NUMERICAL AND ANALYTICAL STUDIES OF CYLINDER / PLANE CONTACT WITH PRESENCE OF DEFECTS IN CERAMICS

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In our work, the effect of presence of defects in the contact between bodies was study. For that the cylinder/plan contact was investigate by using analytical and numerical approach. The results found from Hertz theory and finite element method (FEM) were compared. Two studies were conducted, first between two pairs of biomaterials with and without defects presence. Secondly, the influence of the defects shape and theirs distribution in the material was investigate. Finally, the analysis of the results found from theory and simulation is presented for predicted possible errors, which can occur by applying the analytical formula.

Keywords: Defects, Hertz theory, Hip prostheses, Tribology, Biomaterials, Porous ceramics.

1. Introduction

Ceramics have improved the behavior of artificial articular joints such as hip prostheses [1]. In addition, their chemical inertness and good tribological properties give them an advantage over metals or polymers for a bearing component such as a head of hip prosthesis. However, these materials maintain drawbacks such as defects [2].

Porous ceramics offer a broad range of characteristics that enable them to be used in a wide variety of applications [3]. By selecting a suitable base material for the intended use, and then adjusting the overall size defects, distribution and shape, they can be tailored to suit a diverse range of applications. This generally requires close consultation between the ceramics manufacturer and the customer or user.

This paper aim is to study of the presence of effect (pore) in cylinder /plane contact especially in ceramic material. For that Hertz theory and finite element method was used to investigate the effect of size, shape and distribution of defects in the material on the contact area, contact pressure and von Mises stress between the two bodies [4].

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2. Effect of defects in material strength

The study of the mechanical properties of porous materials generally comes down to determining a correlation between the defects and the mechanical properties: an increase in the volume fraction of pores implies a decrease of the mechanical properties, see figure1. It is recognized that microstructural parameters such as pore shape, size distribution, and the nature of the interconnections between solid particles also influence the correlation between mechanical properties and defects [5-6].

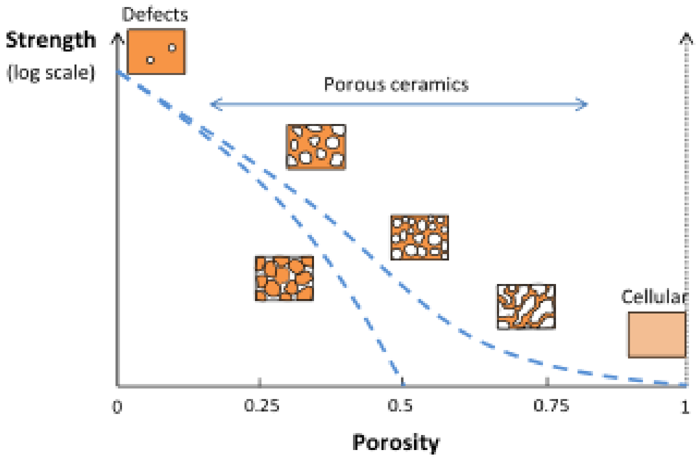


Fig. 1. Defects influence on the mechanical properties of porous material. The two dotted curves describe the evolution of the properties of two different microstructures [7].

3. Static Hertz contact model for cylinder-plane

Hertz was one of the first to study the case of contact of any two surfaces to one another [8]. Hertz assumes that the depth constraints do not exceed the elastic limit of the material and that there is no friction. This allows to calculate analytically the pressure distribution that satisfies the boundary conditions of the massifs, inside and outside the contact area. The singular study of the cylinder / plane contacts is done by considering the plane as a surface with an infinite radius of curvature. In this particular case of contact, we consider a linear loading on a semi-infinite mass. In this case, the problem comes down to the study of a two-dimensional contact and the distribution of the pressure is constant along the y axis [9, 10], see figure 2.
Hertz has expressed the characteristics of the contact by:

Half-width of the contact[m]: $$a = 2\sqrt{\left[\frac{\mathrm{Ln.R}}{\pi.E^*}\right]} \quad (1)$$

Maximum Hertz pressure [Pa]: $p_0 = 2\sqrt{\left[\frac{Ln.E^*}{\pi.R}\right]} = \frac{2.Ln}{\pi.a} = \frac{4}{\pi} p_m$ (2)

