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APPLICATION OF EEMD SINGULAR VALUE ENERGY SPECTRUM IN GEAR FAULT IDENTIFICATION

Dewei GUO[1], Yun WANG[1], Libin YU[1], Yasong PU[1], Jie JIANG[1], Jie MIN[1], Wenbin ZHANG[1,*]

In view of the non-stationary and non-linear characteristics of the vibration signal when the gear fault occurs, a fault identification method based on ensemble empirical mode decomposition (EEMD) and singular value energy spectrum (SVES) is proposed. Firstly, the non-stationary acceleration vibration signal of gear is decomposed into a finite number of stationary eigenmode functions by using the advantage of ensemble empirical mode decomposition, and each order intrinsic mode function (IMF) component is composed of an initial eigenvector matrix. The advantage of EEMD is used to suppress the mode aliasing. Then, singular value decomposition (SVD) is used to solve the problem that it is difficult to determine the number of rows and columns of SVD phase space reconstruction matrix. The singular value energy spectrum is defined and the distribution of singular value energy spectrum under different working conditions is obtained. Finally, because of the good classification effect of grey similarity correlation analysis for small sample pattern recognition, the singular value energy spectrum is used as the element to construct the feature vector, and the working state and fault type of gear are judged by calculating the grey similarity correlation of different vibration signals. The results show that the proposed method can be effectively applied to the fault diagnosis of gear system.

Keywords: EEMD; singular value energy spectrum; grey similarity correlation; fault identification; gear.

1. Introduction

Gear transmission is the main movement and power transmission mode of mechanical equipment, and gear failure is an important factor inducing machine failure. How to extract gear fault feature parameters in the environment of violent speed fluctuation is the key to failure diagnosis of rotating machinery [1-2]. Because the gearbox works in the environment of multiple vibration sources, the background noise is very strong, so the fault signals of gearbox collected on site are mostly nonlinear and non-stationary. The traditional Fourier analysis is only suitable for processing stationary signals, but it can't process multi-component

[1] Assoc Prof. Dewei GUO, College of Engineering, Honghe University, Mengzi, China, e-mail: 28657983@qq.com
* Corresponding Author: Prof. Wenbin ZHANG, College of Engineering, Honghe University, Mengzi, China, e-mail: 190322507@qq.com

non-stationary signals with noise effectively. In the non-stationary signal analysis methods, STFT, WVD, wavelet transform and EMD have been widely used in gearbox fault diagnosis, but these methods have some limitations. For example, the time-frequency resolution of STFT is limited by Heisenberg uncertainty principle, which can not achieve the best at the same time; WVD has cross interference terms; wavelet transform needs to set the basis function and decomposition scale in advance, which is lack of adaptability for the studied signal. In recent years, empirical mode decomposition (EMD) method has been commonly utilized to analyze the non-linear and non-stationary signals of mechanical faults due to its adaptability [3-5]. However, EMD method still has the problem of mode aliasing, which makes it difficult to extract fault characterstic of non-stationary signals under strong background noise. In order to suppress the mode aliasing, Wu et al. [6-7] proposed the ensemble empirical mode decomposition (EEMD) method based on the EMD decomposition of white noise, adding white noise to the signal to supplement some missing scales, which can effectively solve the mode aliasing phenomenon. EEMD is an important improvement of EMD because it can restore the essential characteristics of signals.

After determining the appropriate feature extraction method, how to effectively select the fault feature parameters is very important. At present, singular value decomposition (SVD) is mainly used in signal denoising and periodic component extraction. It can be seen from reference [8] that the energy of data can use the sum of the squares of all the singular values to express. Therefore, the sequence composed of the squares of the singular values of data can be defined as the singular value energy spectrum, so that the relationship between the singular values and the energy distribution of the signal can be established. Generally speaking, the simpler the signal component is, the more concentrated the energy is in a few components; conversely, the more complex the signal component is, the more scattered the energy is. Therefore, the singular value energy spectrum can be used as a measure of the nonlinearity of vibration signal.

However, the embedding time and delay constant need to be determined artificially in the process of singular value calculation. The conventional method is to reconstruct the original signal into Hankel matrix [9-10] or segment the original signal to reconstruct the phase space [8]. In this paper, we try to introduce EEMD into singular value decomposition (SVD). Firstly, the original data is decomposed by EEMD, a matrix is composed of the eigenmode functions obtained from the decomposition. In this way, we can overcome the problem that it is difficult to determine the number of rows and columns of the reconstructed matrix in SVD phase space. Then SVD of the matrix is carried out and the singular value energy spectrum is obtained. When the gear fault occurs, the energy of the same frequency band corresponding to different fault types will be very different, so the energy distribution of the signal represented by the singular value energy spectrum can be obtained. Finally, the grey similarity correlation degree of the feature vector with singular value energy spectrum as the element is calculated for fault identification and classification. The gear is tested under normal condition, mild wear condition, moderate wear condition and broken tooth condition. The results express the proposed way can be effectively used in identification of typical gear failure.

