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OPTICAL SYSTEM OPTIMIZATION FOR PHASE RETRIEVAL USING A GENETIC ALGORITHM APPROACH

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We propose an implementation based on a genetic algorithm approach to optimizing the optical system design for a phase retrieval problem solved using a forward-backward approach. The algorithm uses the desired input and output complex valued optical profiles, the total propagation length, a given number of lenses and up to two phase masks based on which the optimal arrangement of the optical elements is computed. Once computed, the optimal system can be used to compute its corresponding phase mask.

Keywords: phase modulation, optics, genetic algorithm, optimization

1. Introduction

Implementations of the problem of phase mask computation for manipulating optical beams has been presented in a range of applications such as optical tweezers [1, 2], cold atom traps [2, 3], two photon microscopy [4] and novel optical beams [6, 7, 8] to name a few. Solutions to this problem vary from objective specific ones, to more general approaches such as Gerchberg-Saxton[9] and Yang-Gu[10] algorithms.

An algorithm for solving this problem has also been presented by the authors[11] and it is based on a forward-backward propagation approach from which the phase difference at the plane of the phase mask(s) is computed and used to modulate the incident beam. From an experimental perspective, this method requires a manual optimization of the optical system. This can be made by a trial and error approach or using educated guesses, until the quality of the output reaches a desired level of similarity with the desired one.

In order to highlight the problem we address in this article, consider the case of a scenario presented in figure 1 . An optical system is required in order to transform an input optical profile into an output desired one using phase modulation. We assume that in order to implement it, we have access to a limited propagation distance D and we can use one lens and one phase mask. Using the forward-backward propagation method, not all configurations for the

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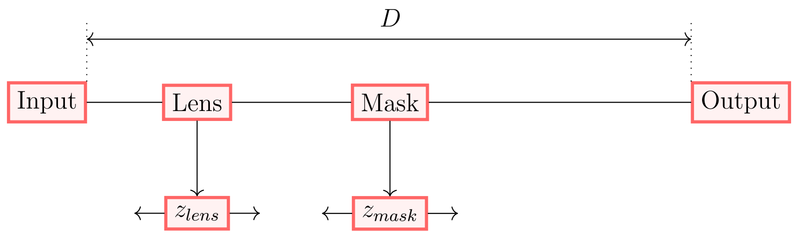

Fig. 1. Diagram of an optical system consisting of a lens and a phase mask. The input profile should be converted into an optical profile that is idealy identical with the desired output profile. In order to achieve this, the positions of the lens and phase mask should be shifted in order to manualy optimize for a better synthesized output profile.

lens and the phase mask return output profiles that are similar to the desired one. This is due to the observation that at the plane of the phase mask an amplitude match has to be achieved between the forward propagated input and the backward propagated output[11]. The same is true for Gerchberg-Saxton where a 2f system is required, while for Yang-Gu the optimal phase mask is computed based on the optical system at hand, although that optical system might not be the optimal one given the input and desired output profiles.

This implies that a method for optimizing the positions of the optical elements constituting the optical system is required. For the forward-backward propagation method we have used a manual optimization approach, where each element has been shifted along the propagation axis until the amplitude match criterion has been achieved, which is often time consuming. In order to limit the time for optimizing the optical system, we have considered developing a software solution based on a genetic algorithm (GA) approach.

GAs[12] can be summarized as a sequenced Monte-Carlo method for computing the output of various mutated offsprings of a given set of states, which are then selected based on a survival of the fittest approach using a fitness function. This process of mutation, selection and reconstruction of the population is repeated until the population is sufficiently adapted to the requirements imposed by the fitness function. This approach has been used in the context of optimizing optical setups for aberration correction[13], integrated optical device design[14] and prism design[15] to name a few.

