A DESIGN METHOD FOR DETERMINNING NON-STANDARD RECTANGULAR SPIRAL SPRING STIFFNESS BASED ON SIMILARITY THEORY AND FEM METHOD

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The stiffness of non-standard spring, which is generally characterized by an unconventional helical shape, is a critical factor to consider in the design of the mechanical structure used in limited space. In this paper a rapid design method for rectangular spiral springs is presented based on stiffness similarity theory and finite element method. We assume a whole non-standard spring is divided into several spring segments, and calculate the elastic similarity coefficient of each segment relative to the standard spring based on the principle of similarity, and the formulation of the total stiffness of the nonstandard spring is finally obtained. This method has been verified by FEM simulation and physical tests. The stiffness performance simulation of the rectangular spring is done by using SolidWorks and ANSYS. On this basis, the elasticity of the rectangular spiral spring with that of conventional standard spring is compared. The designer can quickly and accurately obtain the performance parameters such as the stiffness and deformation of the rectangular spiral spring with the value of similarity coefficient δ. This research is of a certain practical and theoretical value, and provides a guiding method for the design of non-stand spring system.

Keywords: rectangular spiral spring; spring stiffness; stiffness similarity theory; finite element analysis

1. Introduction

Rectangular spiral spring is a non-standard spring, which is often used in limited workspace as an elastic suspension component, which mainly plays the role of buffering and shock absorption [1]. At present, there is no design theory directly related to the rectangular spiral spring. Designers usually design and select the rectangular spring by experience and finite element method. Some researchers [2-5] deduced the approximate calculation formula of large size non-standard dish spring, which is verified by comparing the result of finite element simulation with the result of formula calculation. Manuel Paredes studied the relationship between initial tension, length and load of a cylindrical tensile spring by means of simulation analysis and experiments [6]. The finite element method

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is often used to solve the problem of non-standard spring design, but there are also some problems, such as complex model, low simulation efficiency. In addition, as to the non-standard spring, there is no related fast design method [7-9].

In this paper, a design method of rectangular spring based on sub-model technology is proposed, and the single spring sub-model and theoretical model of rectangular spring are established. A similarity coefficient is introduced into modeling of the rectangular spring based on the standard spring stiffness formula, and the stiffness calculation formula of the rectangular spiral spring is obtained finally.

2. Calculation method of rectangular spiral spring stiffness

2.1 Methods for sub-model creation

Sub-model technology is mainly used to simplify the finite element solution model, which can effectively reduce the amount of calculation and time. The idea of this method is to decompose the object into several parts firstly, analyze the characters of parts, then transfer the results of the sub-model to the overall model, and finally solve the mechanical state of the model [10-13].

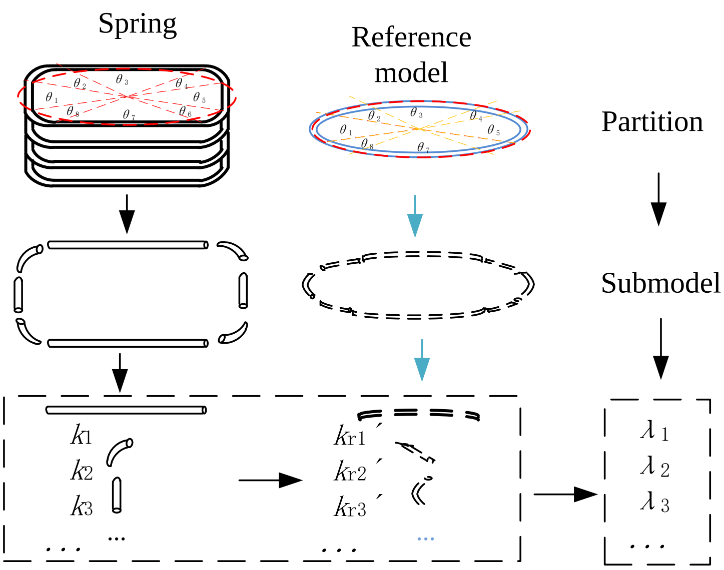

Fig. 1 Integral spring and sub-model theoretical model

It can be seen that the rectangular spring shown in Fig. 1 is structurally similar with normal spiral spring except that shape of each pitch. Therefore, we can decompose the total rectangular spring into several segments by their features, and calculate the stiffness of each part, then get the mechanical properties of the whole spring based on the similarity. The process of creating the sub-model is as follows [14-17].