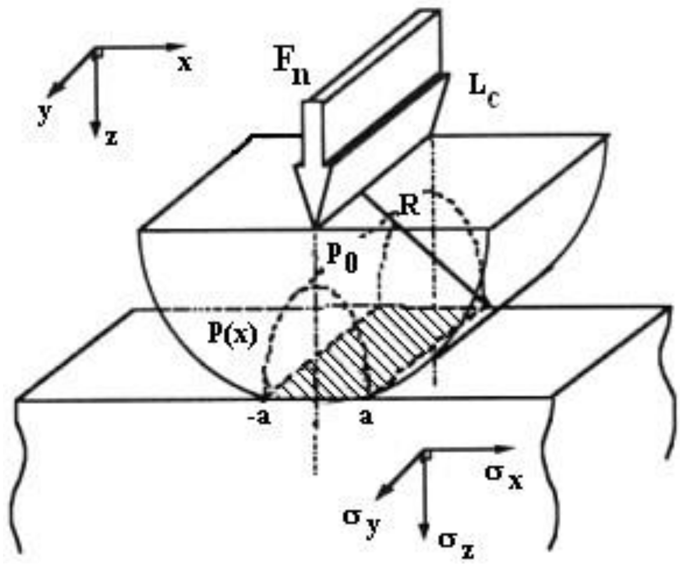


Fig. 2. Particular Hertz contact of type cylinder / plane [9, 11]

Medium pressure: $p_m = \frac{\pi}{4} p_0$ [Pa] (3)

Normal load per unit length: $Ln = \frac{\pi.R}{E^*} p_0^2 = \frac{F_N}{L_c}$ [N/m] (4)

Dent[m]: $\delta = \frac{a^2}{R} = \sqrt{\left[\frac{3F_N.\mathrm{R}}{4.E^*}\right]}$ (5)

Stress: $\sigma_{xx} = p_0.\frac{z}{a}.\left\{2 - \frac{s}{\sqrt{a^2+s^2}} - \frac{\sqrt{a^2+s^2}}{s} - \frac{x^2.s^3a^2}{(a^2+s^2)^{3/2}.(s^4+z^2.a^2)}\right\}$ (6)

$\sigma_{yy} = \frac{2.z.\mu.p_0}{a}.\left\{\frac{\sqrt{a^2+s^2}}{s} - 1\right\}$ (7)

$\sigma_{zz} = -p_0.\frac{z^3.a.\sqrt{a^2+s^2}}{s.(s^4+z^2.a^2)}$ (8)

$\sigma_{xz} = -p_0.\frac{x.z^2.s.a}{\sqrt{a^2+s^2}.(s^4+z^2.a^2)}$ (9)

$\sigma_{xy} = \sigma_{yz} = 0$ (10)

$s^2 = \frac{1}{2}\left[\{(a^2 - z^2 - x^2)^2 + 4.a^2.z^2\}^{1/2} - (a^2 - z^2 - x^2)\right]$ (11)

With :

R: represent the radius of cylinder;
E: Young modulus of material;
E^*: Effective young modulus.

4. Contact modeling by the Finite element method

A simple contact problem occurs when, for example, one elastic curved body with smooth surface comes in contact with no friction with an elastic plane smooth surface under static conditions [4]. The area of contact is a function of the applied load. With the change in contact area with load, the extent of contact is a priori unknown, rendering the problem nonlinear. However, it is of a reversible nature due to the absence of non-conservative forces.

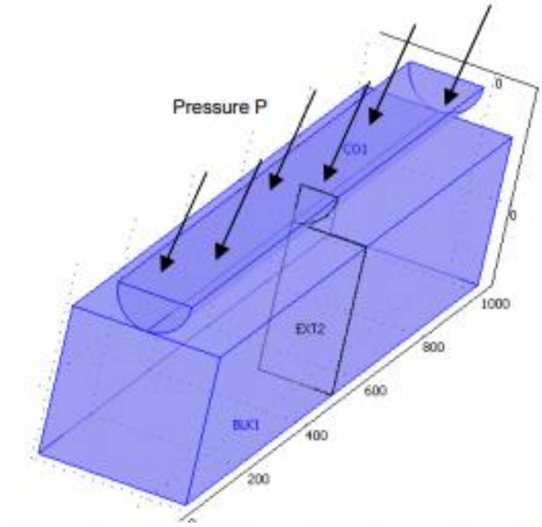


Fig. 3. Static Hertz contact model [12]

A problem of elastic contact between ceramics satisfying radio conditions is simulated with Comsol Multiphysics [13]. The objective is to interpret the contact results of the different study cases by analytical values determined from the Hertz formulation and the numerical computation [14]. The numerical model show the contact between a half-cylinder made with ceramic on a porous ceramic block, as shown in figure 3. The two materials are supposed to be elastic, homogeneous and isotropic. In addition, there is no friction and the small deformations is the supposition of this problem. The conditions of plane deformation are considered.