2. Singular value energy spectrum based on EEMD

According to the principle of EEMD algorithm [7,11], EEMD can decompose the discrete data $x(t)$ (t=1,2,…, N is the observed time series, N refer to the number of sampling points) to obtain k IMF components and a residual component Ra. IMF components represent different frequency components from high to low, and different frequency bands contain different fault information of the signal. Therefore, the k IMF components can be formed into the initial eigenvector matrix B, that is

$$B = [\mathrm{IMF}_1\ \mathrm{IMF}_2 \cdots \mathrm{IMF}_k]^T \tag{1}$$

Then the SVD of matrix B is carried out, and the singular value of the initial eigenvector matrix B is obtained

$$\sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_k\} \tag{2}$$

According to reference [8], the data energy can be

$$E = \sum_{i=1}^{k} \sigma_i^2 \tag{3}$$

Because the singular value of the signal describes the fault characteristics of the signal in each frequency band in the sampling time, the characteristics of gear in various conditions mainly show differences of the singular values in different frequency bands. Therefore, the size of singular value in each frequency band can reflect the difference of various operating conditions of gears.

The SVES of the observed time series can express that

$$q = \{q_1, q_2, \cdots, q_k\} \tag{4}$$

In the formula, $q_i = \dfrac{\sigma_i^2}{\sum_{i=1}^{k} \sigma_i^2}$ represents the proportion of the signal energy of the i component in the whole signal energy, and satisfies the requirement $\sum_{i=1}^{k} q_i = 1$. Therefore, the energy spectrum distribution of singular value can describe the energy distribution of signal in different fault states.

3. Principle of gear fault identification

(1) Under the condition of normal gear system, slight wear, moderate wear and broken gear, *N* samples are taken according to a certain sampling frequency, and a total of 4*N* data samples are obtained.

(2) The data is decomposed by EEMD, and some IMF components are obtained.

(3) According to equation (1), the SVD matrix is constructed for SVD, and the SVES reflecting the fault characteristics of each sample signal is obtained by equation (4).

(4) The mean value $\overline{q}_j$ of singular value energy spectrum vector q_j of N training samples under the same state is obtained as the standard failure mode. Among them, j = 1, 2, 3 and 4 correspond to four states of gear respectively.

(5) The grey similarity correlation degree between the singular value energy spectrum vector *qx* of the signal to be detected and the standard fault mode $\overline{q}_j$ in each state is calculated. The standard fault mode with the largest grey similarity correlation degree with the sample to be identified is considered as the fault type of the sample to be identified. The detailed process of grey similarity correlation degree identification can be seen in the author's previous research results [11].

4. Case analysis

To test the practical effect of the proposed way in gear fault identification, the gear data under four different conditions of normal, mild wear, moderate wear and broken teeth are collected. Figure 1 gives the gear test rig. We use the data acquisition systems to measure the vibration signal. And we use Matlab software to process the data. The rotation frequency of the tested gear is f_r = 23.6Hz, the meshing frequency is f_z = 686Hz, and the f_s=16384Hz [12-13]. Take 20 samples from each conditions, and use EEMD to decompose the signal. From Figure 2 to Figure 5, the signal is decomposed by EEMD, each can obtain 11 IMF components and 1 residual component respectively, as shown in Fig. 2 to Fig. 5.