Our implementation of a GA to the optimization of optical systems for phase modulation using phase masks is given in figure 2 and it starts with a population consisting of randomly generated optical systems structured in cohorts. These systems are generated using user defined types of optical elements that are placed at random positions along the propagation axis, as in the general case from figure 1. A phase mask is computed for each system given the user defined input and output profiles. Then the input profile

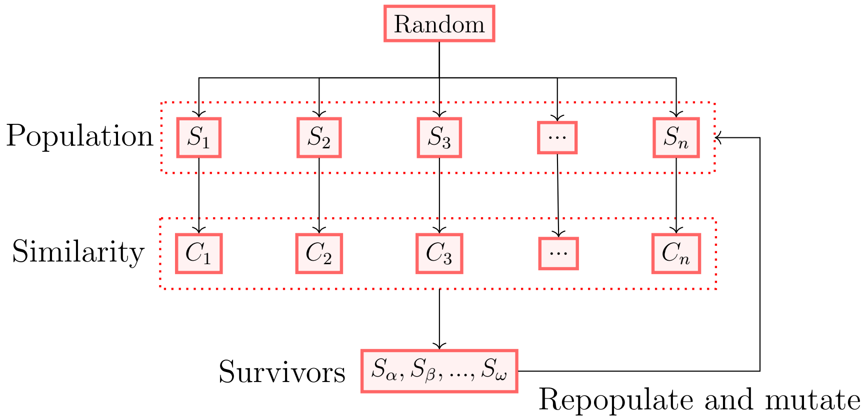

FIG. 2. A simplified flow chart for the GA implementation. A set of randomly generated systems are created which represents the population. The similarity is computed for each system which then allows for the selection of the best fitted cases. The survivors are then used to recreate the population of n systems during which mutations are applied. The process then cycles for a given number of generations.

is propagated through each optical system in which its corresponding phase mask has been introduced, which returns the degree of similarity between the desired and computed output profiles. A fraction consisting of the best optical setups is kept and then used as a starting point for regenerating the population via mutations consisting of slight shifts of the positions of the optical elements of the surviving group. With the population reestablished as the first generation of the initial one, the process is repeated for a user given number of generations. The user can reiterate the process for any number of generations until the computed output profile is similar to the desired one up to the user's requirements.

The main advantage of a GA solution to this problem is that it automates the process of design for the optical system. A user based implementation, independent of its level of expertise, will ultimately rely on heuristics or predefined arrangements such as a 2f system used to implement an optical Fourier Transform for a Gerchberg-Saxton approach. This can limit the method of constructing an optical system since in some instances multiple arrangements are possible, as we will present in the following. By contrast, a GA algorithm uses randomness to search for solutions and thus can find optimal setups that cannot be easily reduced to predefined arrangements.

The GA implementation that we have developed is using an in-house Python 3 module named PyParax[5], which is free for download from Github at **https://github.com/victorcristianpalea/PyParax**. A bried description of PyParax is given in appendix 1 however we highly recommend checking

Palea et al.[5] for extra information on the module, Palea et al.[11] for the phase retrieval approach, and the module documentation and examples on the repository for code description and examples.

2. Implementation of GA algorithm

The GA algorithm implementation can be structured into four main parts: inputs, population structure, evolution, and outputs.

2.1. Inputs

The algorithm uses some fixed parameters that are not altered during the evolution from one generation to the next one. Based on the general case from figure 1, the input and output optical profiles are examples of fixed parameters. Additionally the algorithm requires:

- **number of lenses** - can be any natural number and represents the number of lenses that are to be placed into the system.
- **number of phase masks** - can be set to 1 or 2 and represents the number of phase masks that are to be computed given an optical system.
- **possible focal lengths** - the lenses that are to be introduced into the system have their focal lengths chosen randomly from a list of possible values. This approach we consider helps the user to compute an optimal system that can be implemented using equipment that is at its disposal.
- **maximum propagation distance** - the total free space allowed for the optical system, labeled D in figure 1. We consider this parameter to be relevant for space restricted scenarios where an optimum is intended given a limited propagation distance.

Other parameters that are used during the evolution process include:

- **keep percentage** - the fraction that defines how many specimens survive from one generation to the other.
- **mutation amplitude** - the maximum absolute value by which a mutation can change a given parameter of a specimen. Its value depends on the type of mutation that is applied.

2.2. Population structure

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The population of optical systems is divided in cohorts which represent sets of objects that contain one optical system each. These objects are labeled in the following as specimens. A specimen is an object that contains information such as the optical system structure, the maximum length of the propagation distance, and functions that allow it to mutate. The population and the cohorts are just upper layer groups of specimens. In order to start the evolution of a population, its initialization is required. This is done by choosing the number of cohorts per population and of specimens per cohort, followed by the initialization of each structure.