5. Model Definition

In this investigation, a plane strain problem is considered. In all the cases, we choose a plane cylinder model in contact. All analyzes have been performed with variation of pore shapes, see figure 4. The distribution and distances between the defects are reported in table 1 and 2.

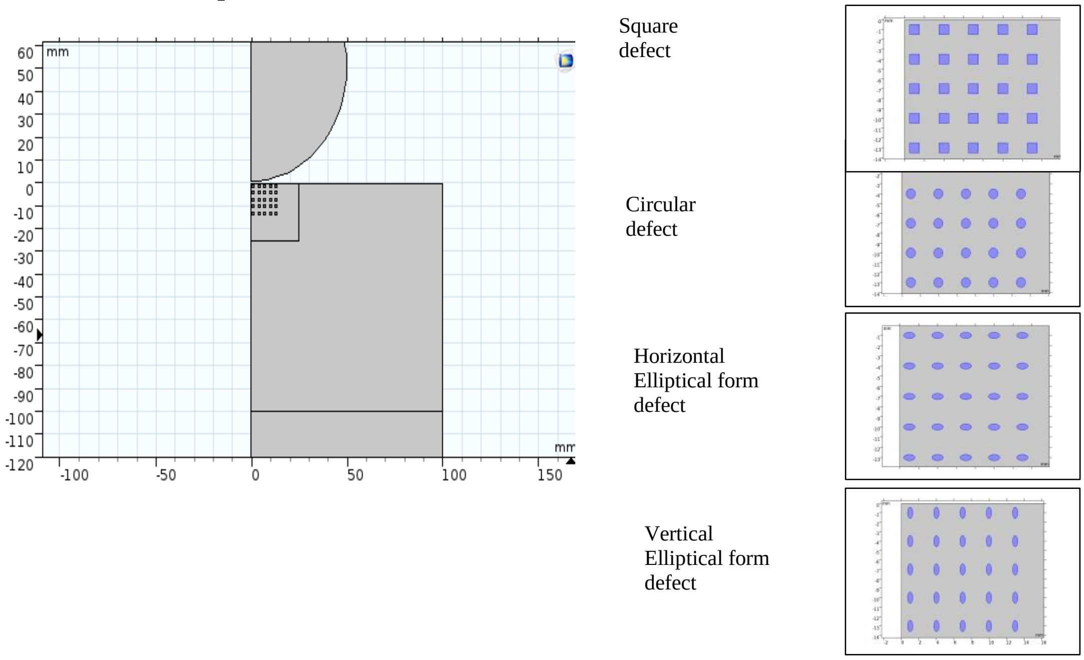


Fig. 4. 2D structures contact

Table 1

The distance between pores for the cases of the models of studies

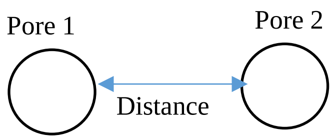
Cases	**Distance (mm)**	
case 1	3 mm	
case 2	3,5 mm	
case 3	4 mm	
case 4	4, 5 mm	

Table 2

Pore size for study model cases

Cases	**Dimension (mm)**
Square defect	1x1
Circular defect	R=0.5
Horizontal Elliptical form defect	0.6x0.3
Vertical Elliptical form defect	0.3x0.6

The materials assigned to the model from the Comsol software database. Another possibility is to create our own material depending on the physics to study, as our case study. The biomaterial used in our study is ceramics. The mesh and boundary conditions applied in this study are showed in figure 5.

