MATHEMATICAL APPROACH OF HYSTERESIS PHENOMENON AND ENERGY LOSSES IN NON-ORIENTED SILICON IRON SHEETS

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In this paper it was studied the mathematical interpretation of the magnetic hysteresis in non-oriented silicon iron alloys. In the framework of loss separation concept the hysteresis, classical and excess energy losses have been analyzed. The hysteresis energy losses were determined using a computationally efficient classical Preisach model. The excess and total energy losses were predicted based on statistical magnetic object theory.

Keywords: Preisach model, magnetic object theory, NO FeSi laminations, energy losses.

1. Introduction

Hysteresis is the principal characteristic of the ferromagnets below the Curie temperature and the description of this important phenomenon is a captivating mathematical problem that is attracting growing attention among the mathematicians. In the industrial applications, material scientists must consider and understand the association between the material microstructure and the magnetization curves, to improve the manufacturing methods and the material quality.

The understanding of the physical mechanisms responsible for hysteresis and the development of adequate mathematical tools to describe it is a real example of a physical and mathematical problem, which is also a source of technological progress.

Hysteresis loops could have a great variety of forms as a function of the material microstructure. A magnetic material can be considered as an assembly of permanent magnetic moments $\mathbf{m}_i$, of quantum-mechanical origin. In iron based materials, iron atoms have a magnetic moment of 2.2 μ_B, where the Bohr-Procopiu

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magneton $\mu_B = \dfrac{hq_e}{4\pi m_e} \cong 9.27 \times 10^{-24}\,\mathrm{Am}^2$ is the natural atomic unit of magnetic moments, h is the Planck's constant, q_e the electron charge and m_e the electron mass [1, 2, 3].

In an ideal paramagnet the individual $\mathbf{m}_i$ vectors do not interact with each other and they are individually influenced by thermal agitation. They will be oriented randomly in space and the total magnetization will be zero. A nonzero net magnetization can be induced by an external field, when the field energy is comparable in order of magnitude to the thermal energy [2].

Weiss has already suggested [4] that ferromagnetic materials could reveal an important spontaneous magnetization, because the magnetic moments are not independent and they must be coupled by an internal molecular field. This field introduces a feedback response, which turns all the magnetic moments to the respective direction. Below the critical Curie temperature T_c the magnetic moments spontaneously achieve a long range order and the material obtains a spontaneous magnetization M_s. In iron based alloys that have $T_c \cong 770$ °C, the value of the internal molecular field will be of the order of 10^9 A/m. As a consequence these materials will exhibit a spontaneous magnetization of 2 T at room temperature. The existence of this high field is not sustained by the experimental results and one would expect to find ferromagnetic materials spontaneously magnetized to saturation. Weiss also proposed that a ferromagnetic material must be divided into regions, called magnetic domains. In each magnetic domain, the molecular field determines the number of magnetic moments, which are aligned parallel to this field, but the orientation of the spontaneous magnetization is different from domain to domain [4, 5]. Nowadays it is considered that the magnetic domains are the result of the balance of several energy terms: exchange energy, which favors uniform magnetization configurations, magnetocrystalline anisotropy that is due to the interactions of the magnetization with the crystalline lattice and favors the orientation of the magnetization along certain preferred directions and magnetostatic energy, which conducts to magnetization configurations with a null average magnetic moment. The magnetization processes, in time dependent external applied fields, determine a rearrangement of the domain structure, through the motion of the domain walls that separate neighboring domains and in some cases through the rotation of the magnetic moments.

2. Hysteresis and energy loss models

The measured hysteresis curve can be modeled with mathematical tools, which contain adjustable parameters and functions [6]. The identification of these functions is made by well-defined magnetic measurements. The mathematical descriptions of hysteresis phenomena can be grouped in three classes.

