FINITE ELEMENTS SIMULATION OF PLANE WAVE REFLECTION BY LIQUIDS FREE SURFACES AT GRAZING INCIDENCE

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In the process of numerical simulation by the Finite Elements Method (FEM) of the ultrasonic acoustic waves propagating in water parallel or quasi-parallel to the free surface have surfaced a series of difficulties. The main issue is the fulfillment of the null pressure boundary condition at the free surface by an incident plane wave with a wavenumber vector parallel to this free surface. The authors deduce the asymptotic evolution of the plane wave in a two dimensional study. As the incidence angle approaches the grazing angle the background pressure input for the FEM analysis is expressed in accordance with this formulation. The acoustic field obtained by the proposed formula is depicted in a numerical analysis software, for a clearer understanding. Then, the implementation in a FEM software package of the results thus obtained, prove to be in good agreement with the analytical solution.

Keywords: Grazing incidence, plane wave reflection.

1. Introduction

The research work in the domain of ultrasonic waves in sea water near the free surface is of interest for many applications. The Finite Elements Method (FEM) is widely used for numerical simulations of ultrasonic waves propagation and scattering in liquids. The incident wave is in general considered to be a classical plane wave, for which the pressure is identical at any given moment of time for all

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the points situated in planes normal on the direction of propagation. When this direction is parallel to the interface water-air (or vacuum, since the air has negligible specific impedance $Z=\rho c$ to that of a liquid, with ρ the mass density and c representing the acoustic wave velocity) a contradiction appears. The acoustic pressure must cancel on the plane surface of the interface, so that the wave amplitude cannot be constant all along the planes normal on this interface. The interface between a fluid medium and vacuum is in fact a free surface. The pressure field of the fluid medium can be expressed using the theory of image sources. The real source in the fluid medium will have an image source that is symmetric with respect to the interface. So, in the case of an incident plane wave with a wave vector at an angle of θ_i, its image will have a wave vector at an angle of θ_r = - θ_i. The image plane wave is in opposite phase to the real plane wave to satisfy the condition of zero pressure imposed by the free surface. This method is widely used to characterize the pressure field scattered by various targets. P. Salaün [1] deals with the effect of the free surface on the far-field pressure on a half-submerged cylinder.

The theoretical aspects of grazing incidence (incident angle $\theta_i \rightarrow \pi/2$) were addressed by Goodier and Bishop in 1952 [2] for an elastic solid half-space. Their solution consisting of acoustic field developed in a series of the emergence angle e, in which $e = (\pi/2 - \theta_i) \rightarrow 0$ and their solution produced an intense debate.

Later, this result was included in classical textbooks. Graff [3] (pp.322) concludes that for grazing incidence $\theta_{\mathrm{i}} \rightarrow \pi/2$ (Fig. 1a) the scalar potential is:

$$\Phi = \left(A_1 - A_2 y\right)\exp\left[i\left(kx - \omega t\right)\right]; \quad y \le 0 \;, \tag{1}$$

in which $k = \omega/c$ [rad/m] is the scalar of the wavenumber, ω [rad/s] is the angular (circular) frequency, t is time and $i = \sqrt{-1}$. Since we opted for vertical upwards (Oy) axis, as required by a FEM software, we set a minus sign in front of the constant A_2, compared with the reference [3]. The author deduces that a linearly increasing amplitude $A_1 - A_2 y$ with depth y < 0, is not a physical solution and for half-space problems "such waves are of little interest". Achenbach [4] includes the same reference [2] for the grazing incidence case, without any other comments. Miklowitz [5] pp.136 presents in detail the results of Goodier and Bishop [2]. Dieulesaint and Royer [6] pp.42 investigate the reflection/refraction at the interface between two fluids, the critical angle and the total reflection, but not at the grazing incidence. More recent researches, up to 2023, on the acoustic waves reflection were published by Rokhlin et al. [7] , Solodov [8] , Caviglia et al. [9] , Kaushik and Gupta [10] or Tsumi et al. [11], but were not referring to FEM simulation problems. The grazing incidence investigated in this paper appears in maritime acoustics problems such as detecting sea ice, e.g. Moreau et al. [12], Liu and Li [13], Sandy et al. [14] or Chotiros [15] in detecting floating objects such as various ships or ice blocks.

