MEAN RUPTURE DEGREE OF GRAPHS

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The vulnerability shows the resistance of the network until communication breakdown after the disruption of certain stations or communication links. We introduce a new graph parameter, the mean rupture degree. Let G be a graph of order p and S be a subset of V(G). The graph G–S contains at least two components and if each one of the components of G–S have orders $p_1, p_2, \ldots, p_k$, *then* $\overline{m}(G-S)=\frac{\sum_{i=1}^{k} p_i^2}{\sum_{i=1}^{k} p_i}$. *Formally, the mean rupture degree of a graph G, denoted mr(G), is defined as* $mr(G)=\max\{\omega(G-S)-|S|-\overline{m}(G-S): S\subseteq V(G),\ \omega(G-S)>1\}$ *where* $\omega(G-S)$ *denote the number of components.*

In this paper, the mean rupture degree of some classes of graphs are obtained and the relations between mean rupture degree and other parameters are determined.

Keywords: Graph Theory, Connectivity, Integrity, Mean integrity, Rupture degree.

1. Introduction

Graphs are often used to model real world problems, such as problem in a communication network. In a communication network, the vulnerability measures the resistance of the network to disruption of operation after the failure of certain stations or communication links. The analysis of vulnerability in networks generally involves some questions about how the underlying graph is connected. When some vertices of a graph are deleted, one wants to know whether the remaining graph is still connected. Moreover if the graph is disconnected, the determination of the number of its components or their orders (the number of vertices of components) is useful. To measure the vulnerability we have some parameters which are connectivity, integrity [2], mean integrity [5], rupture degree [12].

Terminology and notation not defined in this paper can be found in [4]. Let G be a finite simple graph with vertex set $V(G)$ and edge set $E(G)$.
The integrity of a graph G, $I(G)$, is defined to be

$$I(G)=\min\{|S|+m(G-S): S\subseteq V(G)\}$$

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where $m(G-S)$ is maximum order of the components of $G-S$ [2].

Let G be a graph of order p and S be a subset of $V(G)$. When the elements (vertices) of S are deleted from G, the remaining graph is denoted by $G-S$. The graph $G-S$ contains at least one component and if each one of the components of $G-S$ have orders $p_1, p_2, \ldots, p_k$, then $\overline{m}(G-S) = \frac{\sum_{i=1}^{k} p_i^2}{\sum_{i=1}^{k} p_i}$. Formally, the *mean integrity* of a graph G, denoted $J(G)$, is defined as

$$J(G) = \min_{S \subset V(G)} \{ |S| + \overline{m}(G-S) \}.$$

It was introduced as a measure of graph vulnerability by Chartrand, Kapoor, McKee and Oellermann [5].

The *rupture degree* of a noncomplete connected graph G is defined to be

$$r(G) = \max\{\omega(G-S) - |S| - m(G-S) : S \subseteq V(G), \omega(G-S) > 1\}$$

where $\omega(G-S)$ denote the number of components and $m(G-S)$ denote the order of a largest component in $G-S$ [12].

We now introduce a new stability measure. The mean rupture degree is very similar to rupture degree in that vertices are deleted and the number of remaining connected subnetworks. However instead of looking only at size of the largest remaining component, mean rupture degree takes into account sizes of all remaining components, replacing the size of the largest component with the weighted average of all components.

Let G be a graph of order p and S be a subset of $V(G)$. The graph $G-S$ contains at least two components and if each one of the components of $G-S$ have orders $p_1, p_2, \ldots, p_k$, then $\overline{m}(G-S) = \frac{\sum_{i=1}^{k} p_i^2}{\sum_{i=1}^{k} p_i}$. Formally, the *mean rupture degree* of a graph G, denoted $mr(G)$, is defined as

$$mr(G) = \max\{\omega(G-S) - |S| - \overline{m}(G-S) : S \subseteq V(G), \omega(G-S) > 1\}$$

where $\omega(G-S)$ denote the number of components. We see that, $mr(K_n) = 1-n$. A set $S \subseteq V(G)$, is said to be the *mr-set* of G, if $mr(G) = \omega(G-S) - |S| - \overline{m}(G-S)$.

Let G_1 and G_2 be graphs. Now one can ask the following question: Is the mean rupture degree a suitable measure of stability? In other words, does the mean rupture degree distinguish between G_1 and G_2?

We can find that many examples of graphs which suggest that $mr(G)$ is a suitable measure of stability in that it is able to distinguish between graphs. For example, consider the graphs in Fig. 1.

Using the results from Table 1, we have $r(G_1) = r(G_3) = 0$ and $I(G_1) = I(G_3) = 4$. Hence, the integrity and rupture degree does not distunguish between graphs G_1 and G_3 but $mr(G_1) \neq mr(G_3)$ and the mean rupture degree distinguish between graphs G_1 and G_3. In other words, the mean rupture degree takes into account what remains after the graph has been disconnected. So the mean rupture degree gives a better result than the integrity and the rupture degree.

Similarly,

$$J(G_1) = J(G_2) = 4$$

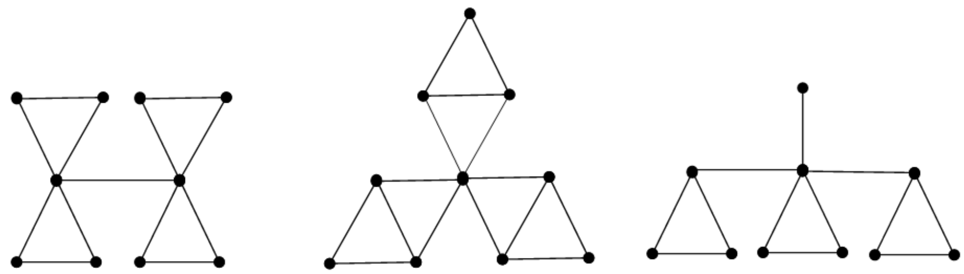

Fig. 1. We also give the table for stability parameters for graphs in Fig. 1 (see Table I).