The first approach considers every magnetic body as a group of interacting particles or hysterons. The hysteresis curve can be determined, if the distribution of switching and interaction fields of each elementary unit is known. This concept was elaborated by F. Preisach [7] and developed later by I. Mayergoyz [8]. A modified model is the dynamic Preisach model, which extends the classical one by introducing a rate dependent factor for each elementary unit and taking into account the delay time of the magnetic flux density behind the magnetic field [9, 10]. The dynamic Preisach model can be applied for the calculation of energy losses in electrical machine cores [11, 12].

The second approach was introduced by D. Jiles [5]. He considers continuous differential equations for the description of a general ferromagnet, into which certain measured functions or parameters are inserted.

The simulation of the hysteresis phenomenon based on the dynamic interactions of Barkhausen jumps and statistical arguments can be considered the third approach and was introduced by G. Bertotti [13]. This theory describes the frequency dependence of the energy losses, using the statistical model of loss separation.

These techniques use certain concepts and arguments from magnetic domain theory, either by identifying elementary units, which switch under the effect of the different external, interaction and eddy current fields or by focus on individual domain walls that move over a series of pinning centers.

3. Barkhausen jumps in non-oriented silicon iron steels

A given volume of a ferromagnetic material contains a number of pinning centers (dislocations, grain boundaries, voids, precipitates), which hinder the free movement of the domain walls and cause localized variations of the magnetostatic energy. The Barkhausen jumps appear when the applied field is increased sufficiently enough to move the domain wall over the lattice defects and further increase the magnetization value of the electrical steel. The magnetization vector **M** changes its value as a function of the applied magnetic field in increasingly random steps. It is coupled to the external field by the quantity $-\mu_0\mathbf{M}\mathbf{H}_a$, which continuously modifies the system energy balance as $\mathbf{H}_a$ is varied in time, (μ_0 is the vacuum magnetic permeability and $\mathbf{H}_a$ is the applied magnetic field strength). The stability of the magnetic domain structure is changed by the applied field variation. The local energy minimum is transformed into a saddle point thus makes the domain structure unstable and spontaneously evolves to a new configuration. This rearrangement may be localized in space, where a domain wall segment jumps to a neighboring stable position or, as in the case of the nucleation of new magnetic domains with reversed magnetization, it may change a large magnetic domain structure in a substantial part of the material [14].

The magnetization processes are reversible and irreversible in non-oriented silicon iron steels. The Barkhausen jumps are usually associated with the irreversible process like domain wall translation. In the case of intermediate to high magnetic fields the rotation of the magnetization vector becomes irreversible, when the magnetic moments change their direction from the original low energy axis to the applied magnetic field direction. This phenomenon occurs when the field energy overcomes the anisotropy energy [2].

In non-oriented silicon iron sheets, when the sample is magnetized along the rolling direction, the Barkhausen jumps are due only to the irreversible domain wall movement. In the case of the applied magnetic field, which is oriented perpendicular to the rolling direction, the Barkhausen jumps occur as a consequence of the irreversible domain wall movement and the irreversible rotation of magnetization at the saturation point [15].

Regarding the hysteresis loop, the most important Barkhausen activities appear in the area around the coercive field, where the rate of change of magnetization is highest [16].

4. Statistical interpretation of energy loss separation

The prediction of the energy losses as a function of magnetizing frequency f and peak magnetic polarization J_p presents a great interest in the industrial applications of the electrical steel. This dependence is related to the structural parameters like grain size that defines the microstructure of a given material and to the domain size that characterizes the magnetic domain structure. Based on [13] the specific energy losses can be decomposed into the sum of two principal components: hysteresis energy losses W_h and dynamic energy losses W_{dyn}.