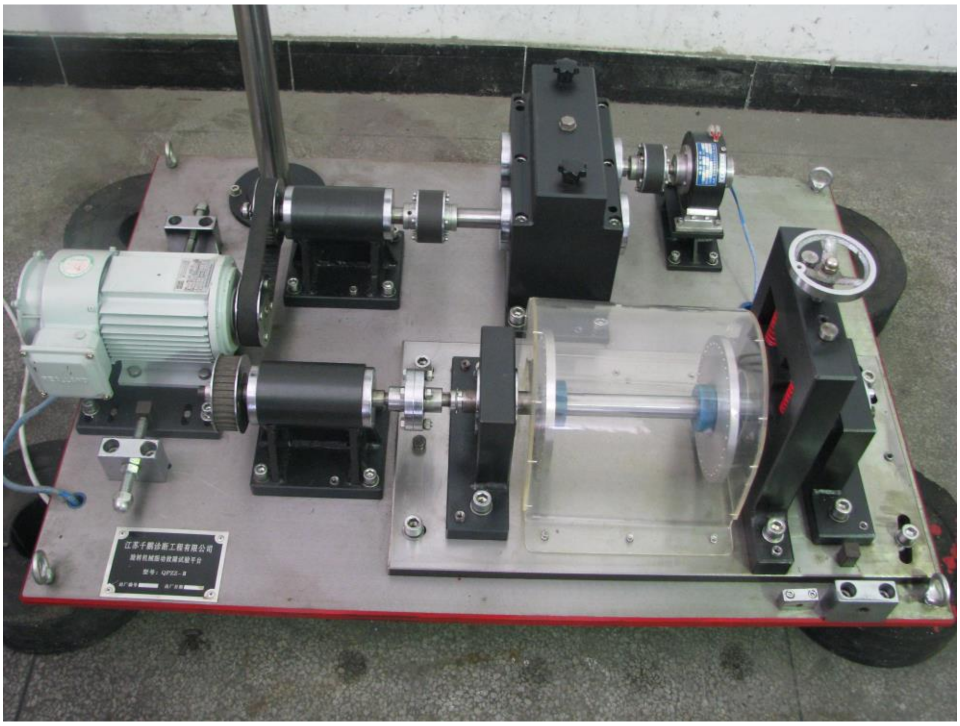
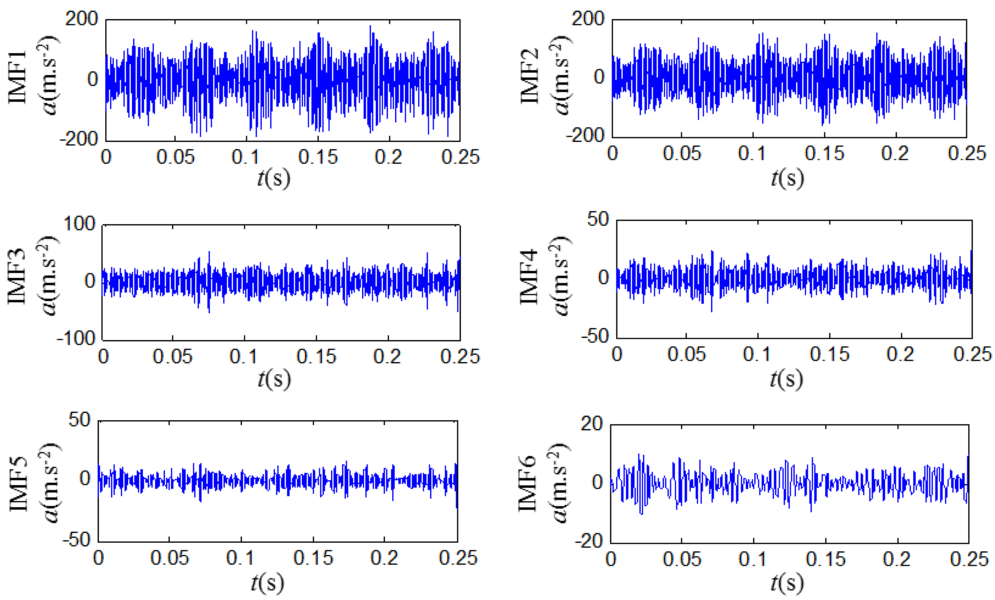