Considering the stiffness of the normal spiral spring is always known and the elasticity similarity of two different springs, the complete model can be intercepted into sub-models as follows:

1) Divide the total rectangle spring into segments according to the geometrical features, and decompose the reference spring in the same way.

2) Analyze the stiffness of the rectangle spring parts and reference spring parts, and get the coefficient stiffness of the sub-model of the rectangle spring.

3) Calculate the stiffness of the overall spring based on the mechanical relationship between the sub-model and the overall model.

2.2 Stiffness of sub-model and integral spring

Each divided segment of the spring section is essentially a small elastic component. Therefore, the total spring can be thought of as a combination of several elastic elements with stiffness k_1, k_2 and k_i respectively and the mechanical model is shown in Fig. 2.

The relationship between stiffness of rectangle spring and reference spring is

$$k_i = \lambda_i k'_{ri} \tag{1}$$

where,

k_i: the stiffness of spring segment i;

k'_i: the stiffness of reference spring segment i;

λ_i: coefficient stiffness of spring segment i.

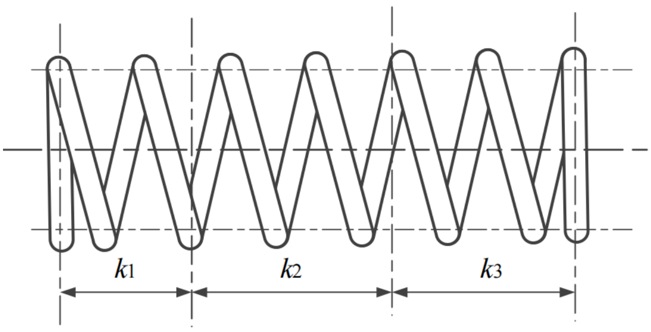

Fig. 2 Integral spring and sub-model theoretical model

Spring segments are connected with each other to form the integral spring, so the sum of the elastic deformation of each section of spring is equal to the deformation of the integral spring. Therefore,

$$\frac{F_1}{k_1} + \frac{F_2}{k_2} + \frac{F_3}{k_3} + \cdots = \frac{F}{k} \tag{2}$$

where,

F_i: The load on the spring segment i;

k: Stiffness of integral spring;

The force exerted on each spring segment is the overall load *F*,

$$F_1 = F_2 = \cdots = F \tag{3}$$

Then the relationship between the global stiffness and the sub-model stiffness is obtained.

$$\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots = \frac{1}{k} \tag{4}$$

If the stiffness of sub-models is equal, then the effective stiffness formula of single section spring is as follows.

$$k = k_i / n \tag{5}$$

where, *n*: The number of effective turns of the whole spring.

2.3 Simulation for rectangular spiral spring

Combined with the SolidWorks software modeling and the finite element calculation software ANSYS Workbench [18-21], the simulation operation flow of the spring sub-model is shown in Fig. 3.