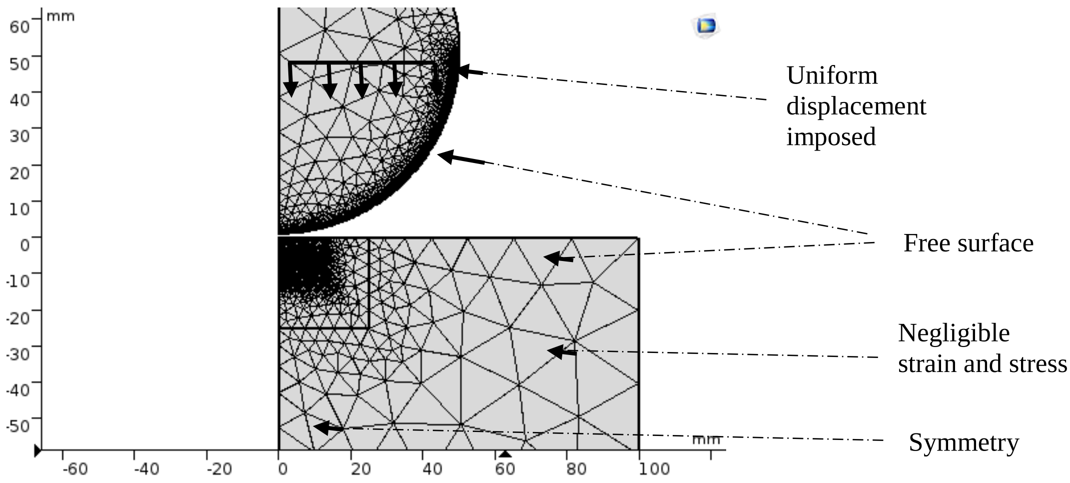


Fig. 5.Geometry study

6. Results and discussions

For the interpretation of the results, we present the results of the simulation of the holy contact, showing the variation of the von Mises stress, thus the graphical representation of the numerical with analytical Hertz pressure, and finally we represent the four cases of studies with and without defects.

For contact pressure results, we note that the maximum contact pressure is found in square cases, 2, 3 and 4. The minimum contact pressure is found in the case of vertical elliptical form. Therefore, we note for the case of the circular pores that there is a uniform growth in contact pressure from one case to another case according to the distance between the pores.

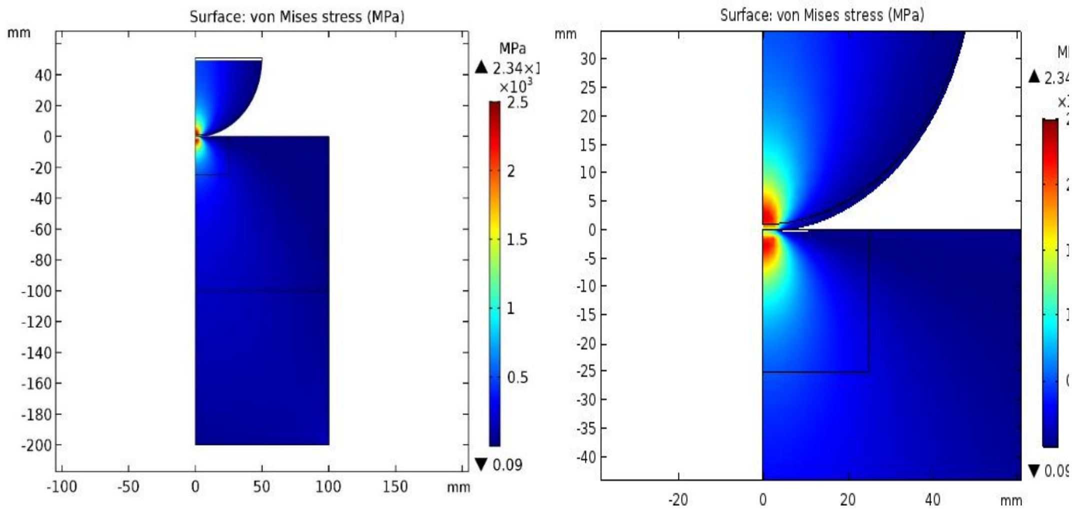


Fig. 6. The von Mises stress map in contact without defects

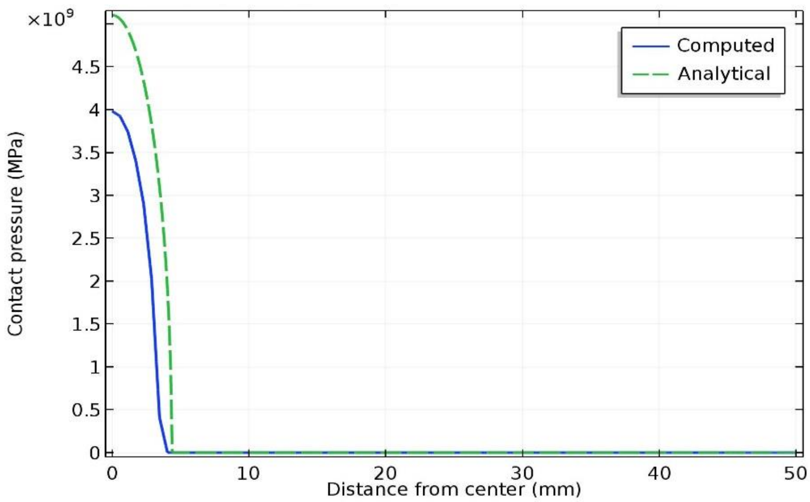


Fig. 7. Analytical and numerical results of contact pressure for the case without defects