Fig. 1. The gear test rig



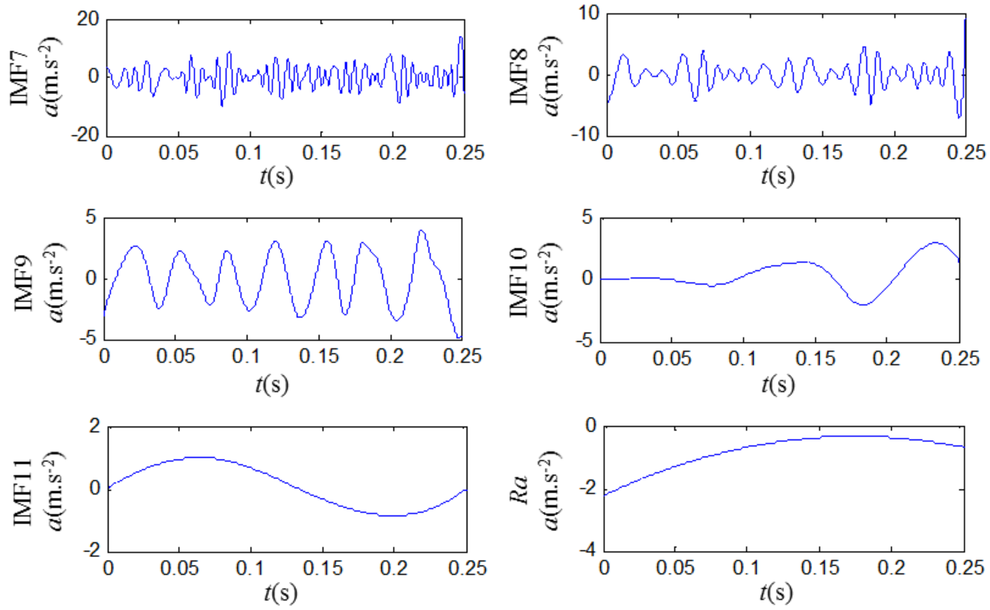
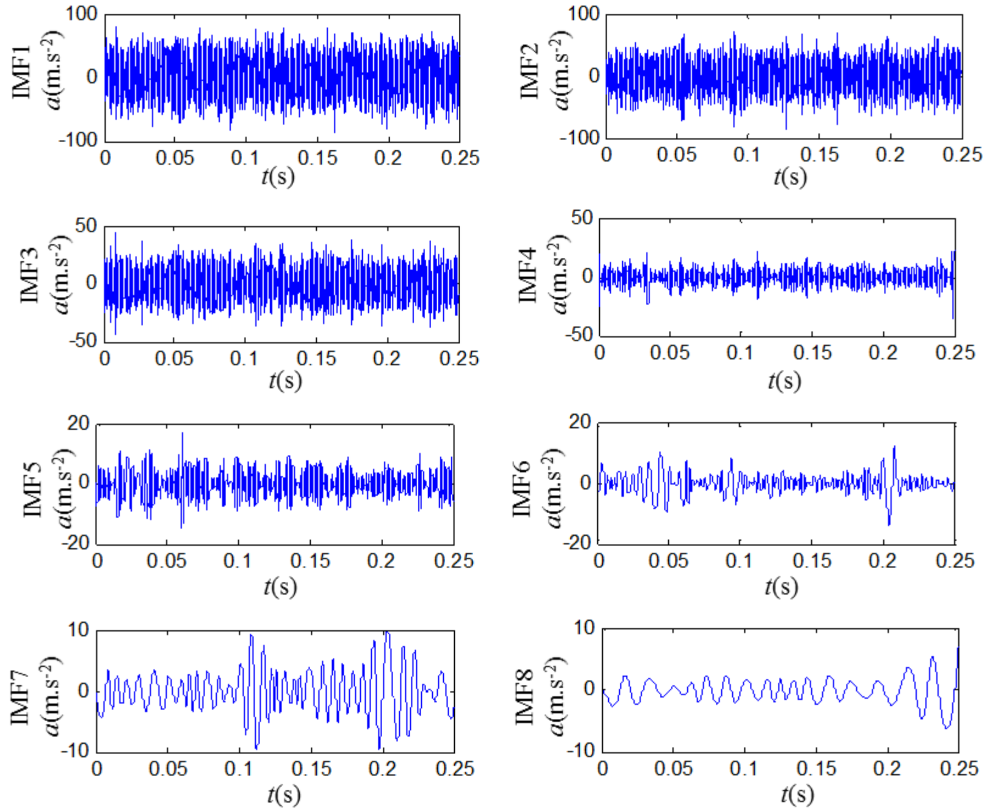


Fig. 2. EEMD decomposition results of normal signal



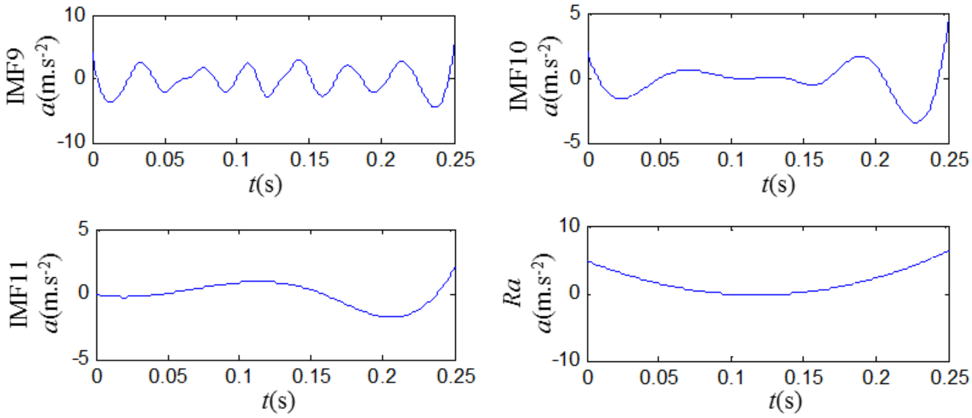
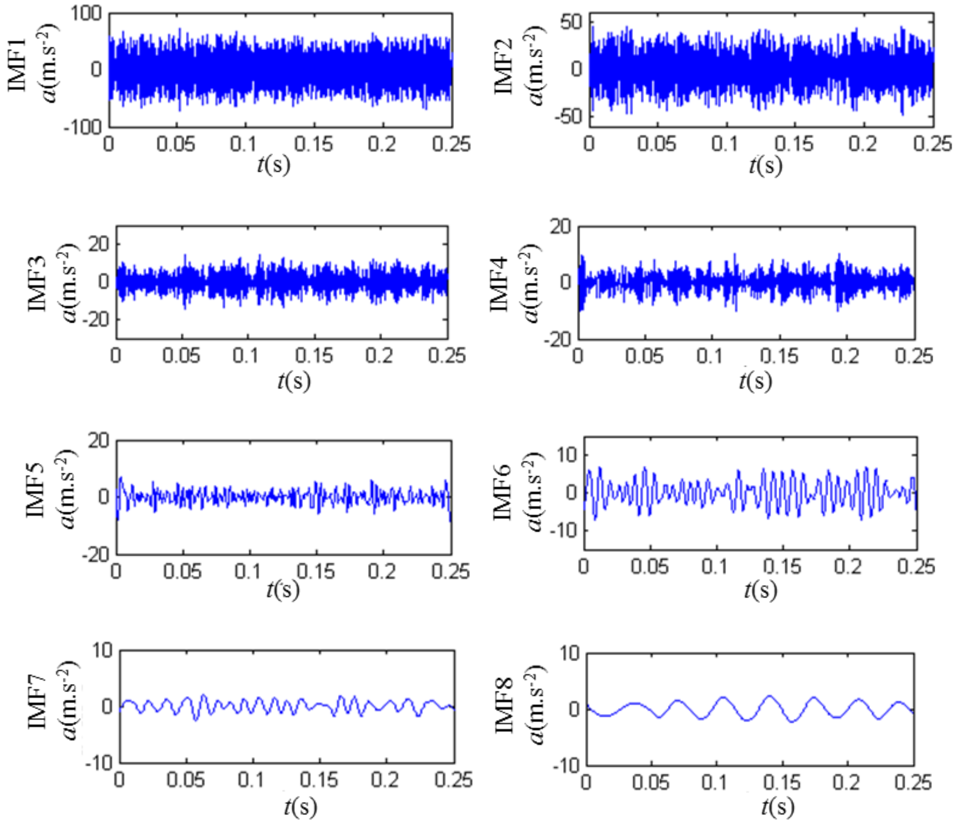


