided that it be statistically significant. The standard error, the t-statistic and the p-value are commonly used to test for individual statistical significances of the variables [19]. If the p-value is small, one can reject the hypothesis of statistical insignificance and conclude that the specific variable has a significant effect on the dependent variable. For example, a p-value of 0.05 means that there is only a 5% chance that the correct null hypothesis will be rejected, and therefore the variable is statistically significant.

From Eq.(5’) one should remark the higher the frequency, the higher the phase shift. Since the purpose of the present work is to reveal sub-daily time shifts, the behavior in the range of the short cycles is more relevant than in the longer ones. The short-run is in the sense of the usual partition of the trends of the stock exchange markets [20]:

$$T_0 \leq 4 \text{ days, or } k_0 \geq 6 . \tag{8}$$

The main time-frequency correspondences and the medium- short-run partition are given in Table 2.

Table 2

Time-frequency correspondences

Time cycle T_f (days)	24	12	8	6	4	3	2.7	2.4	2.2	2
Real frequency f (day^{-1})	1/24	1/12	1/8	1/6	1/4	1/3	3/8	5/12	11/24	1/2
Numeric frequency k	k_{min}=1	2	3	4	k_0=6	8	9	10	11	k_{max}=12
Cycles partition	Medium run				Short run					

The relative phase spectrum $\alpha_{xy}(f)$, like the coherence spectrum, is only useful as an estimate. It is only valid at frequencies where there is a significant coherence value. A significant coherence value indicates that the relative phase is indeed concentrated around the mean. If it is not significant, then the relative phase distribution is not significantly concentrated, and the mean relative phase is not meaningful. Therefore the investigation of RPF does make sense only if MSC has significant value. Since the interest is focused here on finding the average time shift irrespective of the frequency, but preferring the higher ones, a magnitude coherence index (MCI) of the coherence function is defined as a mean value over the shortest cycles (equivalent, the highest frequencies)

$$\Gamma = \frac{1}{k_{\max} - k_0} \cdot \sum_{k=f_0}^{k=f_{\max}} \left| \gamma_{xy}(k) \right| , \tag{9}$$

where MCI is denoted by the capital letter Γ. The lower threshold value of MCI that validate the computation of time shift is given by the cross spectrum of synthesized Gaussian series with Hurst exponent of 0.5 [21]:

$$\Gamma > \Gamma_{\text{TH}} = 0.100 . \tag{9'}$$

2.2 Calibration

The method allows for calibration by using a simulated time shift. When putting $x \equiv y$ in Eq.(2) and delaying one of the series with one step Δt, the relative phase should be shifted with a quantity depending linearly on the frequency f (see Fig.1):

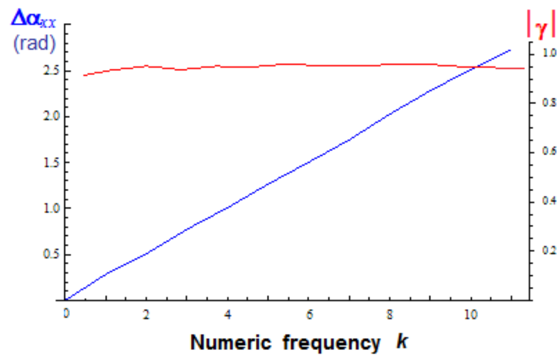

Fig.1 Calibration: relative phase shift (diagonal line and left axis) and magnitude coherence function (horizontal line and right axis) for simulated time shift $\Delta t = -1$day

$$\Delta\alpha_{xx}(f, \Delta t) \propto -2\pi \cdot f \cdot \Delta t, \quad f = 1, \ldots, f_{\max}, \tag{10}$$

because

$$\gamma_{xx}(f, \Delta t) = \frac{< X(f) \cdot \left(X(f) \cdot e^{\mathrm{i}2\pi \cdot \Delta t \cdot f}\right)^* >_\tau}{\sqrt{P_x(f)}\sqrt{P_x(f)}} = \gamma_{xx}(f, 0) \cdot e^{-\mathrm{i}2\pi f \cdot \Delta t}. \tag{10'}$$

Obviously $|\gamma_{xx}(f,0)| = 1$ in the whole band, i.e. $\alpha_{xx}(f,0)=0$. The value of $|\gamma_{xx}(f, \Delta t)|$ is smaller than the theoretical unitary value provisioned by Eq.(10') because of the finite length of the series and of the truncating errors especially at higher frequencies when working to the limit of Shannon-Nyquist condition. To conclude, Eq.(10) allows for calibrating the slope $2\pi \cdot \Delta t$ in (f, α) coordinates.