The W_h depends on J_p and on material characteristics, therefore can be calculated by means of Preisach model of ferromagnetic hysteresis, because the direct connection between the elementary rectangular loops and the discontinuous character of the magnetization process (Barkhausen jumps) at a microscopic scale. In non-oriented silicon iron sheets the material behavior is described by a function $p\ (a, b) = p_1(a)p_2(b)$, which represents the density distribution of the elementary loops of different reversal field a and b, with $b < a$, $a \leq H_s$, $b \geq -H_s$, where H_s is the saturation magnetic field strength [17]. In order to obtain this density distribution function, it is necessary to determine experimentally the normal magnetization curve and the saturation hysteresis cycle. The experimental data of the normal magnetization curve must have N points equally spaced, obtained by varying the applied magnetic field strength H from zero to saturation value H_s. Also, in the case of the saturation loop it is necessary to have an equally spaced distribution of

N points from positive saturation H_s to zero value and another N points uniform distributed from zero to negative saturation value $-H_s$. Because of the symmetry of the Preisach distribution the function p_1 is defined by N values and p_2 is characterized by $2N$ values. Hence the entire Preisach distribution p will have $3N$ equations. The knowledge of the function $p(a, b)$ is sufficient to determine the macroscopic behavior of the material. The magnetic polarization can be calculated for a given sequence of H at the end of the path. The Preisach density $p(a, b)$ exists, is unique and it depends only on the nature and structure of the material [18 - 23]. The set of the $3N$ equations can be described as follows, for $0 \le k \le N - 1$:

$$\begin{cases} J_{k+1} - J_k = \dfrac{H_s^2}{N^2} p_1(a_{k+1}) \sum\limits_{i=0}^{k} \left[p_2(b_{N-i}) + p_2(b_{N+i+1}) \right]; \\ J_{N+k} - J_{N+k+1} = \dfrac{H_s^2}{N^2} p_2(b_{k+1}) \sum\limits_{i=0}^{k} p_1(a_{N-i}); \\ J_{2N+k} - J_{2N+k+1} = \\ = \dfrac{H_s^2}{N^2} p_1(a_{k+1}) \left[\sum\limits_{i=0}^{k} \left[p_2(b_{N-i}) + p_2(b_{N+i+1}) \right] + \dfrac{H_s^2}{N^2} p_2(b_{N+k+1}) \sum\limits_{\substack{i=k+1 \\ i \le N}}^{N} p_1(a_i) \right]. \end{cases} \tag{1}$$

To solve the above system of equations one have to impose the value for $p_1(a_N)$ as equal to 1.

Based on the $p(a, b)$ values it can be determined the calculated magnetic polarization:

$$J = \iint\limits_{S_+} p(a,b)\,da\,db - \iint\limits_{S_-} p(a,b)\,da\,db, \tag{2}$$

where S_+ and S_- are the areas in the Preisach plane oriented positive and negative.

The classical Preisach model is a formal one and it is mainly a general concept to describe, by differential calculus, the irreversible magnetization processes. It can be associated with the hysteresis energy losses W_h, which are due to the irreversible displacements of the domain walls and can be calculated according to [24]:

$$W_h = \iint\limits_{S_+} p(a,b)(a-b)\,da\,db - \iint\limits_{S_-} p(a,b)(a-b)\,da\,db. \tag{3}$$

The explanation of W_{dyn} can be made using macroscopic domain models, without taking into account the fine-scale wall motion, but this classical interpretation is a very gross simplification. It was noticed that W_{dyn} is usually higher than W_{cl}, which are the classical energy losses, determined when the presence of the magnetic domains is disregarded and one assumes that the whole material has a uniform and homogenous magnetization [25, 26]. For a silicon iron sheet of thickness d and electrical conductivity σ, in the case of a sinusoidal magnetic flux waveform the classical energy losses can be calculated according to eq. (4):

$$W_{cl} = \frac{\pi^2 \sigma d^2 J_p^2 f}{6}. \tag{4}$$

The difference between the dynamic energy losses and the classical losses is called the excess losses, and they are a consequence of the magnetic domain effects [27]. This type of energy losses is usually connected with Barkhausen noise [28] or it is due to the continuous rearrangements of the domain configuration [29]. The energy excess losses were analyzed in [30, 31] and it has been made a statistical interpretation of them, in which the magnetization process could be described in terms of the dynamic properties of n simultaneously magnetic objects (MO), randomly situated in the sheet cross section. Each magnetic object corresponds to a group of strongly interacting magnetic domain walls. The time evolution of these can be identified in terms of a single equivalent object. This theory can be applied to a wide variety of materials, by investigating the physical meaning of the concept of MO in each case.