Fig. 3. EEMD decomposition results of mild wearing of tooth surface signal



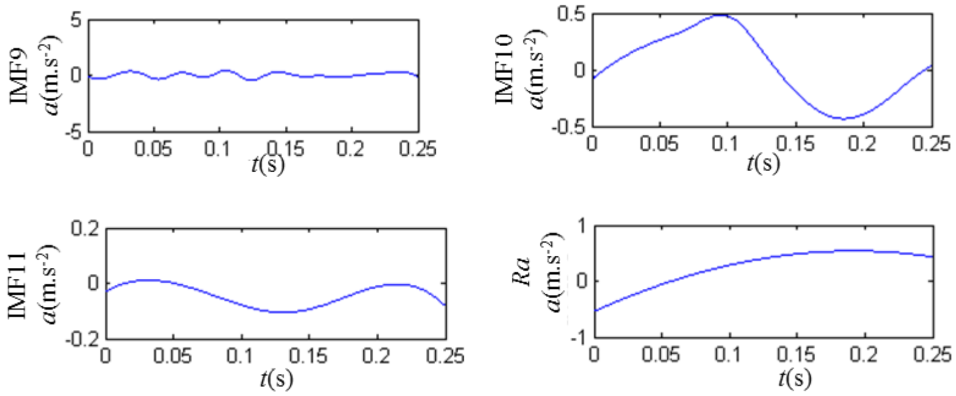
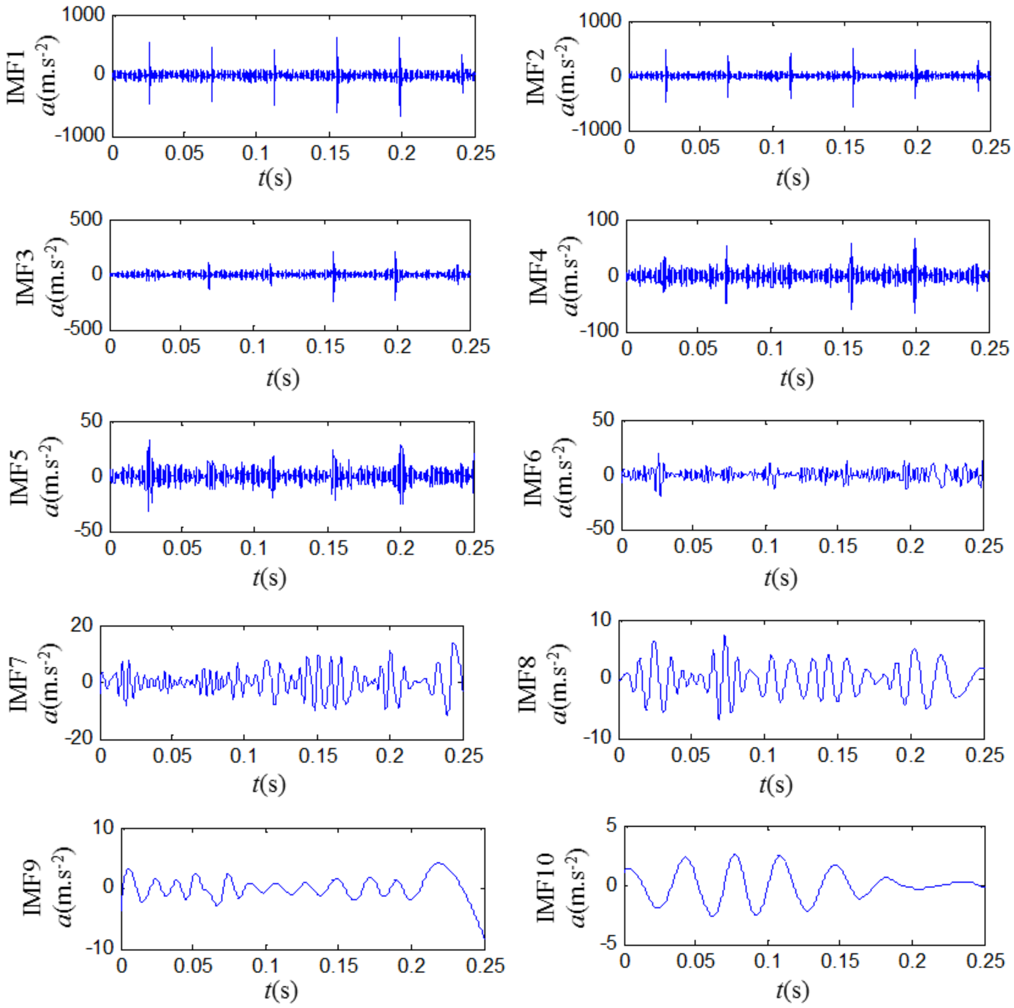