According to Eq.(7), the slope behaves anti-symmetrically with Δt, and its theoretical value for $\Delta t = \pm 1$ is:

$$\hat{b}\Big|_{\Delta t = \pm 1} = \mp \frac{2\pi}{\tau}. \tag{11}$$

3. Data

The exchange rates were acquired from the site Forex Trading and Exchange Rates Services [22] and correspond to the interval 1 January 1999-31

December 2012, i.e. 5114 values. The aggregate BET index was taken along the same interval from the site of the Bucharest Stock Market (BVB) [23]. Experimentally we found the smaller the Hurst exponent of the series, and consequently the smaller or absent the long run correlations in the series, the better the theoretical value given by Eq.(11) [24]. That is why the method is working better on the derivative of series. Keep in mind that in order to have reliable results, identical operations should be performed over both series in the pair; if not otherwise mentioned, returns of the series were taken.

4. Results and discussion

Hereafter several examples are presented using BET composite index and some ROL exchange rate series.

4.1 Calibration

The calibration is consisting in shifting the second series with one step $\Delta t = \pm 1$, then estimating the corresponding slope $\hat{b}$ and finally computing the time shift according to Eq.(7) in the form:

$$\widehat{\Delta t} = -\hat{b} \cdot \frac{\tau}{2\pi}. \qquad (12)$$

The computed time shift $\widehat{\Delta t}$ is compared to the theoretical value given by Eq.(11) and corrections could be operated, if the case.

The calibrations of BET composite index and of Romanian Leu (ROL) vs. most used reference currencies in the world like United States Dollar (USD), Euro Zone currency (EUR), Swiss Franc (CHF), British Pound (GBP), and Japanese Yen (JPY) are summarized in Table 3. All results are highly statistically significant according to the standard error *s* and the *p*-values computed by the built-in Mathematica software. All results are validated by high values of MCI given by Eq.(9) that fulfill Eq.(9').

Table 3

Computed Δt for simulated time Δt = ±1 day

	$\Delta t = -1$ day					$\Delta t = 1$ day				
	Γ	Slope			$\widehat{\Delta t}$ (days)	Γ	Slope			$\widehat{\Delta t}$ (days)
		$\hat{b}$	*s* ($\times 10^{-3}$)	*p*-value ($\times 10^{-19}$)			$\hat{b}$	*s* ($\times 10^{-3}$)	*p*-value ($\times 10^{-19}$)	
BET	0.960	0.252	0.5	0	–0.963	0.960	–0.253	0.5	0	0.966
ROL/USD	0.956	0.252	0.5	0	–0.963	0.956	–0.252	0.5	4.8	0.963
ROL/EUR	0.963	0.250	0.5	0	–0.955	0.964	–0.250	0.5	2.8	0.955
ROL/CHF	0.962	0.251	0.6	0	–0.959	0.962	–0.251	0.6	18	0.959
ROL/GBP	0.959	0.252	0.6	0	–0.963	0.959	–0.252	0.6	7.4	0.963
ROL/JPY	0.952	0.255	0.8	0	–0.974	0.952	–0.255	0.8	97	0.974

All computed (absolute) values of the time shifts estimates $\hat{\Delta t}$ are systematically smaller than the theoretical value of Δt of one day to the limit of –5%, i.e. approximately one hour. The *p*-values are extremely small therefore the results are of extreme confidence accordingly. There is a biasing toward the negative valued slopes meaning an apparent small advance of the second series with respect to the first one. The biasing diminishes with the increase of the overlapping factor of the sliding window so that the anti-symmetry $\hat{\Delta t}\Big|_{-\Delta t} = -\hat{\Delta t}\Big|_{\Delta t}$ was reached for a factor of 0.75 in Table 3.

For the scope of the present work we assume the theoretical value given by Eq.(12) is reliable enough in order to use it in the form

$$\hat{\delta t} = -3.82 \cdot \hat{b}\,. \qquad (12')$$

4.2 Examples

It is not a trivial issue to find relevant time shifts as average values over a certain length of a pair of time series. There are several conditions to be fulfilled in order the findings be significant, like the existence of coherence above the threshold (9’) and a favorable *t*-statistic. Secondly, the calendar synchronization of the series is a necessary condition for a proper interpretation of the small time shift, if any. The delay or advance of the second series is referred to the first one in the pair. If too large, the time shift could overcome the coherence time and the study doesn’t make sense. And in the third place, if such a yet unusual short and persistent time shift is detected, care should be taken to what extent the finding is meaningful and could be explained in the theoretical framework of the domain the series are belonging – here the financial domain. Since we focus on the method, the last group of explanations will be not emphasized here.