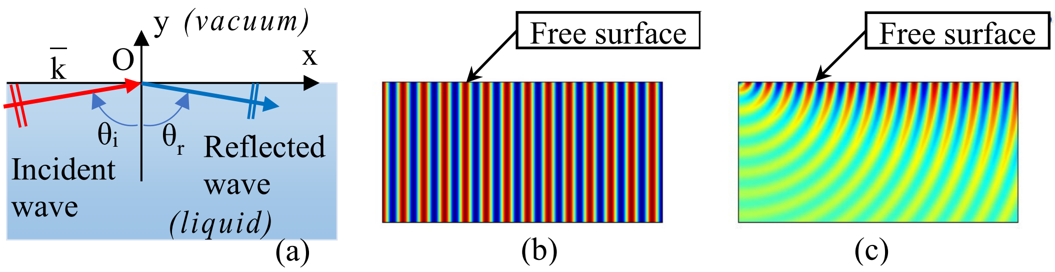

Fig. 1. (a) The wave incident on the free surface (y=0); (b) Incident plane wave pressure, pattern of blue(min) yellow, and red (max) stripes. (c) Spurious scattered acoustic pressure (c) (color online)

To better understand the investigated problem, the configuration is described by a 2D cross-sectional view of COMSOL Multiphysics software. First of all, in a FEM simulation, an incident plane wave is sent with the selection "background pressure field" parallel to the free surface imposed by the selection "Sound Soft Boundary" for which the total pressure is zero (Fig. 1b). In the context of a semi-infinite medium, unwanted reflections are avoided by surrounding the computation domain by wave absorbing domains (PML Perfectly Matched Layer) on the three boundaries (left, right and bottom). However, a spurious solution appears for the scattered pressure (Fig. 1c). Moreover, when plotting the total pressure field, we clearly observe a dependence on the x coordinate. We expected the pressure field not to vary along the free surface because the medium is considered semi-infinite, and the excitation plane wave originates from infinity by definition. As a matter of fact, it is the combination of the two selections "background pressure field" (for the plane wave) and the selection "Sound Soft Boundary" (for the free surface) that causes a conflict in the model. This leads to non-physical results and proving nonexistent to this date of a validated FEM implementation of the classical acoustic pressure field.

In most FEM models, the acoustic pressure, which is a scalar quantity, can be applied on a straight boundary segment and will generate a plane wave with the exception of the two ends of this boundary. The spurious cylindrical waves produced by these two boundary points will alter the intended acoustic pressure field. Moreover an inclined boundary will generate a finite extent acoustic filed, leaving incorrect results in the lower-right corner of the FEM domain.

To avoid the issues mentioned above, the present work is focused on the problem of incident pressure waves in fluids at grazing incidence against a free surface, developing an asymptotic solution for the propagating wave, which has an amplitude increasing with increasing distance from the interface (depth). Numerical simulations in commercially available FEM package COMSOL [16], will prove that this approach is adequate. The deduced acoustic pressure field (paragraph 2), shown in paragraph 3, can be directly and efficiently implemented in FEM packages (paragraph 4) in order to simulate any acoustic problems with grazing incidence.

2. Theoretical aspects

This paragraph focuses on finding the equations that govern the incident and reflected waves (as shown in Figure 1.a) using the classic theory of wave reflection. The equation governing the wave propagation in fluids is [5], [4], [3], [6] :

$$\frac{\partial^2 p}{\partial t^2} = c^2 \Delta p \quad , \tag{2}$$

simplifies in the two-dimensional cartesian case to:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) . \tag{3}$$

We are generalizing the search for possible solutions of this equation, choosing:

$$p(x,y,t) = P(y) \exp\left[i (k_x x - \omega t) \right]. \tag{4}$$

Injecting the expression (4) in (3) and leaving aside the harmonic common factor $\exp\left[i (k_x x - \omega t) \right]$, one gets:

$$\frac{\partial^2 P(y)}{\partial y^2} + \left(k^2 - k_x^2 \right) P(y) = 0 \tag{5}$$

Using the common notation for the component of the wavenumber along (Oy): $k_y = \sqrt{k^2 - k_x^2}$, the general solution of (5) is:

$$P(y) = A_1 \exp\left(i k_y y \right) + B_1 \exp\left(-i k_y y \right). \tag{6}$$