The initialization of each structure requires the following parameters:

- specimen: **number of lenses**, **number of phase masks**, **possible focal lengths** and **maximum propagation distance**.
- cohort: number of specimens per cohort, 1 set of parameters required for specimen.
- population: number of cohorts, 1 set of parameters required for cohort.

When initializing a cohort, the set of parameters for defining a specimen are applied to all specimens initialized inside the cohort. The same stands for initializing a population.

This structure assures that multiple types of solutions survive inside a population, while also allowing for a computational speed up via the parallel implementation of the evolution process.

2.3. Evolution

The evolution is the process of going from one existing generation to the next one at a population level. During this process the following steps are applied:

(1) The phase mask(s) is(are) computed for each specimen using the forward-backward approach[11].

(2) An output profile is computed using the previous phase mask(s).

(3) A comparison between the computed output and the desired one is made using the cross-correlation of the L^2 normalized profiles resulting a number in-between 0 and 1 which is attributed to the specimen. The normalized profiles are computed using

$$\psi \to \frac{\psi}{\sqrt{\sum_n |\psi[n]|^2}} \tag{1}$$

and the cross-correlation is given by

$$(\psi_1 \star \psi_2)[l] = \sum_n \psi_1[n]\psi_2^*[n+l] \tag{2}$$

where ψ, ψ_1 and ψ_2 are discrete functions that describe optical profiles on a equally spaced grid.

(4) A search for the best fitted specimens is made by computing

$$F = \max(|\psi_{computed} \star \psi_{desired}|) \tag{3}$$

which is the fitness function for each specimen. The fitness function F is computed using normalized profiles, so it can take values in-between 0 and 1. The similarity between the two profiles is greater if F approaches 1 for the shifts along the transverse axes being 0. Under these conditions F is then used in order to identify the fraction of survivors based on the **keep percentage**. This evaluation is carried out at a cohort level e.g. for a cohort of 100 specimens, if the **keep percentage** is 10%, then the best 10 specimens in terms of similarity values survive, while the others are discarded.

(5) The cohort is repopulated until it reaches the same number of specimens that it had initially. The repopulation consists first of saving the surviving specimens as they are, followed by the occupation of vacant slots with mutated versions of them. This approach guarantees that if mutations degrade the quality of the surviving specimens, the progress made in terms of optimization is not lost from one generation to the next one.

The above mentioned steps are repeated for a given number of generations. Additionally, at each step from one generation to the next one, data regarding the best specimen inside each cohort is recorded.

The mutation mentioned in the previous steps consists of randomly selecting an optical element from each specimen except free spaces, and changing its position randomly. This shift is done by an arbitrary amount limited by the user through the **mutation amplitude** parameter.

The second type of mutation we have considered affects the **maximum propagation distance** parameter. This type of mutation acts at a cohort level by multiplying all the free spaces of each specimen by a randomly generated number limited by the user through the **mutation amplitude parameter**. The result of this mutation is that the optical elements are separated or approached proportionally, thus increasing or decreasing the **maximum propagation distance** parameter associated with the specimen.

The evolution as described above motivates the structure of the population by allowing different cohorts to evolve independently. If only one cohort is considered, then by running the evolution procedure for a given number of iteration, all the specimens tend to only one arrangement due to the elimination of a fraction. For example a cohort of 100 specimens evolving with a **keep percentage** of 10% could maintain mutated copies of only the best fitted specimen from the original population in 2 generation.

In terms of computational performance, the cohorts can be evolved in parallel since they are independent from each other which reduces the computational time significantly.

2.4. Outputs

The implicit outputs of the algorithm are the optical systems that have been selected during the repeated evolution process in order to satisfy the conditions imposed by the fitness function. These optical systems are described by the properties and positions of the optical components they consist of. The corresponding phase mask can be computed based on the optical positioning of each known optical element and the position at which the phase mask is supposed to be positioned. Other outputs are:

- a record of the best specimen - at each step along the evolution process the best specimen per cohort is recorded in order to follow the evolution of the optical system.
- a record of the maximum and minimum values of the cross-correlation comparison - at each step along the evolution process the maximum and minimum values of the comparison analysis are recorded per cohort. The maximum values can be used to follow the convergence of the specimens to an optimum, while the minimum values show the overall tendency of the entire cohort to move towards an optimum.