Table 4

Measured time shifts in the case of exchange rates involving ROL

	Time interval	Γ	Slope			Computed δt (days)
			$\hat{b}$	s	*p*-value ($\times 10^{-3}$)	
HUF/ROL-PLN/ROL	1Jan.’99 - 31Dec.’12	0.602	0.028	0.007	3.3	0.11
SEK/ROL-PLN/ROL	1Jan.’99 - 31Dec.’12	0.596	0.024	0.008	13.3	0.09
HUF/ROL-SEK/ROL	1Jan.’99 - 31Dec.’12	0.694	0.004	0.005	467	irrelevant

In table 4 are presented three cases. The most relevant result is the one involving the pair HUF/ROL-PLN/ROL where the computed time shift benefits from a favorable *p*-value and small standard error. The second case is suffering from a lower confidence, while the third is considered irrelevant because of the too large p-value (HUF stands for Hungarian Forint, PLN for the Polish Zloty, and SEK for the Swedish Krona).

Therefore the two hours advance of PLN/ROL compared to HUF/ROL might indicate indeed synchronous dynamic in the exchange rate values during the investigated interval. Even if the series are sampled at a repetition rate of one day, and the result might appear a weird one, the exchange rates are varying during the trading days according to the market rules and it is not a contradiction to detect average values of time shifts smaller than the sampling rate. This kind of information could be helpful for speculative traders.

Another example is related to the behavior of BET index compared to the exchange rates of ROL with respect to the reference currencies EUR and USD. They were investigated in the time intervals preceding and following the financial shock in August 2007, as well as for the whole interval 1 Jan.1999-31 Dec.2012 (see Fig.2).

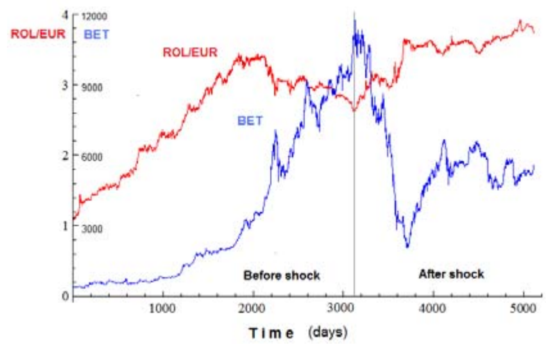

Fig.2 ROL/EUR and BET time series; the beginning of the financial shock in August 2007 is marked with vertical line

The results are given in table 5.

Table 5

Comparison before and after the shock in August 2007

	Time interval	Γ	Slope			Computed δt (days)
			$\hat{b}$	s	p-value ($\times10^{-3}$)	
BET-ROL/USD	1Jan.’99-1Aug.’07	0.113	–0.236	0.240	354.1	irrelevant
	31Aug.’07-31Dec.’12	0.277	–0.259	0.043	0.3	0.98
	1Jan.’99 - 31Dec.’12	0.175	–0.241	0.040	0.3	0.92
BET-ROL/EUR	1Jan.’99-1Aug.’07	0.069	–0.223	0.175	238.7	irrelevant
	31Aug.’07-31Dec.’12	0.231	–0.234	0.059	4.1	0.89
	1Jan.’99 - 31Dec.’12	0.109	–0.233	0.047	1.0	0.89

The facts indicate the presence of relevant time shift in the aftershock. The aftershock interval is imprinting the apparent synchronization over the whole 13-year interval with the cost of diminishing the coherence down to the noise limit established by Eq.(9’). The exchange rates lag BET with almost one day – more exactly 22 hours. This value is in good agreement with the interval between 4PM – the closing time of BVB, and 1PM – the time when the Central Bank is announcing the reference exchange rates in the following day.

5. Conclusions

The paper presents a method to detect time synchronization between processes embedded in two time series using the magnitude and relative phase of the complex-valued cross coherence function. To the limit of the theoretical hypotheses and operational settings assumed in the present work, temporal shifts smaller than the sampling interval of the series could be measured after a proper calibration that allows for a theoretical resolution of one hour.

Several conditions have to be fulfilled in order that the findings be meaningful, like significant value for coherence index and an acceptable *t*-statistic in line with the particular objective of the analysis.

Examples of time shifts of around 2 hours were found in some couples of financial time series involving ROL and two currencies from neighbouring countries (PLN and HUF). When shortening the averaging interval the likelihood to detect the shifts increases, especially in the vicinity of historical events. This is the case of the financial shock that occurred in August 2007. By partitioning time axis in “before shock” and “after shock”, significant delays were detected among BET index on one side, and ROL/USD and ROL/EUR exchange rates in supporting the idea the stock exchange dynamic is an important driving force of the evolution of ROL in the aftershock interval.

In spite of existence of calendar synchronization, the evidence of time synchronization among processes belonging to distinct series in the short run of 2-4 days is relatively rare. The electronic transactions are diffusing information in real time and at global scale bringing the efficient market paradigm closer to real life.

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