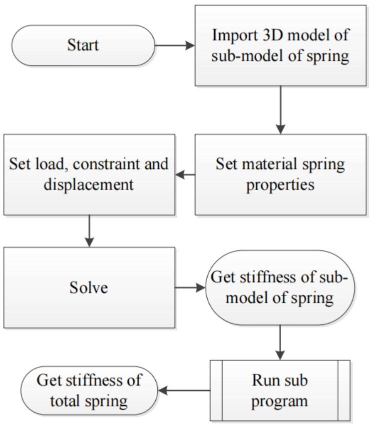

Fig. 3 Simulation operation flowchart

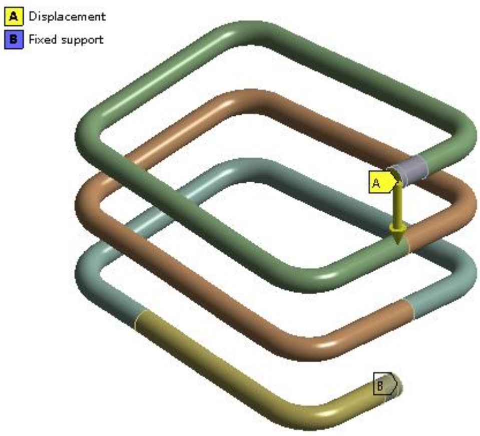

Fig. 4 Sub-model of spring imposed by Constraints

A fixed constraint is applied to the lower part of the spring sub-model, while the vertical displacement constraint is applied to the upper part. Fig. 4 shows the constraint imposed by the spring sub-model.

3. Experiments and verification

3.1 Verification of stiffness of rectangular spiral spring

Taking the rectangular spring shown in Fig. 5 as the research object, the stiffness test set-up is shown in Fig. 6.

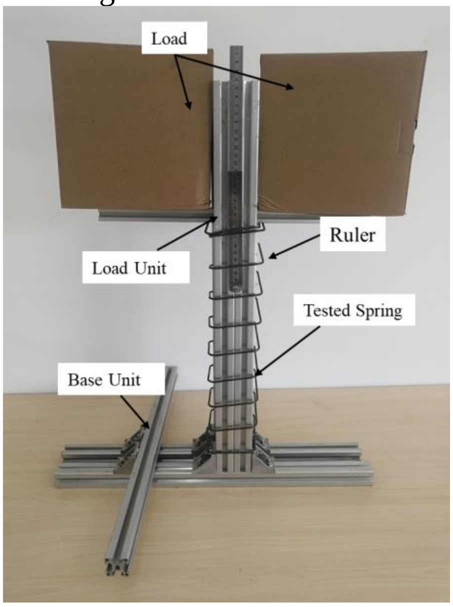

Fig. 5 Rectangular spring stiffness testing device

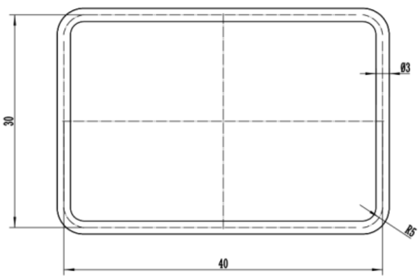

Fig. 6 Dimensions of rectangular spring

ANSYS Workbench was applied to carry out finite element simulation of the two models, and 12 mm displacement constraint was applied to the spring sub-model respectively, and 40 mm displacement constraint was applied to the overall spring model. Fig. 7 and Fig. 8 show the shear stress neutralism of the integral spring model and the sub-spring model. By comparing the data of the two figures, it can be seen that the maximum shear stresses of the spring sub-model and the overall model appears inside the rounded corners of the rectangular spring, which are 277.2 MPa and 284.12 MPa respectively, with an error of about 2.4%.

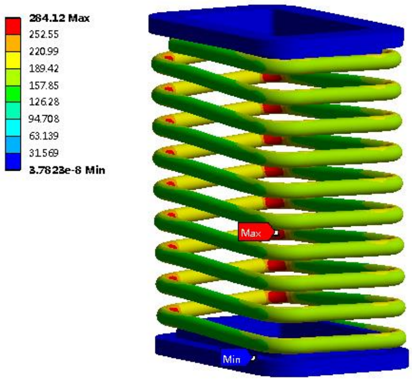

Fig. 7 Shear stress nephogram of whole spring

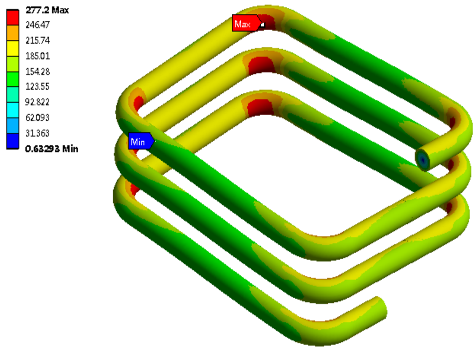

Fig. 8 stress nephogram of Spring Sub-model

Using formulas (6) and (8) are used to further analyze the finite element results of the sub-model, and the results calculated in 6 are compared with the analysis results of the global finite element method, as shown in Table 1. It can be seen from the comparison results in Table 1 that the error is about 1.0%, whether it is the overall stiffness of the spring or the reaction force.