These general solutions indicate that in the fluid domain can propagate two possible oblique waves relative to the system of axes shown on Fig. 1 , one towards positive (Oy) (incident wave on Fig. 1) with amplitude A_1, and another towards negative (Oy) (reflected wave on Fig. 1) with amplitude B_1, in full agreement with the classical theory of wave reflection. The grazing incidence is a limit case for the classical wave reflection at a free surface, but actually with no reflecting waves. For this reason, we prefer to write the solution (6) as:

$$P(y) = A \cos\left(k_y y \right) + iB \sin\left(k_y y \right) \tag{7}$$

For the incident wave, the (Oy) component of the wavenumber is $k_y = k \cos\theta_i$, which for grazing incidence $\theta_i \rightarrow \pi/2$, tends to zero: $k_y \rightarrow 0$. Consequently:

$$\cos\left(k_y y \right) \rightarrow 1 \ ; \ \sin\left(k_y y \right) \rightarrow k_y y , \tag{8}$$

so that the acceptable solution for grazing incidence is

$$P(y) = A + iBk_y y \tag{9}$$

Obviously in the case of a free surface, the pressure field must have A = 0 and the solution for the wave propagation is:

$$p(x, y, t) = iBk_y y \exp\left[i(k_x x - \omega t)\right]. \tag{10}$$

Apparently, being a linear function of the depth coordinate y, the pressure amplitude increases indefinitely which "violates physical intuition" according to ref. [3]. In fact, being a development around zero of $\sin(k_y y)$, the pressure amplitude Bk_y will remain finite even far from the free surface. The solution is no longer a classical plane wave since the amplitude increases with increasing distance from the free surface, in each plane normal on the propagation direction.

3. Numerical analysis

The obtained solution is numerically analyzed using a MATLAB [17] code, for the classical case of plane wave reflection, progressively approaching the grazing incidence. The general solution (6) valid for arbitrary incident angles is considered for the total pressure field generated by superposed incident and reflected plane waves, defined by (4):

$$p(x, y, t) = \left[A_1 \exp(ik_y y) + B_1 \exp(-ik_y y)\right]\exp\left[i(k_x x - \omega t)\right]. \tag{11}$$

For the free surface $y=0$ the pressure field must cancel so $B_1= -A_1$ and for an incident pressure of amplitude P_0, the solution is:

$$\begin{aligned} p(x, y, t) &= P_0\left[\exp(ik_y y) - \exp(-ik_y y)\right]\exp\left[i(k_x x - \omega t)\right] \\ &= i2P_0 \sin(k\, y \cos\theta_i)\exp\left[i(k_x x - \omega t)\right] \end{aligned} \tag{12}$$

A plane wave of P_0 =1 Pa, frequency f =1 MHz is sent at incident angles θ_i = 75° (Fig. 2) and θ_i = 89.99° (Fig. 3). On the left of each figure are shown the real parts of the incident pressure, which is a plane wave for a certain incident angle. On the right, is presented the real part of the total acoustic pressure respecting the boundary condition at the free surface: $P=0$ at $y = 0$.

It is important to notice that the total acoustic pressure shown on Fig. 3 is in agreement with the linear expression (10) obtained in the previous paragraph. Moreover, the total acoustic pressure maximum amplitude is considerably less than the theoretical maximum total amplitude shown on Fig. 2 which is $2P_0$ = 2 Pa in this case. The explanation comes from the fact that this maximum total amplitude $2P_0 \sin(k\, y \cos\theta_i)$ from (12) will be reached at a large distance from the interface:

$$y_{\max} = \frac{\pi}{2k\cos\theta_i}, \tag{13}$$

which in this case (θ_i = 89.99°) is y_{max} = 2.12 m. Naturally as $\theta_i \rightarrow \pi/2$ the distance $y_{\max} \rightarrow \infty$. It is interesting to mention that in our MATLAB [17] code, the input incidence angle $\theta_i = \pi/2$ generates a plot very close to the one in Fig. 3 (right) which might surprise an unexperimented user. In fact, due to roundoff errors, the value used in the plots has a numerical value k_y=2.5996e-13 rad/m, but not zero. If the user will set directly $k_y=0$ rad/m, then the returned solution will be an almost null total pressure over the computation domain, corresponding to a distance $y_{\max} \rightarrow \infty$