The dynamic behavior of a MO is described by a viscous type equation $H_{exc} \propto \frac{d\Phi}{dt}$, where $H_{exc} = \frac{W_{exc} f}{\dot{J}}$ is the excess dynamic field acting on the MO, $\dot{J} = 4 J_p f$ is the average magnetization rate in a given cross section of the sample, and $\frac{d\Phi}{dt}$ is the magnetic flux change provided by the MO. The proportionality constant between H_{exc} and $\frac{d\Phi}{dt}$ is determined by the damping effect of the eddy currents induced by the MO [30]. In the case of n simultaneously active MO's, the magnetic flux change $\frac{d\Phi}{dt}$ is a fraction $\frac{1}{n}$ of the total flux rate $S\dot{J}$ imposed to the sample, where S is the cross section area of the sample. One could notice also the dependence:

$$H_{exc} = \frac{\sigma G S \dot{J}}{n} = \frac{H_w}{n}, \tag{5}$$

where G is a dimensionless coefficient assumed equal to the one which can be calculated when the whole flux change is due to a single planar Bloch wall passing through S. G is determined according to [30, 31]:

$$G = \frac{4}{\pi^3} \sum_k \frac{1}{(2k+1)^3} = 0.1356 \tag{6}$$

The number of the magnetic objects n is a function of the excess field H_{exc} and of a set $\{P\}$ of parameters linked to the material microstructure and magnetic domain structure. The non-oriented silicon iron alloys have a linear dependence:

$$n = n_0 + \frac{H_{exc}}{H_0}, \tag{7}$$

where n_0 represents the number of simultaneously active MO's when the frequency tends to zero and H_0 is the corresponding magnetic field. The field H_0 is a constant value for a given material almost independent of J_p and f, and dependent on the structural characteristics of the material.

It can be obtained some important information on the microstructure of non-oriented silicon iron alloys, or on the evolution of the domain structure just changing the value of the peak magnetic polarization and using the representation $n(H_{exc})$ [32].

The knowledge of n_0 and H_0 permits one to predict through (8) the behavior of excess losses ($W_{exc-calc}$):

$$W_{exc-calc} = 2J_p\left(\sqrt{4H_0H_w + (n_0H_0)^2} - n_0H_0\right) \tag{8}$$

4. Experimental results and discussion

Measurements of the total energy losses have been performed on four non-oriented silicon iron sheets NO FeSi M400-65A industrial grade. The physical properties of this alloy are listed in Table 1.

Table 1

Physical and geometrical properties of M400-65A samples

Sample ID	Mass [g]	Density [g/cm^3]	Length [mm]	Width [mm]	Thickness [mm]	Resistivity [Ωm]
1	41.56	7.65	303	30.02	0.6	47.7×10^{-8}
2	43.8				0.638	
3	45.44				0.66	
4	41.94				0.62	

The material has been tested between 3 Hz and 200 Hz by means of an industrial Single Sheet Tester (Brockhaus Messtechnik) at controlled sinusoidal magnetic flux waveform. This device can perform fast measurements, offers an AC frequency characterization and provides the hysteresis cycle, the relative magnetic permeability and the total power loss data. The values of the magnetic properties, used in the numerical analysis, have been calculated by averaging the four experimental sets of data.

The hysteresis losses have been obtained by extrapolating the dynamic losses down to $f \rightarrow 0$ and compared with those, determined through Preisach classical model.