Table 1

Comparison of calculation results between global model and sub-model

	Integral spring model	Spring sub-model	Error (%)
Effect of reaction (N)	49.907	49.394	1.0
Spring stiffness (N/mm)	1.248	1.235	1.0

Comparative analysis of computer usage of the two methods. As can be seen from table 2, using the sub-model analysis method, the number of nodes and the number of elements in the grid can be reduced by more than 72% , the computing time was reduced by 76.3 %, the memory footprint and the size of the resulting file were reduced by more than 74 %, and the reductions were significant.

Table 2

Comparison of computing resource usage between the overall model and the sub-model

	Integral spring mode	Spring sub-model	Percent
Node number	987383	267168	72.9%
Number of element	626512	141121	77.5%
Computation time (s)	131	31	76.3%
Memory footprint (MB)	4961.28	987	80.1%
Result file size (MB)	391.19	98.188	74.9%

Table 3

Comparison of stiffness between the model testing and simulation

	Testing (N/mm)	*Simulation(N/mm)*	*Percent*
1	*0.420*	*0.465*	*9.7%*
2	*0.435*	*0.465*	*6.4%*
3	*0.430*	*0.465*	*7.5%*
4	*0.432*	*0.465*	*7.1%*

The stiffness of testing and simulation data is shown in table 3. It can be seen that the error between the experimental data and the simulated data is maintained at 10%.

4. Case analysis and discussion

4.1 Comparison between rectangular spring and standard spring

65 Mn rectangular spiral spring and circular spiral spring are selected as the comparison objects, The geometric parameters and material parameters are shown in table 4 and table 5.

Table 4

Geometric parameters of spring

project	rectangular spring	circular spring
Dimensions	*88 mm × 66 mm*	*ϕ66 mm*
Fillet (mm)	5	-
Wire diameter (mm)	3	3
Pitch (mm)	25	25
Cycle number	3	3

Table 5

65Mn material parameters

project	numerical value
Density (kg/m^3)	7800
Elastic modulus (MPa)	198000
Shear modulus (MPa)	79000
Poisson's ratio	0.282

(a) Sub-model modeling and stiffness calculation

According to the design parameters in Table 3, 3D simulation modeling is established and performed. Fig.9 show the reaction force at the fixed constraint position of the two spring sub-models respectively.

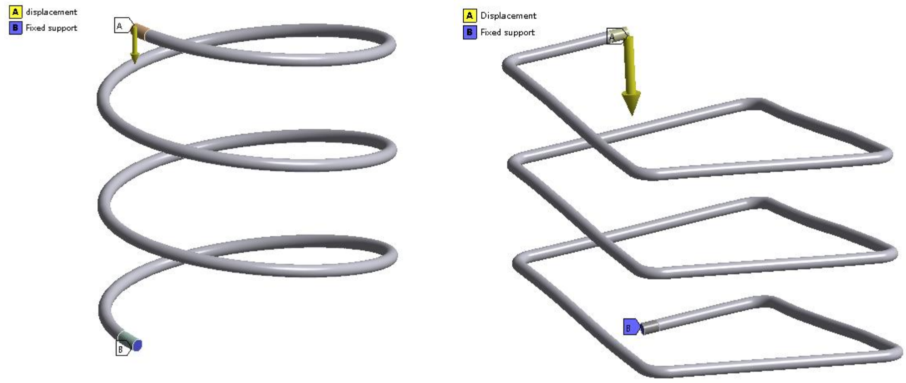

Fig. 9 The reaction force at the fixed constraint position of the circular spiral spring sub-model and the rectangular spiral spring sub-model

The reaction force at the fixed constraint position of the circular spiral spring sub model is 26.948 N in Fig. 9, the reaction force at the fixed constraint position of the circular spiral spring sub model is 11.041 N in Fig. 9. It is calculated by formula (3), the stiffness of the circular spiral spring sub model is 0.898 N/mm, The stiffness of the rectangular spring sub model is 0.368 N/mm.