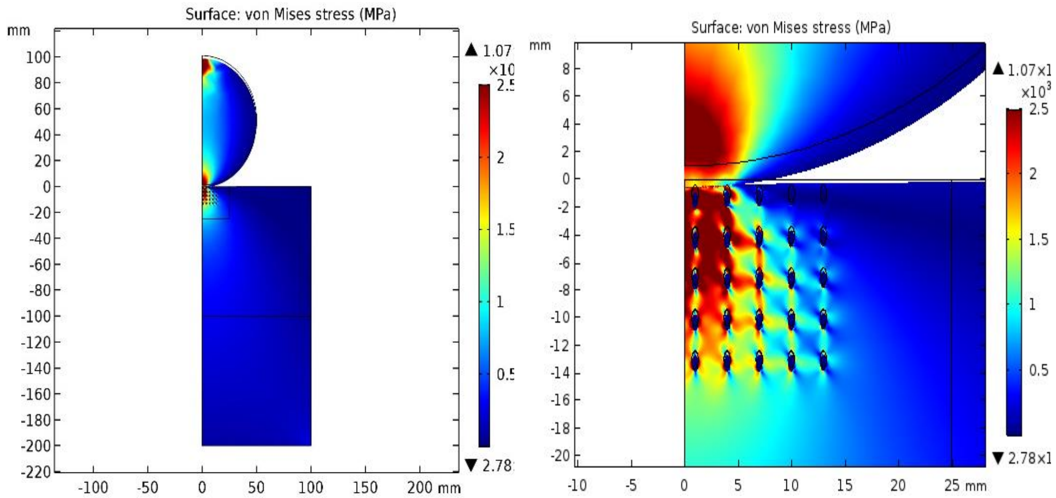


Fig. 8. The von Mises stress map in contact with defects "Vertical ellipse 1 case"

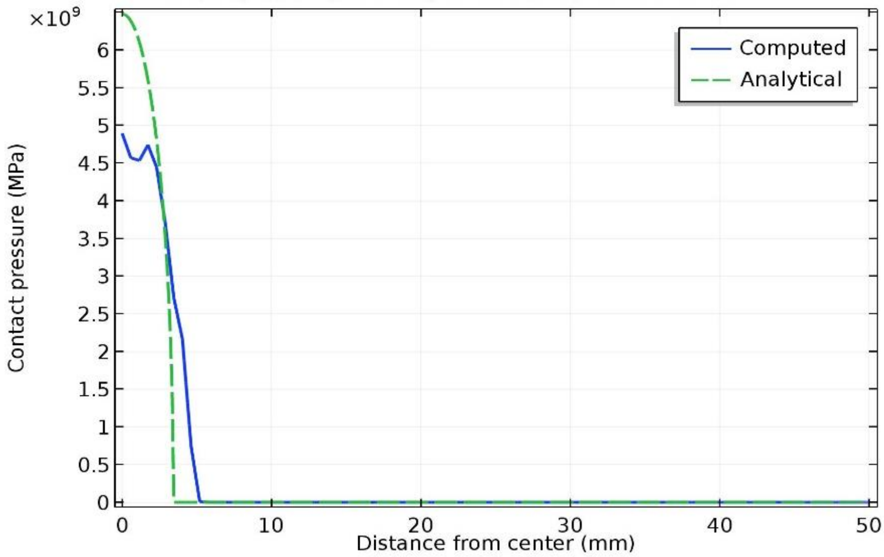


Fig. 9. Analytical and numerical results of the contact pressure for "Case 1 Vertical elliptical form"

In general, the value of the contact pressure for all the cases studied with defects is less than that free of defects, see figures 7, 8 and 9., except for cases of square pores in cases 2, 3 and 4 and those of pores in the circular form from case 4, see figure 10.