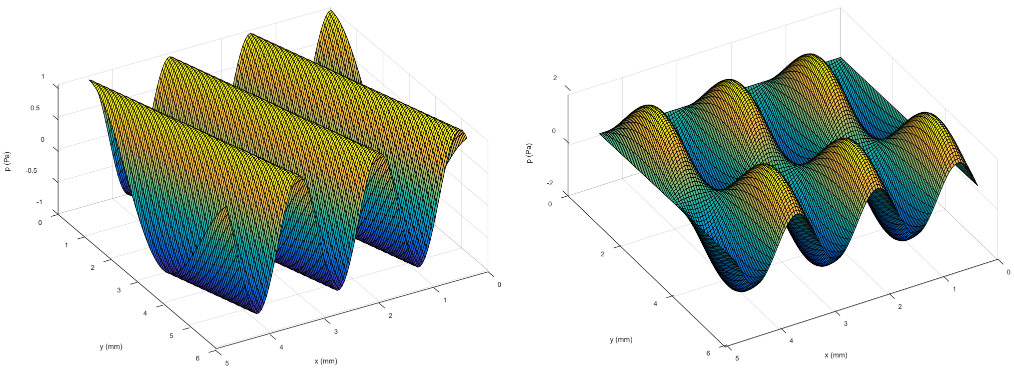

Fig. 2 Incident acoustic pressure (left) and total acoustic pressure (right) for θi = 75°

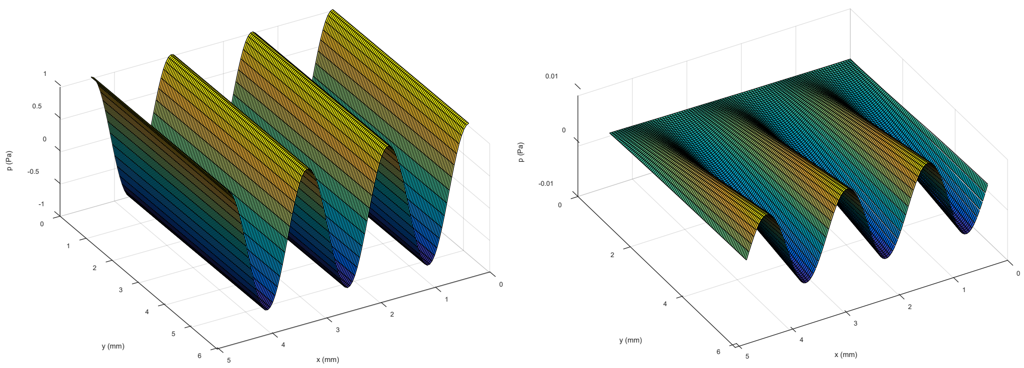

Fig. 3 Incident acoustic pressure (left) and total acoustic pressure (right) for θi = 89.99°

4. Finite Elements model for wave reflection with background incident pressure

The objective is to obtain a FEM model such that the total acoustic pressure corresponds to the formula (10), or equivalently validating the limit case $\theta_i \rightarrow \pi/2$

as defined by formula (12), providing an output as in the numerical simulation presented in the previous paragraph. For this purpose, we develop a FEM model using the "pressure acoustics, frequency domain" provided by COMSOL Multiphysics software [16].

a. FEM domain geometry

The FEM domain capable of modeling a problem covering a half-space, poses the initial problem of geometrical dimensions. The geometry in this case must cover several wavelengths in both (Ox) (direction of the interface) and (Oy) normal direction. Selecting a frequency $f = 1$ MHz, the wavelength in water (ρ=1000 kg/m^3, c=1480 m/s) is $\lambda = 1.5$ mm. A domain of 20 x 10 mm was selected for all FEM simulations at this frequency, bounded on three sides by 10 mm thick PML.

b. Boundary conditions

Obviously, the free surface must satisfy the condition for the total acoustic pressure: $p_t = p_{inc} + p_{refl} = 0$ which can be set as “sound soft boundary” in the FEM model. As for the other three edges, there is no physical boundary condition applicable to simulate a non-reflecting wave condition. These three boundaries must not influence the waves propagating in any direction. The most appropriate way to solve this problem is to surround on three edges the fluid domain, by PMLs, as mentioned in the Introduction. These PML are absorbing any waves passing through these domain. On Fig. 4 are presented the domains with their respective dimensions in meters.