Fig. 4. EEMD decomposition results of moderate wearing of tooth surface signal



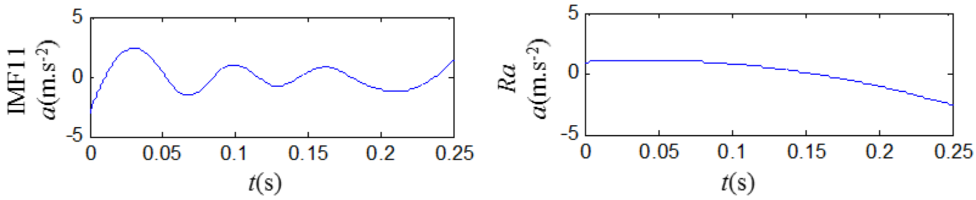
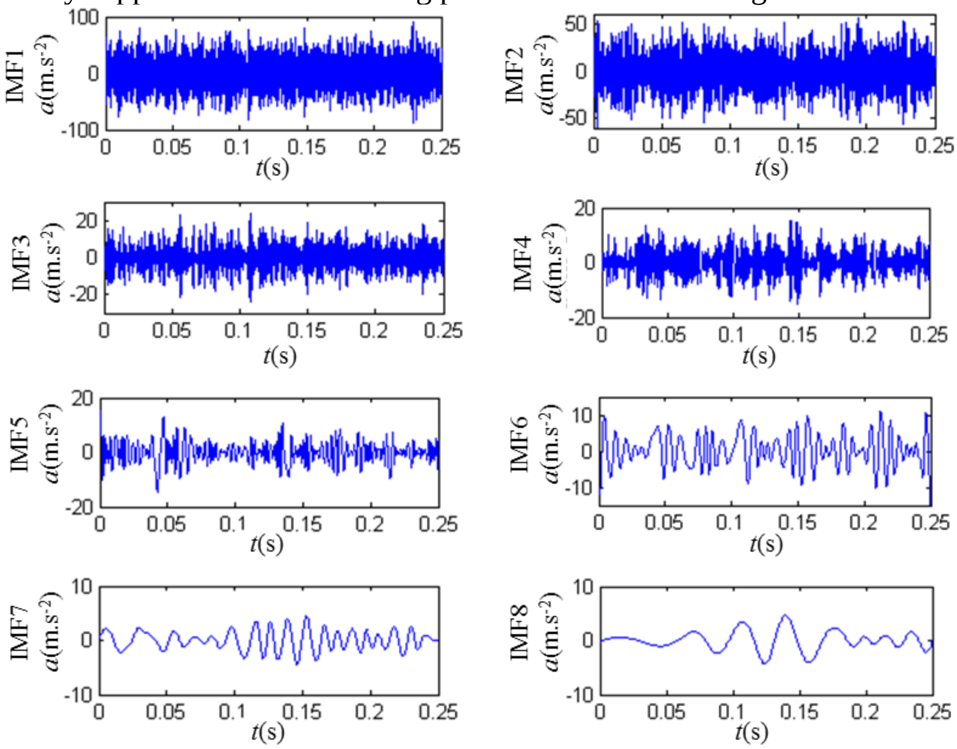