3. Numerical case studies

3.1. Similarity optimization

This is the most straightforward result that can be tested using the implementation described in section 2 since the survivability of a specimen depends on the similarity of the computed output to the desired one. For the purpose of this case, we have considered three scenarios consisting of a **number of lenses** of 0, 1 and 2. The rest of the parameters are **number of phase masks** = 1, **possible focal lengths** = 10mm, 30mm and 50mm, **maximum propagation distance** D = 100mm, **keep percentage** = 50%, **mutation amplitude** = 1mm. The only mutation that has been applied implied the shift of an optical element and not the propagation distance.

This analysis is carried out for a 1-dimensional transverse spatial domain with the input profile being a Gaussian and the output a truncated Airy function. The population consists of 10 cohorts, each of which having 20 specimens. As it can be seen in figure 3, after 10 generations all the cohorts have evolved to a similarity close to 1, with the number of generations required for convergence increasing with the number of lenses.

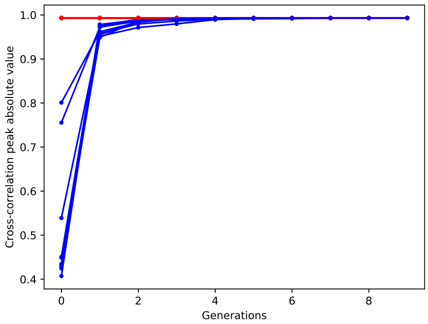
In terms of positioning of the optical elements, it can be seen in figure 4 that on the case where no lenses are considered, all cohorts evolve such that the position of the phase mask goes towards a common value. This is not the case for 1 and 2 lenses since multiple configurations are possible.

3.2. Propagation distance optimization

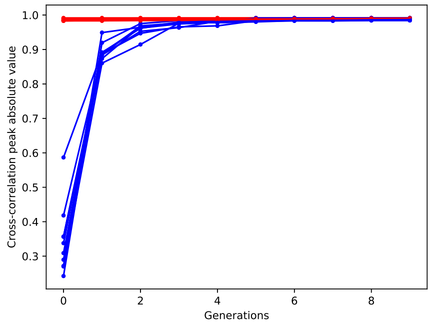
For the purpose of propagation distance optimization we consider a scenario where we assume that an optimal space for the entire optical system is unknown so a guess of 10mm for **maximum propagation distance** is used as a starting point. The selection is made for a **number of lenses** = 0. The **mutation amplitude** for the propagation distance is 0.2 which changes the distances d by the formula

$$d \to d \cdot (1 + 0.2 \cdot RNG(\min = -1, \max = 1)) \tag{4}$$

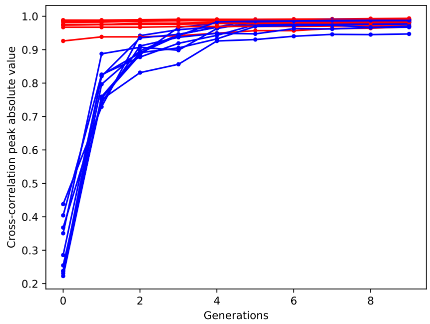
where RNG is a random number generator function that returns a value from a uniform distribution defined on the interval $(-1, 1)$.



(A) No lens.



(B) 1 lens.

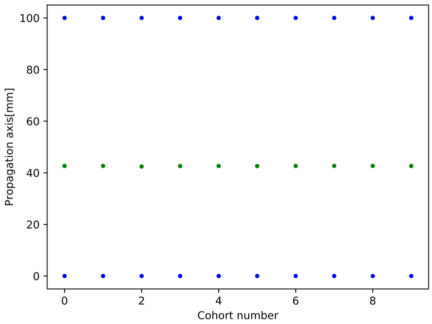


(C) 2 lenses.