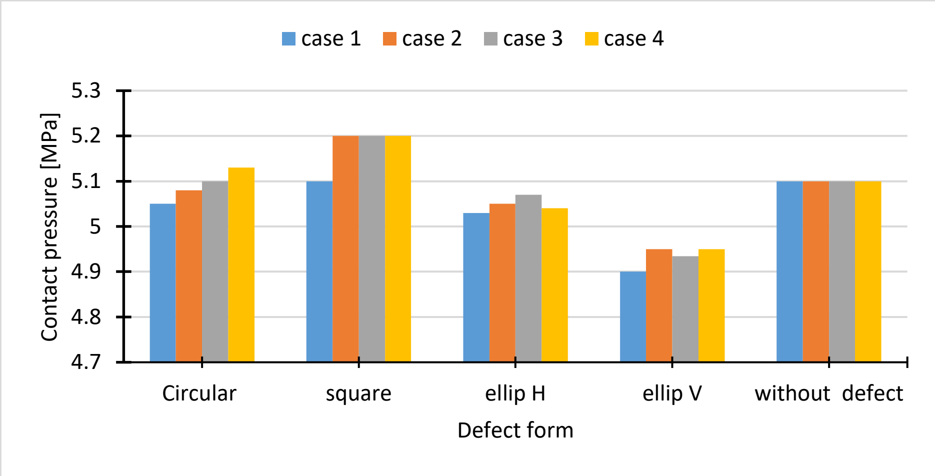


Fig. 10. Numerical results of the contact pressure

For von Mises stress, the value in the cases without defects, see figure 6, is lower than the stress of the four cases with defects. We also note that the maximum value of the von Mises stress is found in the cases of horizontal elliptical form, and the minimum value is in the case of a square form, see figure 11.

For the effect of the distance between pores, whatever its shape, does not affect the value of the constraint "remains constant", except in the case of vertical and horizontal elliptical form.

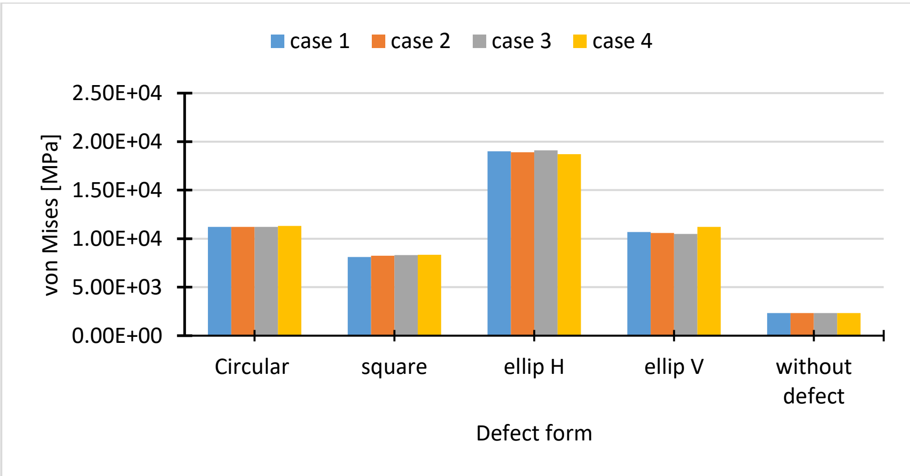


Fig. 11. The von Mises stress for all the cases with and without defects

For the results of the contact length, the contact radius between the cylinder body and the porous plane is strictly superior to the contact radius without defects see figure 12.

. The maximum value of the contact radius is in the case of square form, in cases 1 and 2, as well as the horizontal elliptical form, in cases 1 and 2.We also note that the contact radius values for the cases, circular and elliptic forms, are almost equal. Finally, the results show that the distance between the pores, whatever their shape, has no effect on the length of the contact, in the all the cases treated.

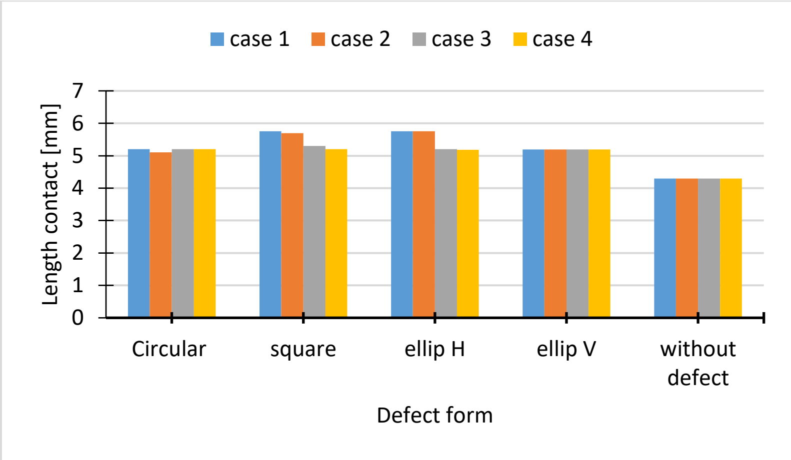


Fig. 12. Representation of numerical results of the contact length

Comparison between numerical and analytical solution:

For the comparison, the Hertz solution [8] is also plotted in the same graph for proving the precision of the Finite elements solution.

For the first case, contact without defects, the analytical solution proposed by Hertz in [8] and equation (3) gives an over-evaluation of the contact pressure, compared to the numerical solution; on the other hand, the length of contact remains almost the same, see figure 9.