(b) Comparative analysis of overall stiffness

From the previous step, through the simulation, the stiffness of the rectangular spiral spring is obtained 0.368 N/mm, the stiffness of circular spiral spring is 0.898 N/mm. As you can see, the rectangular spiral spring cannot meet the requirements of high elasticity, but compared with the circular spiral spring, the mechanical properties are poor.

Through simulation analysis, the sub model shear stress nephogram of rectangular spiral spring and circular spiral spring is shown in Fig. 10, as can be seen from the Fig. 10, the maximum stress distribution of circular spiral spring is more uniform inside the spring, the maximum stress of the rectangular spiral spring appears in the inner side of the bending position, compared with the conventional standard spring, the utilization rate of elastic parts is lower.

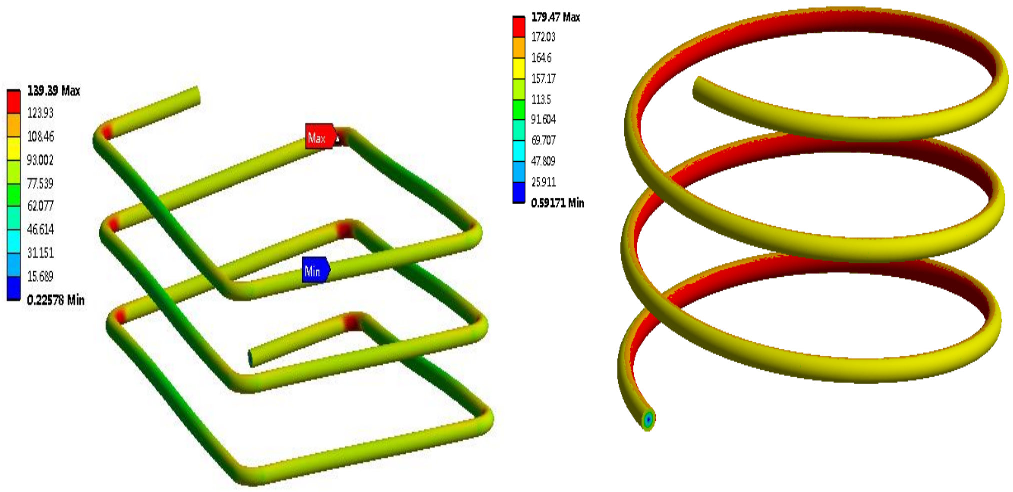

Fig. 10 Shear stress nephogram of rectangular spiral and circular spring sub-model

4.2 Similarity coefficient of rectangular spiral spring

Although the shape of rectangular spiral spring is different from that of standard circular spiral spring, the essence of both is to use the elasticity of material itself to play its role. If the fillet part of the rectangular spiral spring is enlarged continuously, the rectangular spring can become a circular spiral spring. Set k_r is the stiffness of rectangular spiral spring, k_c is the stiffness of circular spiral spring.

According to reference [11], the stiffness k_c of standard circular spiral spring is calculated as follows,

$$k_c = \frac{F}{S} = \frac{Gd^4}{8D^3n} \tag{7}$$

where,

F: The load on the spring;

s: The deformation of spring under load;

G: Shear modulus of spring material;

d: Wire diameter of spring wire;

D: Spring diameter;

n: Number of effective spirals of spring;

According to formula (7), the stiffness formula of rectangular spiral spring can be obtained as follows:

$$k_r = \frac{F}{S} = \lambda \frac{Gd^4}{8D^3n} \tag{8}$$

It can be seen from formula (8) that the similarity coefficient δ is the key parameter to calculate the stiffness of rectangular spring, which is directly related to the shape of rectangular helical spring.