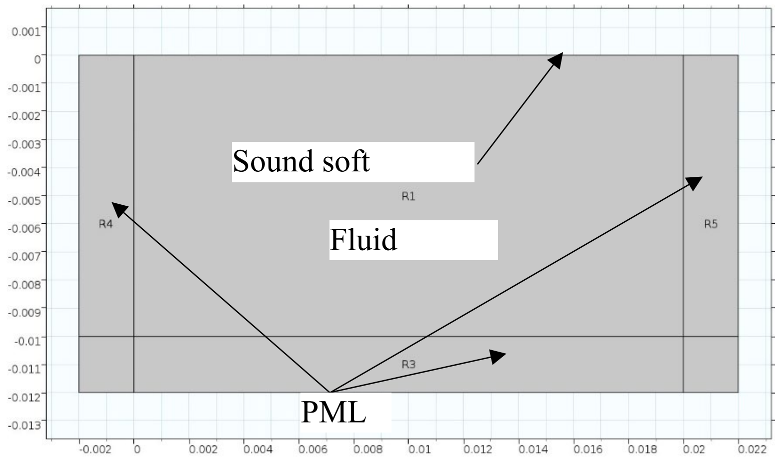

Fig. 4 Geometry of the FEM model. Dimensions in [m].

c. Incident wave

For such “Pressure acoustics” problems there is an option available to set a “background acoustic pressure” indicating the incident pressure amplitude (e.g. 1

Pa), frequency (e.g. 1 MHz) and direction of the incident wavevector. There is no possibility to set an incident pressure on the left boundary for two reasons: 1) the boundary is between the water domain and the left PML altering its functionality and 2) the normal pressure being a scalar value, the boundary must be inclined according to the incident angle, generating a bounded acoustic beam in the domain, leaving an unacceptable low-right corner non-insonified.

d. FEM mesh

It is recommended to set a minimum of 10 elements per wavelength. Moreover, since computing the reflection at the free surface requires high accuracy, the mesh will be denser near the free surface (Fig. 5) and in the PML regions. Overall, there are around 48000 quadrilateral elements.

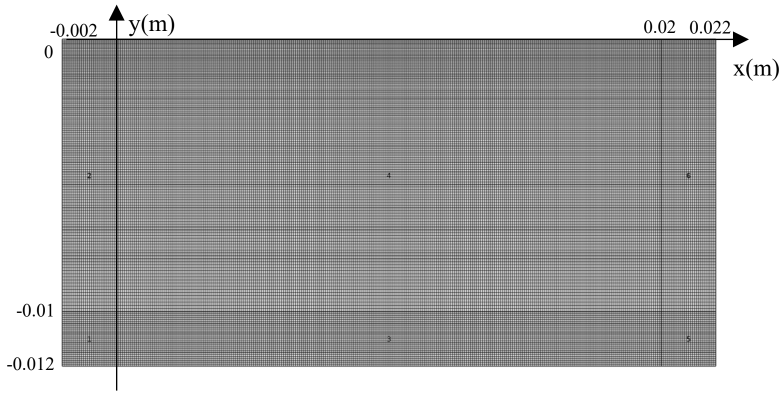

Fig. 5 FEM mesh for this problem. Same dimensions as in Fig. 4.

e. FEM simulation results

The first simulation concerns the incident angle $\theta i = 45°$. This angle was chosen for the clear pattern of interfering waves: incident and reflected square-shaped nodes and antinodes (Fig. 6).

For better clarity, the computed total acoustic pressure is shown only in the fluid domain (0.02 by 0.01 m), the PML domains being discarded from this and the following plots.

It is clearly visible the expected pattern, but in the lower left corner of the total acoustic pressure (Fig. 6 right) appears a strange plot close to the pattern of the incident wave and the square-shaped pattern of nodes and antinodes is missing. The same FEM model was tested for incidence angles $\theta i = 75°$ () and $\theta i = 89.99°$ (Fig. 8) with similar results.

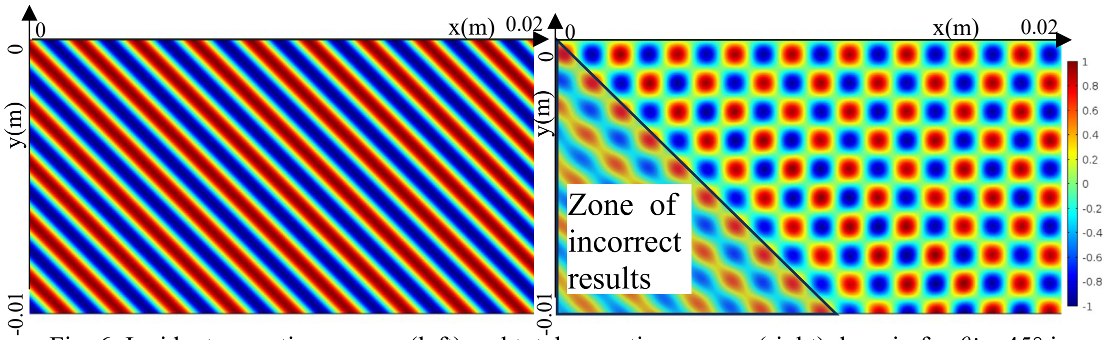

Fig. 6. Incident acoustic pressure (left) and total acoustic pressure (right) domain for $\theta i = 45°$ in the 0.02x0.01m FEM domain, common color bar (Pa) (online)