Fig. 5. EEMD decomposition results of tooth missing signal

We know that EEMD decomposes the non-stationary gear fault signal into several stationary IMF components, each IMF component contains different time characteristic scale and energy distribution. For easy comparison, Figure 6 shows the result of adaptive decomposition of moderate wear signal of the same gear by EMD method. In the same coordinate range, the sixth IMF component in Fig. 2 have less modal aliasing than the sixth IMF component in Fig. 6. The 7th and 8th IMF components in Fig. 4 and Fig. 6 are similar in waveform, but their amplitude fluctuation ranges are different, and the mode aliasing degree with small amplitude fluctuation is obviously lighter, which fully shows that EEMD can effectively suppress the mode aliasing phenomenon after adding white noise.



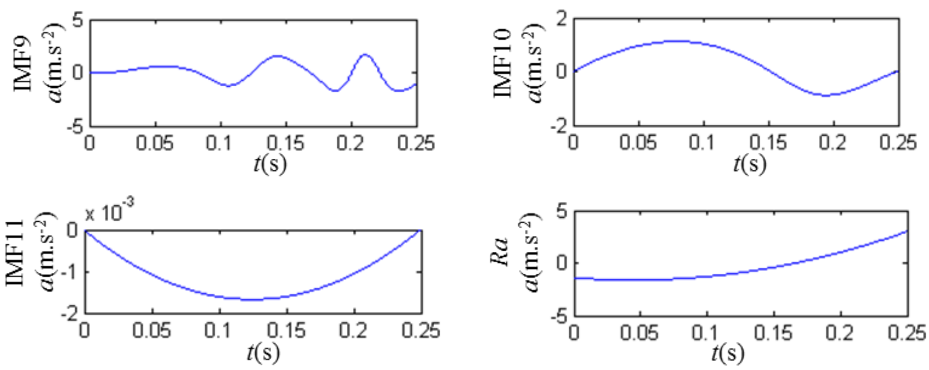
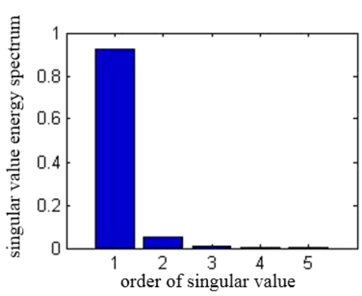
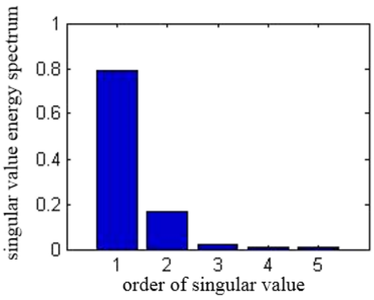


Fig. 6. EMD decomposition results of medium-worn signal

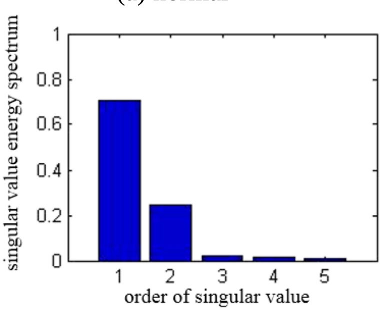
Ten samples of each state are randomly selected as training samples. According to the steps of fault feature extraction, the corresponding singular value energy spectrum distribution of each state can be obtained. As shown in Figure 7. The values in the figure are the average values of the singular value energy spectrum of 10 training samples. After many experiments, it is found that the first five orders of the energy spectrum vector of the singular value of the signal account for a large proportion, which indicates that the useful signals are concentrated in the first five singular values after the singular value decomposition. The proportion of other singular values is small, which also reflects that singular value decomposition can separate useful signal from noise. Therefore, the first five orders of singular value energy spectrum vector are taken as feature vectors for classification and recognition. From Figure 7 we know that the singular value energy spectrum of different gear fault states has obvious differences. Under normal conditions, the data is dominated by meshing frequency and rotation frequency, and the energy is concentrated in few modes. Therefore, after EEMD decomposition of the signal, the first order singular value obtained by SVD of the matrix composed of each IMF is larger, and other orders are smaller. With the appearance of gear fault, more frequency components appear in the vibration signal. Besides the first order singular value, the singular values of other orders also increase gradually. In the case of broken tooth fault, the main vibration signals are rotation frequency, higher harmonic and meshing frequency, but the frequency range of main vibration modes is narrower than that of wear fault. Therefore, the energy of the first singular value is more concentrated than that of the wear fault, and its value is between the normal state and the wear fault. Limited to space, three samples are randomly selected from the remaining 10 samples of each state as the samples to be tested. Table 1 shows the singular value energy spectra of the sample to be tested.