Because the length-width ratio of rectangular spiral spring is an important structural parameter of rectangular spiral spring, the calculation result of the similarity coefficient δ can be obtained by calculating the ratio of the overall stiffness of the rectangular spiral spring with length-width ratio to standard circular spring with equal diameter.

A rectangular spiral spring with the length-width ratio of 4:3 and the diameter of steel wire of 3 mm is used to establish several groups of models of rectangular springs and standard springs. Through simulation and comparative analysis, the calculation results in Table 6 are obtained.

Table 6

Stiffness comparison of circular spiral spring and rectangular spiral spring

diameter of circular spiral spring (mm)	50	60	70	80	90
stiffness of circular spiral spring	3.160	1.835	1.160	0.775	0.545
rectangular spiral spring length* width (mm*mm)	66.7×50	80×60	93.3×70	106.7×80	120×90
stiffness of rectangular spiral spring (N/mm)	1.315	0.755	0.475	0.317	0.222
similarity coefficient δ	0.416	0.411	0.409	0.408	0.407

It can be seen from Table 6 that the maximum similarity coefficient δ is 0.416, the minimum is 0.407, take the average value as 0.410, Substituting δ = 0.410 into formula (8), we get,

$$k_r = \frac{F}{S} = 0.410\,\frac{Gd^4}{8D^3 n} \tag{9}$$

Substituting the sub-model parameters of rectangular helical spring in Section 2.2 into formula (9), and get the stiffness k_r = 0.380 N/mm, which is 3.2% less than 5% compared with the simulation result of 0.368 N/mm, within the allowable error range.

5. Conclusions

(1) The calculation time and operation of the method based on the rectangular sub model can be greatly reduced.

(2) Under the same utilization space, the utilization ratio of rectangular spring is lower than that of conventional standard spring, and the rigidity of rectangular spring with the length-width ratio of 4:3 is about 0.4 times that of conventional standard spring.

(3) According to the characteristics of rectangular spring and the stiffness calculation formula of conventional standard spring, the stiffness calculation formula of rectangular spiral spring proposed in this paper can effectively guide

designers to quickly estimate the performance parameters such as stiffness and deformation of rectangular spiral spring.

REFERENCES

[1]. *Yan Hongwei, Sun Ming*, "Design for Rectangular Spiral Spring Automatic Molding Equipment" ,Journal of Machine Design and Research, vol. 32, no. 6, 2016, pp. 148-152

[2].*Singh, HarmeetBrar, Gurinder Singh,* "Characterization and Investigation of Mechanical Properties of Composite Materials used for Leaf Spring", 7th International Conference of Materials Processing and Characterization (ICMPC), vol. 5, no. 2, 2017, pp. 5857-5863

[3]. *Jenarthanan, M. P.Ramesh, Kumar. S.Venkatesh, G.Nishanthan, S.,* "Analysis of leaf spring using Carbon / Glass Epoxy and EN45 using ANSYS: A comparison", International Conference on Advanced Functional Materials (ICAFM), vol. 5, no. 6, 2018, pp. 14512-14519

[4]. *Pulkit Solanki, Ajay Kumar Kaviti,* "Design and computational Analysis of Semi-Elliptical and Parabolic Leaf Spring",8th International Conference on Materials Processing and Characterization (ICMPC), vol. 5, no. 9, 2018, pp. 19441-19455

[5]. *He-chuan Song, Yi-du Zhang, Qiong Wu, et al*, "Low-stiffness spring element constraint boundary condition method for machining deformation simulation", JOURNAL OF MECHANICAL SCIENCE AND TECHNOLOGY, vol. 34, no. 10, 2020, pp. 1-12

[6]. *Krishan Kumar, M.L. Aggarwal,* "Optimization of Various Design Parameters for EN45A Flat Leaf Spring", 5th International Conference on Materials Processing and Characterization (ICMPC) ,vol. 4, no. 2, 2017, pp. 1829-1836

[7]. *Radhakrishnan, K.Antony, A. GodwinRajaguru, K.Sureshkumar, B.,* "Torsional vibration analysis of torsion bar spring for off road vehicle driver seat ", International Conference on Recent Trends in Nanomaterials for Energy, Environmental and Engineering Applications (ICONEEEA) ,vol. 21, no. 1, 2019, pp. 669-672