The explanation comes from the fact that correct reflected waves only appear from the upper-left corner of the domain and propagate as expected from the direction of the reflected wave θ_r with the origin in the upper-left corner. The scattered wave pressure, which is not shown here, is wrong in the lower-left part of the domain being determined by the presence of reflected waves in the FEM domain.

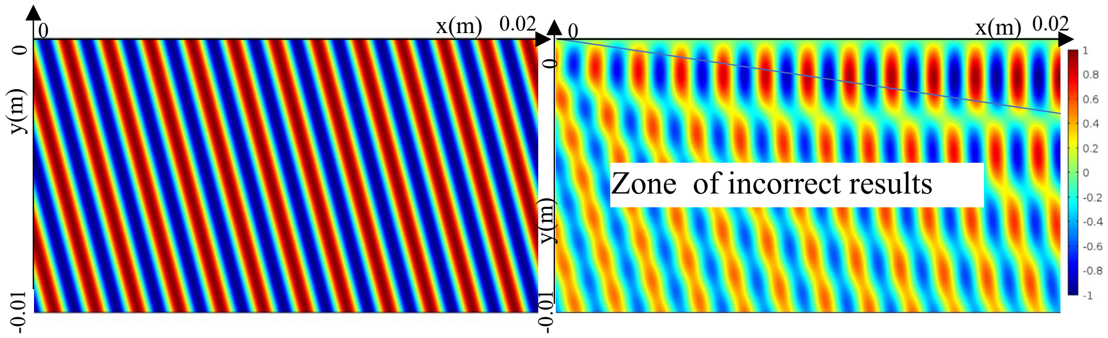

Fig. 7. Incident acoustic pressure (left) and total acoustic pressure (right) for $\theta i = 75°$, common color bar (Pa) (online)

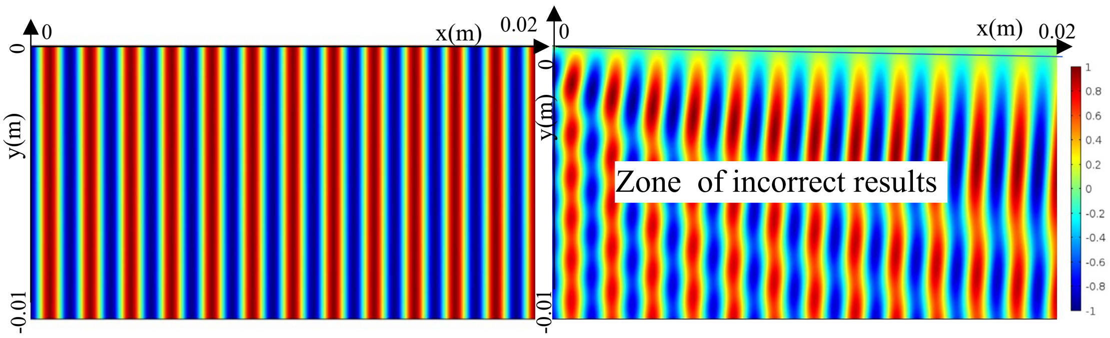

Fig. 8. Incident acoustic pressure (left) and total acoustic pressure (right) for $\theta i = 89.99°$, common color bar (Pa) (online)

These FEM simulations provide several conclusions:

- The incident acoustic pressure is distributed in the entire acoustic FEM domain like a stationary wave (no time dependence in the simulation), whereas the correct reflected (scattered) wave is only computed and superposed starting from the upper-left corner of the domain limited by the direction of the reflected wave. In the lower-left part of the domain, which becomes dominant for large incidence angles, the scattered pressure corresponds to spurious "ripples" of the reflected wave field.
- The correct total acoustic pressure field can only be expected to occur in the subdomain above the line of reflection direction, drawn from the upper -left corner of the acoustic domain (Fig. 6). For large incident angles the horizontal extent of the fluid domain becomes prohibitive to expect a correct pattern in part of the FEM domain. For grazing incidence, this extent tends to infinite.
- For the particular case of grazing incidence, it is not possible to extend the domain such that a significant subdomain represents the correct total acoustic field. Consequently, the presented model of "background acoustic pressure" cannot be used for grazing incidence angles ($\theta_i \rightarrow \pi/2$) but only for limited subdomains at small incidence angles (e.g. $\theta_i = \pi/4$).