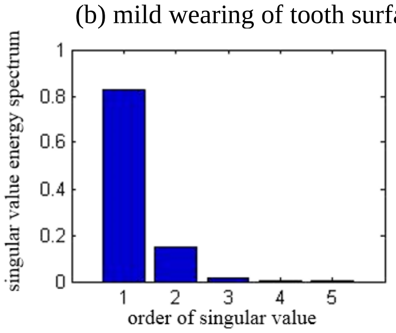
(a) normal



(b) mild wearing of tooth surface



(c) moderate wearing of tooth surface



(d) tooth missing

Fig. 7. Chart of singular value energy specturm of the first five order for gear's four conditions

Table 1

Singular value energy spectrum of the first five order for gear's four conditions

Working conditions	Sample	q_1	q_2	q_3	q_4	q_5
normal	1	0.9232	0.0604	0.0078	0.0032	0.0028
	2	0.9234	0.0607	0.0073	0.0031	0.0025
	3	0.9237	0.0598	0.0080	0.0030	0.0027
mild wearing of tooth surface	1	0.7858	0.1643	0.0227	0.0109	0.0080
	2	0.7924	0.1602	0.0235	0.0093	0.0073
	3	0.7897	0.1594	0.0230	0.0098	0.0081
moderate wearing of tooth surface	1	0.7020	0.2509	0.0207	0.0111	0.0098
	2	0.6977	0.2499	0.0225	0.0139	0.0099
	3	0.7103	0.2423	0.0210	0.0111	0.0090
tooth missing	1	0.7903	0.1837	0.0153	0.0040	0.0032
	2	0.8202	0.1493	0.0172	0.0052	0.0038
	3	0.8294	0.1383	0.0170	0.0053	0.0040

Finally, according to equation (6), the grey similarity correlation degree between the singular value energy spectrum of the sample to be detected and the standard eigenvector in each state is calculated. According to the value of similarity correlation degree, the fault identification is carried out. Table 2 shows the results.

Table 2

Grey similarity correlation between fault sample and standard fault pattern

Sample	normal	mild wearing of tooth surface	moderate wearing of tooth surface	tooth missing	Result
1	0.9580	0.6695	0.5700	0.6954	normal
2	0.9396	0.6514	0.5592	0.6937	normal
3	0.9460	0.6741	0.5381	0.7129	normal
4	0.8040	0.9652	0.8641	0.8063	mild wearing of tooth surface
5	0.8095	0.9461	0.8133	0.8098	mild wearing of tooth surface
6	0.8066	0.9410	0.8399	0.8045	mild wearing of tooth surface
7	0.7758	0.8269	0.9517	0.8262	moderate wearing of tooth surface
8	0.7702	0.8139	0.9718	0.8307	moderate wearing of tooth surface
9	0.7790	0.8270	0.9665	0.8355	moderate wearing of tooth surface
10	0.8102	0.6159	0.5447	0.9287	tooth missing
11	0.8178	0.7080	0.6633	0.9116	tooth missing
12	0.8010	0.7336	0.6747	0.8929	tooth missing

Table 2 shows that grey similarity correlation degree has achieved ideal effect on gear fault pattern recognition, which shows that the grey similarity correlation degree can accurately classify small sample fault recognition problems. The correct classification results can also be obtained by identifying the remaining 28 samples.

In order to test the advantages of grey similarity correlation analysis in small sample classification and recognition, the recognition performance of grey similarity correlation analysis and BP neural network is compared. Table 3 shows the results. It can be seen that in the case of small samples, the grey similarity correlation analysis has good classification and recognition ability.

Table 3

Recogniton performance comparison of BP neural network and grey similarity correlation

Recognition method	Training sample	Test sample.	Recognition performance %			
			normal	mild wearing of tooth surface	moderate wearing of tooth surface	tooth missing
BP neural network	10	10	100	70	80	100
Grey similarity correlation	10	10	100	100	100	100

5. Conclusions

(1) EEMD method is an adaptive signal processing method. By adding white noise sequence, the mode aliasing phenomenon in EMD decomposition is effectively solved, which is more conducive to signal feature extraction.

(2) Each IMF construction matrix after EEMD decomposition is decomposed by singular value decomposition. This can solve the problem that the number of rows and columns of SVD phase space reconstruction matrix is difficult to determine.

(3) The singular value of the signal is closely related to the energy of the signal. Each IMF obtained by EEMD contains frequency information of different frequency bands. Therefore, it is reasonable and feasible to classify different fault types by calculating the singular value energy spectrum distribution of each IMF.

(4) The method of grey similarity correlation makes up for the deficiency of traditional grey correlation method, eliminates the influence of the value of resolution coefficient, and can truly reflect the similarity between data series.

(5) Because EEMD singular value energy spectrum reflects the change of signal energy. To describe this change quantitatively, information entropy theory can be introduced to construct singular value entropy, and the gear fault types can be classified and identified by different singular value entropy.

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