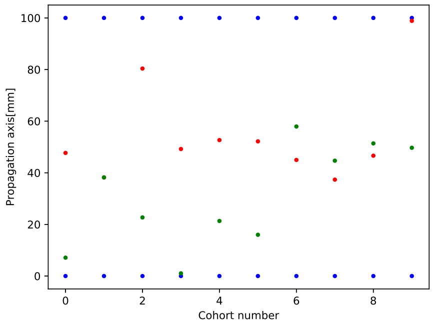
Fig. 3. Evolution of the maximum (red) and minimum (blue) absolute values of the peak cross-correlation as a function of generation number for similarity comparison. Each pair of lines corresponds to a different cohort.

The choice for the parameters have been set in order to guarantee that an optimum cannot be found simply by shifting the positions of one optical element, thus rendering the propagation distance change via mutation a requirement. The implementation consists of evolving the population in steps of 10 generations followed by a propagation distance mutation. This cycle then repeats until at least 1 cohort reaches a cross-correlation peak absolute value of 99%. The evolution of the similarity analysis is represented in figure 5.

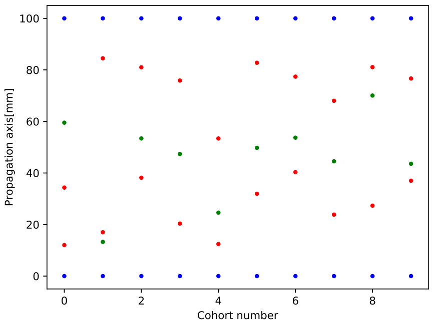
As it can be seen in figure 5, a stair-like pattern emerged where each jump indicates the introduction of a propagation distance mutation. Thus for the first 10 generations the similarity optimization has selected for the optimal solution given the conditions imposed by the parameters. After the 10th generation a propagation distance mutation is applied which immediately translates into an increase of the similarity value when the similarity optimization is resumed. The process of similarity optimization followed by



(A) No lens.



(B) 1 lens.



(C) 2 lenses.

FIG. 4. Phase mask (green) and lenses (red) positions of each cohort for the last generation.

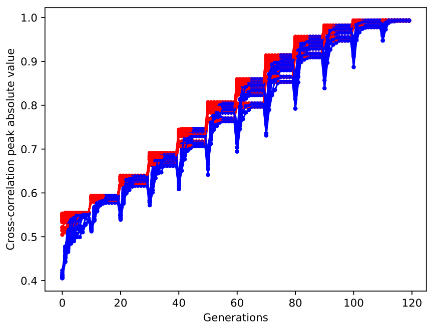


FIG. 5. Results of the similarity comparison for propagation distance optimization.

propagation distance mutation is repeated generating noticeable jumps in the similarity plot until the 99% threshold is reached. The whole process can be

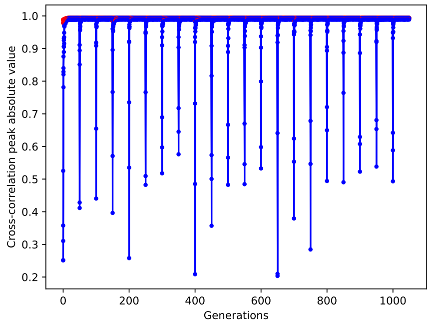


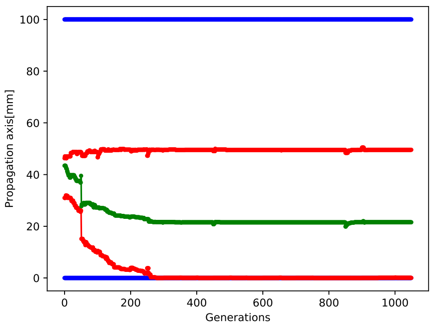
Fig. 6. Cross-correlation peak amplitude values for robustness analysis.

accelerated by choosing a higher value for **mutation amplitude** parameter but overshooting the minimum required value for the propagation distance is more probable.