5. Finite Elements model for grazing incident wave on a free surface

The solution proposed in this paper to the limit case of grazing incidence ($\theta_i \rightarrow \pi/2$), is to set as background pressure wave, the formula (12) deduced in paragraph 2 and validated in paragraph 3, valid also for the total acoustic pressure, in the form:

$$p(y) = 2iP_0 \sin\left(\frac{y}{y_{\max}} - \frac{\pi}{2}\right)\exp(ik_x x), \quad (14)$$

in which P_0 is the incident pressure amplitude in the fluid domain and y_{max} is given by formula (13). The FEM simulation provides the following results (Fig. 9).

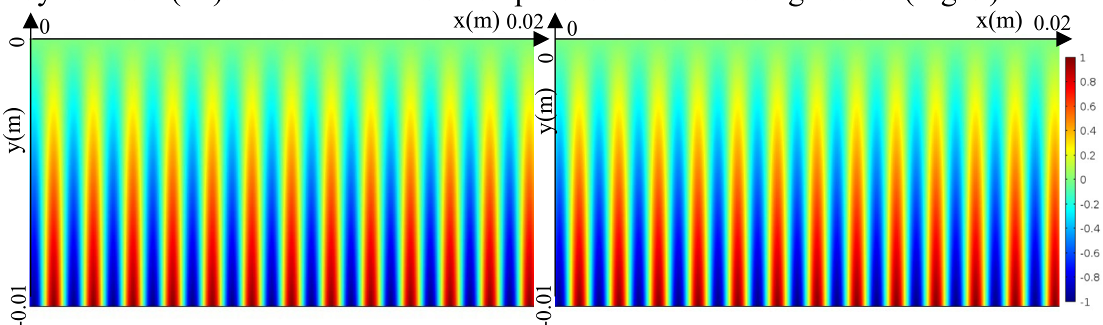

Fig. 9. Incident acoustic pressure (left) and total acoustic pressure (right) for $\theta i = 90°$, common color bar (Pa) (online)

This incident (background) pressure field is in agreement with the deduced analytical solution, so it is natural to lead to a computed stationary wave pattern, without interaction with the free surface. It means that the total acoustic pressure is identical to the incident pressure field and the scattered acoustic pressure is null in the entire FEM domain. This result is in agreement with the propagation along the free surface ($\theta i = \pi/2$). Moreover, the total acoustic pressure amplitude is in good agreement with the values deduced by the analytical model and corresponding numerical results shown in the previous paragraphs.

6. Conclusions

The Finite Elements Method (FEM) is capable of providing accurate solutions to ultrasonic acoustics problems. However, using incident plane waves as "background pressure field" reduces even drastically the subdomain in which the total acoustic pressure is correctly determined. The limit case of waves propagating at grazing incidence in a fluid requires special attention since the reflected (scattered) acoustic pressure fills the entire FEM domain with spurious values.

This problem was not adequately addressed in well appreciated textbooks or published papers, to the authors knowledge. In the present paper, the problem of grazing incidence against a free surface is solved. An analytical solution is provided and a numerical validation is provided for $\theta_i \rightarrow \pi/2$.

The plane wave reflection by a free surface can be studied by FEM with imposed "background pressure field" at incidences $\theta_i << \pi/2$, leaving the user with a certain subdomain of correct total acoustic pressure (upper-right domains on Fig. 6 .. Fig. 8). Increasing this domain of validity of results requires a different approach, which is beyond the scope of the present work.

At grazing incidence which a special case of incidence, the proposed formulation for the incident acoustic field using "background pressure field", as deduced in the present paper, can be transposed into FEM models, representing a fluid half-space. The FEM results correspond to the analytical solution.

Once the total acoustic pressure field produced by a grazing incident wave is accurately determined by the deduced formula, the entire FEM domain model is valid and can be used for further studies of ultrasonic waves scattered by various obstacles which might be present in such a fluid half-space.

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