For the second case of study with defects, we note an overestimation of the analytical contact pressure compared to the results of finite elements method and an underestimation of the analytical contact length compared to the numerical study by FEM.

From the results, we note that the expression proposed by hertz in [8] cannot represent all the cases of contact especially the contact between bodies with defects or with non-linearity in geometry.

The finite element method can gives response to those problems of contact with defects or with non-linearity in geometry.

Effect of distance between pores and the free surface:

To see if the distance of the pores with respect to the center of the contact surface influences the maximum stress and the pressure, a complementary case study is made for the case of study with low contact pressure "1st case vertical defects", by minimalizing the distance that separated the pores and the free surface, see figures 13 and 14.

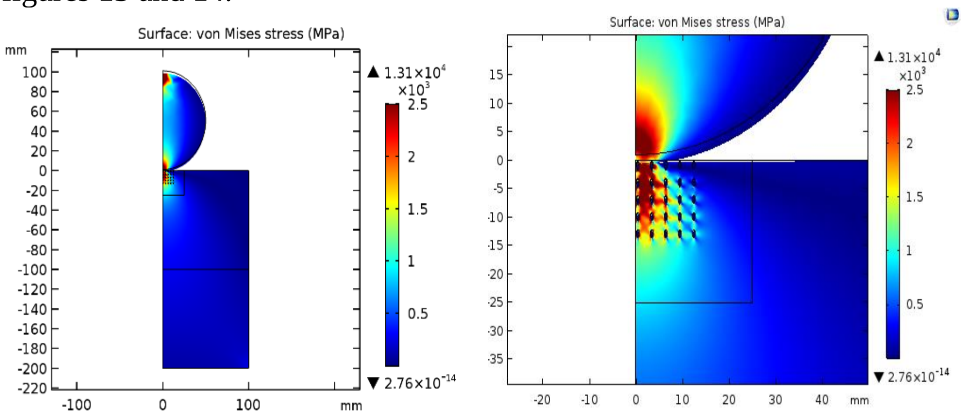


Fig. 13. The von Mises stress map in contact with defects "vertical elliptical form 1 case"

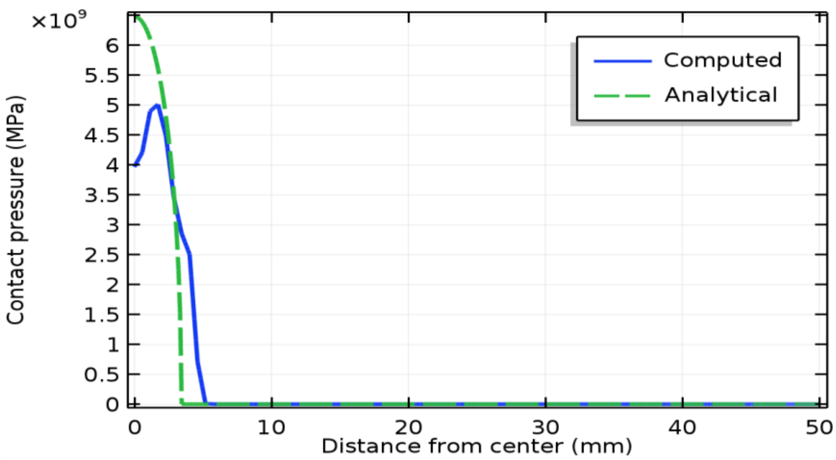


Fig. 14. Analytical and numerical of the contact pressure in case 1 for vertical elliptical pore

The von Mises stress increases considerably when the pores approaches the center of the contact surface, increases proportionally until it reaches the actual wear, despite Hertz assumes that the depth stresses do not exceed the elastic limit of the material and that it there is no friction.

The initial pressure ($p_0 = 4x10^9$MPa), in the case approaching the center of contact area is less than that of the Vertical elliptical form 1 case, but the maximum pressure is almost the same thus the radius of contact.

Effect of element type and mesh density

The sensibility study was done to know the variation of results as function of mesh density and elements type. We note that element type affect only the von Mises stress with a difference between free triangular and free quadrature elements about 0.02e4 MPa. Contact pressure and contact length are insensible to the variation of elements type.

Figures 15, 16 and 17 show the variation of von Mises stress, contact length and contact pressure as the function of mesh density.