[8].*Wahed, M. A.Gadi, V. S. R.Gupta, A. K.Supradeepan, K.Singh, S. K.Kotkunde, N. R.,* "Finite element analysis of spring back in Ti-6Al-4V alloy",9th International Conference of Materials Processing and Characterization (ICMPC), vol. 18, no. 7, 2019, pp. 2693-2699

[9]. *Niranjan Hiremath, K.S,*Tarun, "Static and modal analysis of diaphragm spring used in micro depth sensing indenting Machine",3rd International Conference on Recent Research Emerging Trends in Materials and Mechanical Engineering (ICRRETMME), vol. 20, no. 2, 2019, pp. 161-166

[10]. *R. Naresh, V.B.S. Rajendra Prasad, G. Venkata Rao,* "Static and Transient Response of a Leaf Spring with EPDM Rubber Sandwiched Between Steel Leaves",2nd International Conference on Materials Manufacturing and Modelling (ICMMM), vol. 22, no. 4, 2020, pp. 3250-3260

[11]. *Akhileshwar Nirala, Navneet Kumar, Desh Bandhu Singh, et al,* "Simulation analysis of composite helical spring for compression, torsional and transverse mode", 2nd International Conference on Advances in Mechanical Engineering and Nanotechnology, ICAMEN 2020, vol. 28, no. 4, 2020, pp. 2263-2267

[12]. *Seralathan, S.Prasanna, HemanthArun, J.Guhanesh, B.Prasanth, D.Hariram, VPremkumar, T. Micha,* "Finite element analysis of hybrid composite material based leaf spring at various load conditions", International Conference on NanoTechnology-Ideas, Innovation and Industries, vol. 33, 2020

[13]. *Huang Hongduan, Yuan Daimin, Wei Chaozhong, Lu Yichu,* "Strength Calculation and Fatigue Life Prediction of Stopper Leaf Spring in Sliding Door ",Automobile Parts, vol. 19, 2019, pp. 50-52

[14]. *Geng Kaihe, He Jingliang, Chen Yong, Li Juanmei,* "Fatigue optimization analysis of torsion bar spline based on sub-model",Journal of Beijing Information Science & Technology University, vol. 34, no. 4, 2019, pp. 170-75
[15]. *Paredes, ManuelStephan, ThomasOrcière, Hervé,* " Enhanced Formulae for Determining the Axial Behavior of Cylindrical Extension Springs ", ArXiv, 2019
[16]. *Liu Zhipeng, Zhou Jie, Wang Shilong, Wang Sibao, Yang Wenhan,* "Fatigue Life Prediction of Stranded-Wire Helical Springs Based on Finite Element Method",China Mechanical Engineering
[17]. *Zhao Linyan, Shi Yuqing, Jiang Chunxia,* "Research on the Dynamic Characteristics of the Piecewise Smooth Vibration System with a Clearance and Subjected to Friction", Journal of Lanzhou Jiaotong University, vol. 33, no. 3, 2014, pp. 41-45
[18]. *Wang H, Li A Q, Hu R M, et al,* "Accurate stress analysis on steel box girder of long span suspension bridges based on multi-scale submodeling method",Advances in Structural Engineering, vol. 13, no. 4, 2010, pp. 727-740
[19]. *Zhou Ze, Li Guangyao, Cheng Aiguo, Zhao Min,* "A New Method for Body Structure Analysis Based on Stress Recovery of Submodel",China Mechanical Engineering, vol. 24, no. 12, 2013, pp. 1671-1675
[20]. *Xie Zhigang, Chen Xiaoqin,* "Finite Element Analysis and Theoretical Calculation of Stress and Strain for Large Non-standard Disc Spring" , Mechanical Engineer, no. 5, 2014, pp. 143-145
[21]. *Mao Jiabing.* "Calculation of stress intensity factor of stiffened wall panel based on Abaqus and Submodel analysis",China Science and Technology Information, 2016， pp. 37-38+23.