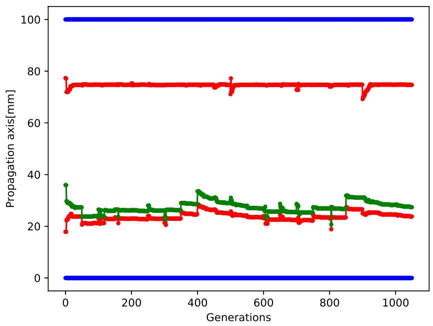
3.3. Robustness of the optimal system

In this case study we compare the robustness of the optical system obtained through evolution in order to check if the GA returns to the same solution if various mutations are applied on the solution. The parameters for this scenario have been similar to the ones from subsection 3.1 with the differences being **number of lenses** = 2, the number of cohorts is 5, and 50 generations cycles on which shifting mutations are used to optimize. In-between two successive cycles two random shifting mutations with **mutation amplitude** = 20mm are applied on each specimen. Due to the high value of the mutation between cycles, a significant change is expected in the outcomes of the similarity function and the return to the original setup can be investigated.

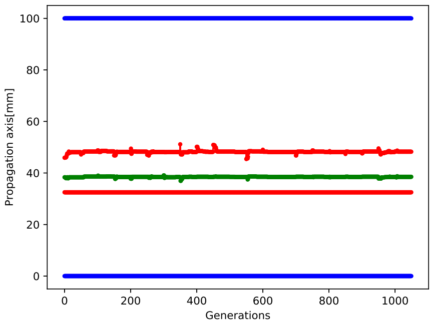
In figure 6 the similarity analysis is plotted showing that between each two consecutive cycles the minimum cross-correlation peak absolute values drop significantly, followed by a rapid recover. Next in figure 7 the system evolution of the best specimen per cohort is plotted individually. For cohorts 1, 3, 4 and 5 the evolution reached a stable system since at generations that are multiples of 50 a shift of the optical elements can be observed, followed by a return to the optimal arrangement. The case in cohort 2 is similar but with the added observation that the system is shifted from the optimum much more than the others, so convergence to a steady state does not occur completely during the 1100 generation, but a constant tendency to it is obvious due to the shift of the phase mask and first lens towards the approximately 21mm position.



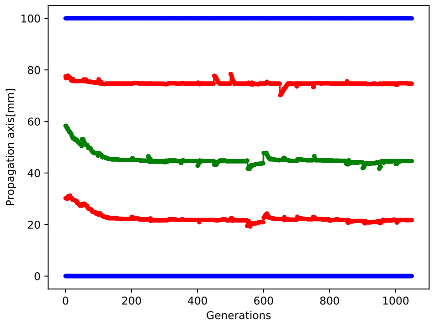
(A) Cohort 1.



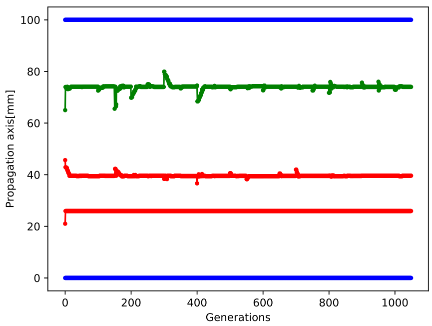
(B) Cohort 2.



(C) Cohort 3.



(D) Cohort 4.



(E) Cohort 5.

FIG. 7. Phase mask (green) and lenses (red) positions at each generation for robustness analysis.

4. **Experimental results**

For validating the GA approach we have made an optical system similar to the one from figure 1 consisting of a lens with focal length 300mm and a Holoeye LC2002 spatial light modulator for the phase mask. The input is a Gaussian profile and the expected output is a 2-dimensional Airy profile. A Motic 1SP camera has been used for recording the intensity profile in the output plane, which limits the propagation distance to 500mm.

The optimal system given by the GA algorithm required that the phase mask is placed at approximately 49.5mm and the lens at 450mm, both values being referenced to the input plane. The computed phase mask for this setup has been validated numerically and experimentally. The numerical case generated an output profile with similarity of 99.79%. For the experimental result, we have just placed the optical elements according to the results of the GA. The first step in comparing the numerical and experimental results has been to check the scaling between the recorded image and the one used for the GA, with approximately $36\mu m$ for the numerical one and $38 - 40\mu m$ for the experimental one. These lengths describe the distances from the global intensity peak to its upper and left neighbors, so being relatively close implies that the two images are similar in scale. Furthermore the similarity between the numerical and experimental intensity outputs has been computed after setting both images at the same scale, returning 90.07%. The actual intensity profiles are shown in table 1.