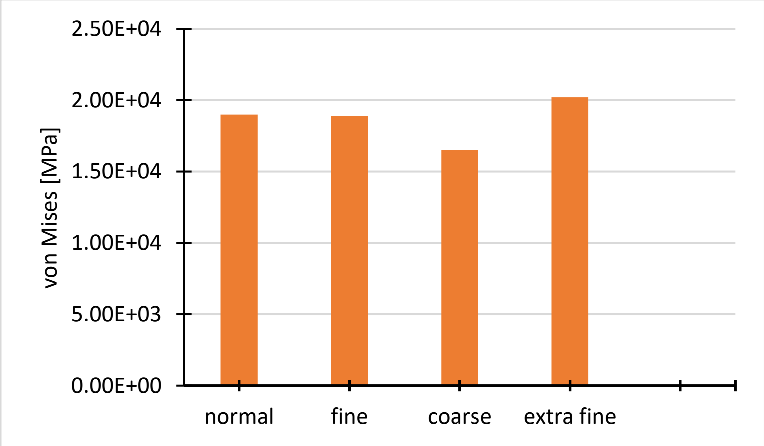


Fig. 15. The von Mises stress as the mesh density

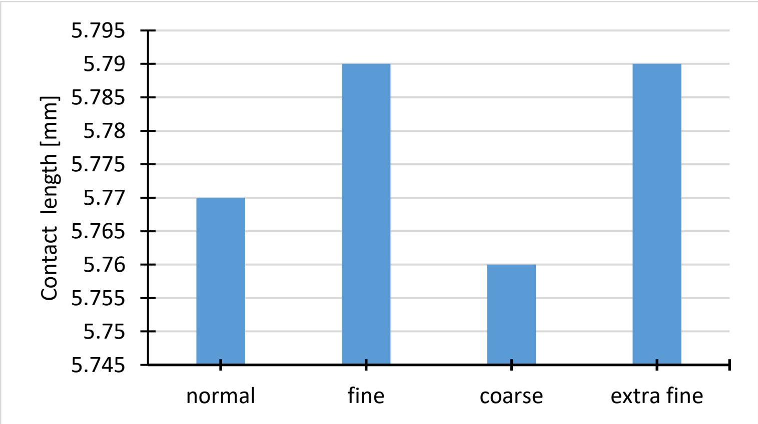


Fig. 16. The contact length variation as the mesh density

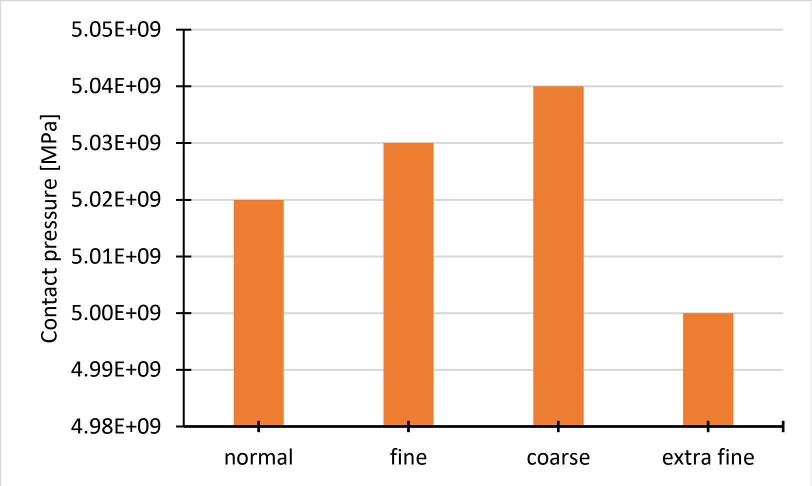


Fig. 17. The contact pressure variation as the mesh density

The error in the computation between coarse and extra fine mesh, of contact length as the function of mesh density is about 0.5%. For the contact pressure, it is about 0.7% and for the von Mises stress, it is about 20%. We assume that the choice of mesh density affects considerably the von Mises stress and have no influence on the value of the contact pressure and contact length.

7. Conclusions

The ceramics we have studied here are more particularly porous ceramics whose mechanical and physical characteristics are more interesting than those of so-called standard materials especially for biomedical uses.

The main objective of this work was to study the influence of the presence of the pores and their shapes on the mechanical behavior of contact components. The contact pressure in the case of elliptical form studies here is the smallest, particularly those of vertical shapes, which is strictly inferior to that of the crop without defects, with stress values and radius of contact close to that of contact of holy biomaterials.

Bringing the pores closer to the free center of the contact surface increases the contact pressure, as well as the inclination of the pores towards the center the results gathered in this contact study are acceptable in the absence of sufficient data and effective experimental means and protocols. Our initiative, which is in comparative studies, between analytical and numerical; it is therefore a sketch for further research in this fertile field.

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