In the case of the Preisach classical model, the input data for the calculation of the density matrix are based on eq. (1) and $N = 99$ points measured on the normal magnetization curve and 198 points on the descendent part of the magnetization curve obtained at $J_p = 1.5$ T (Fig. 1).

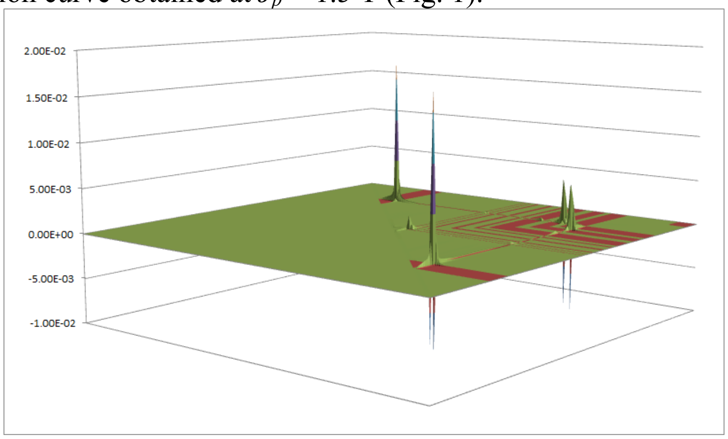

Fig. 1. Preisach density distribution (matrix of 198 × 198 points) in the case of NO FeSi M400-65A alloy based on experimental data measured for a magnetic polarization $J_p = 1.5$ T

Using the Preisach density matrix and eq. (2) have been obtained the modeled hysteresis cycles in the case of two peak magnetic polarization (Fig. 2). One can notice that the classical model does not calculate very accurately the saturation point, but it approximates well the area of the hysteresis cycle, that makes it suitable to determine the energy hysteresis losses W_h according to eq. (3).

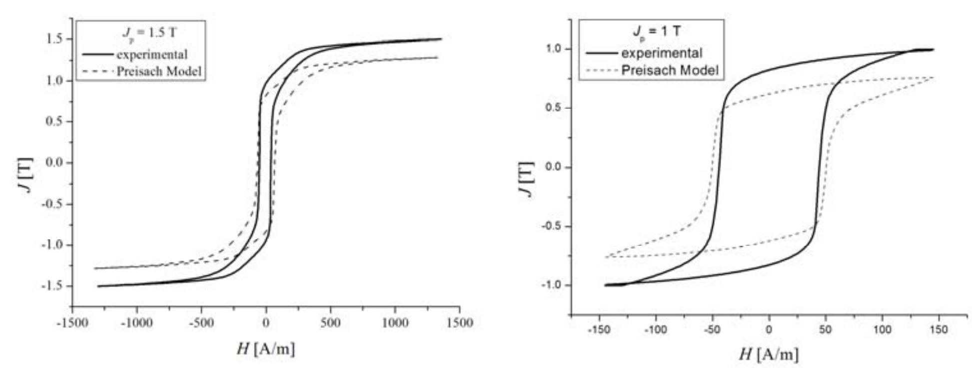

Fig. 2. Comparison between the experimental and modeled hysteresis cycles in the case of two values of the peak magnetic polarization

The hysteresis energy losses, determined from experimental data and through Preisach modeling, are presented in Table 2.

Table 2

Histeresis energy losses W_h [J/kg]

	Method		
Peak polarization	Experimental	Preisach model	Error* [%]
$J_p = 1.5$ T	0.03746	0.03824	2.082
$J_p = 1$ T	0.01524	0.01593	4.527

$* \; Error = \dfrac{Experimental - Preisach\ model}{Experimental} \times 100.$

The dynamic losses W_{dyn} have been subtracted from the total energy losses W_{tot} and the hysteresis losses W_h. The classical losses W_{cl} were determined according to Eq. (4), in order to calculate and make a statistical interpretation of the excess energy losses W_{exc}, based on magnetic objects theory. The dynamic behavior of the active magnetic objects is described by the n number of MO's and the excess dynamic field H_{exc} (Fig. 3). The values of the two phenomenological parameters n_0 and H_0 have been estimated by mathematical extrapolation of the $n(H_{exc})$ dependencies [33].