In order to showcase that this setup is optimal at least locally, we have shifted the lens by 5cm and 10cm respectively towards the phase mask. This comparison is also shown in table 1 where it can be seen visually that the numerical case shifts diagonally and has an increase in the spatial frequency of the local intensity peaks, while the experimental case loses any resemblance with the wanted profile.

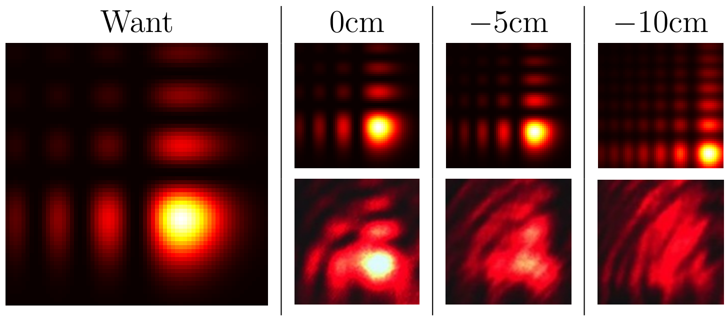


Table 1. Intensity profiles of wanted output compared with numerically and experimentally retrieved ones (0cm), along with outputs for shifted lens showcasing that the system configuration is optimal at least locally (−5cm and −10cm).

5. **Conclusions**

The GA approach to the optimization of an optical system has been presented in terms of three criteria, namely output similarity, propagation distance and robustness, with experimental validation. The method also indicates the existence of multiple optical setups that can transform the input into an output that is similar to the desired one under certain conditions. These results suggest that a GA approach can be used for optimizing an optical system for phase mask computation and modulation and reducing the time needed for designing an optical system intended for phase modulation.

Appendix - Brief PyParax description

PyParax simulates the propagation of optical beams using the paraxial approximation of the wave equation

$$\partial_z \psi = \frac{i}{2k}(\partial_x^2 \psi + \partial_y^2 \psi)$$

where ψ is the envelope of the electric field component of the electromagnetic wave which is assumed to vary slowly along the propagation axis, $k = 2\pi/\lambda$ where λ is the wavelength, $i = \sqrt{-1}$ and ∂_α^β is short-hand notation for the partial derivative of order β with respect to the variable α.

This propagation model allows for the introduction of some optical elements along the propagation axis z, namely phase and amplitude masks, one particular example of phase mask being lenses. These are implemented in PyParax such that an optical system made of various phase and amplitude masks can be created and used for simulating the propagation of a beam through it.

The computation of the numerical solution is done in Fourier space on the transverse axes where the solution for the propagation equation is given by

$$\tilde{\psi}(\xi, \eta, z) = \tilde{\psi}(\xi, \eta, 0) \exp\left(-\frac{2\pi^2 i(\xi^2 + \eta^2)}{k} z \right)$$

with $\tilde{\psi}$ being the Fourier transform of ψ along the axes x and y with the corresponding spatial frequencies ξ and η.

The solution is computed on a discrete grid using the Fast Fourier Transform algorithm as a numerical equivalent of the Fourier transform. Thus a profile at position z along the propagation axis consists of a 2-dimensional complex valued array $\psi[n, m]$ with sizes N and M along each axis. From a physical perspective, the relevant quantities intensity I and the phase ϕ are linked to ψ via

$$I[n, m] = |\psi[n, m]|^2$$
$$\phi[n, m] = \arg(\psi[n, m])$$

both of which being $N \times M$ arrays. All the calculations are done using data from the $N \times M$ arrays ψ from propagation to analysis using cross-correlation. For instance, a mask of shape $N \times M$ is applied to a profile ψ with the same shape using an element-wise product between the two corresponding arrays.

The above mentioned cases describe a 2-dimensional transverse profile, but they can also be reduced to 1-dimensional transverse profiles by eliminating the y variable, along with its corresponding spatial frequency η. In section 3 we have used the 1-dimensional transverse case, while for the experimental result from section 4 we have used the 2-dimensional case. Thus, in subsection 2.**3** the optical profiles consist of 1-dimensional discrete complex valued arrays.

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