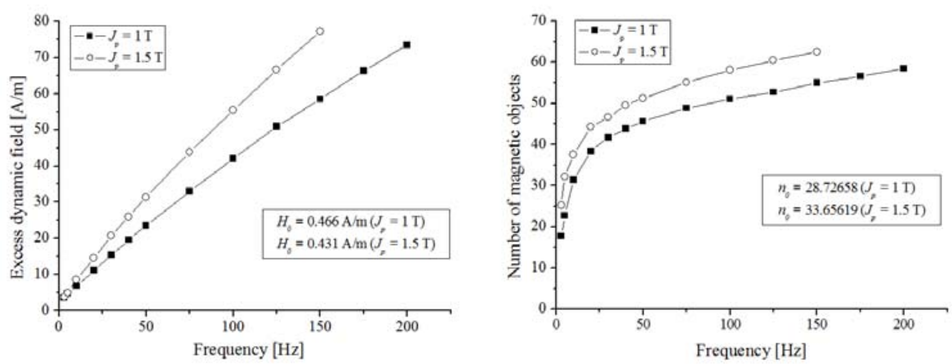

Fig. 3. Excess dynamic field and number of magnetic objects versus frequency in the case of two values of the peak magnetic polarization

The excess energy losses W_{exc_calc} can be predicted through the Eq. (8). Adding the values of the energy hysteresis losses W_h, calculated with the Preisach model, the classical energy losses W_{cl} and the predicted excess losses W_{exc_calc} one can determined the analytical total energy losses W_{tot_calc}. Also the predicted losses have been compared with the one obtained from the experimental data (Fig.4).

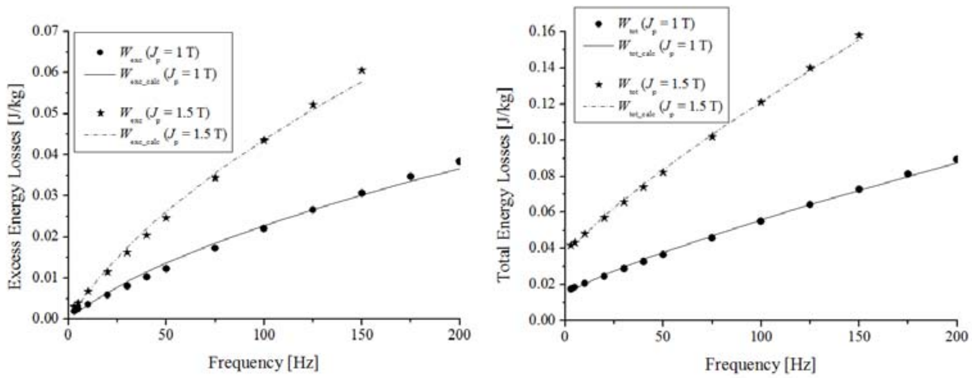

Fig. 4. Comparison between the experimental values (W_{exc}, W_{tot}) and the predicted (W_{exc_calc}, W_{tot_calc}) behavior of the excess and total energy losses in M400-65A NO FeSi

6. Conclusions

In this paper it was described a detailed presentation of magnetization phenomena, hysteresis and energy loss models. It was explained the concept of the Barkhausen jumps and their correlation with the irreversible magnetizing processes. The authors implemented a computational efficient scalar Preisach model, in order to determine the hysteresis energy losses. Magnetic object theory was used for the prediction of the excess losses. The total energy losses were reconstructed from statistical and analytical data and they are in good agreement with the experimental ones.

Acknowledgment

The work has been funded by the Sectorial Operational Programme Human Resources Development 2007-2013 of the Ministry of European Funds through the Financial Agreement POSDRU PERFORM/159/1.5/S/138963.

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