Table 1.

	I	J	r	mr
G_1	4	4	0	0
G_2	4	4	−1	−1
G_3	4	3,56	0	0,44

i.e., the mean integrity does not distunguish between graphs G_1 and G_2. But the mean rupture degree distinguish between graphs G_1 and G_2, while $mr(G_1) \neq mr(G_2)$.

The comparison of mean rupture degree to integrity, mean integrity and rupture degree of graphs G_1, G_2 and G_3 indicates that the mean rupture degree can be a useful measure of graph stability. Therefore the mean rupture degree gives us more knowledge about the network to disruption.

2. Mean rupture degree of several classes of graphs

In this section, we consider the mean rupture degree of some graphs.

Theorem 2.1. *For n≥3, then,*

$$mr(P_n) = \begin{cases} 0\,, if\ n\ is\ odd, \\ \dfrac{-4}{n+2}\,, if\ n\ is\ even. \end{cases}$$

Proof. Let S be a cut set of P_n and $|S| = r$. We have the following two cases, depending on n.

Case 1. We consider the case when n is odd. Removing r vertices from P_n leaves $r+1$ components and these components must be size of $\left\lfloor \frac{n-r}{r+1} \right\rfloor$ or of size $\left\lceil \frac{n-r}{r+1} \right\rceil$.

Therefore, if $r \le \frac{n-1}{2}$, then $\omega(P_n - S) \le r+1$ and $\overline{m}\ (P_n - S) \ge \dfrac{r\left\lfloor \frac{n-r}{r+1} \right\rfloor^2 + \left\lceil \frac{n-r}{r+1} \right\rceil^2}{n-r}$.

Hence,

$$\omega(P_n - S) - |S| - \overline{m}(P_n - S) \le \quad r+1-r-\frac{r\left\lfloor \frac{n-r}{r+1} \right\rfloor^2 + \left\lceil \frac{n-r}{r+1} \right\rceil^2}{n-r}$$

$$\omega(P_n - S) - |S| - \overline{m}(P_n - S) \le 1-\frac{r\left\lfloor \frac{n-r}{r+1} \right\rfloor^2 + \left\lceil \frac{n-r}{r+1} \right\rceil^2}{n-r}$$

the function $f(r)$ takes its maximum value at $r = \frac{n-1}{2}$ and we get

$$\omega(P_n - S) - |S| - \overline{m}(P_n - S) \le 1-1 = 0. \qquad (1)$$

It is easy to see that there is a cut set S^* such that $|S^*| = \frac{n-1}{2}$ then $\omega(P_n - S^*) = \frac{n+1}{2}$ and $\overline{m}(P_n - S^*)=1$. From the definition of mean rupture degree, we have

$$mr(P_n) \ge \omega(P_n - S^*) - |S^*| - \overline{m}(P_n - S^*) = \frac{n+1}{2} - \frac{n-1}{2} - 1 = 0. \qquad (2)$$

If $r \ge \frac{n+1}{2}$, then $\omega(P_n - S) \le n - r$ and $\overline{m}(P_n - S) \ge 1$. Hence

$$\omega(P_n - S) - |S| - \overline{m}(P_n - S) \le n - r - r - 1 = n - 2r - 1$$

the function $f(r)$ takes its maximum value at $r = \frac{n+1}{2}$ and we get

$$\omega(P_n - S) - |S| - \overline{m}(P_n - S) \le n - 2.\frac{n+1}{2} - 1$$

$$\omega(P_n - S) - |S| - \overline{m}\ (P_n - S) \le -2 \qquad (3)$$

By (1), (2) and (3) we have $mr(P_n)=0$.

Case 2. We consider the case when n is even. If $r \le \frac{n}{2} - 1$, then $\omega(P_n - S) \le r+1$ and $\overline{m}(P_n - S) \ge \dfrac{r\left\lfloor \frac{n-r}{r+1} \right\rfloor^2 + \left\lceil \frac{n-r}{r+1} \right\rceil^2}{n-r}$. Therefore,

$$\omega(P_n - S) - |S| - \overline{m}\,(P_n - S) \leq \quad r+1-r-\frac{r\left\lfloor \frac{n-r}{r+1}\right\rfloor^2+\left\lceil \frac{n-r}{r+1}\right\rceil^2}{n-r}$$

$$\omega(P_n - S) - |\mathrm{S}| - \overline{m}(P_n - \mathrm{S}) \leq 1-\frac{r\left\lfloor \frac{n-r}{r+1}\right\rfloor^2+\left\lceil \frac{n-r}{r+1}\right\rceil^2}{n-r}$$

the function $f(r)$ takes its maximum value at $r = \frac{n}{2}-1$ and we get

$$\omega(P_n - S) - |S| - \overline{m}(P_n - S) \leq \frac{-4}{n+2}. \tag{4}$$

It is easy to see that there is a cut set S^* such that $|S^*| = \frac{n}{2}-1$ then $\omega(P_n - S^*) = \frac{n}{2}$ and $\overline{m}(P_n - S^*) = \frac{n+6}{n+2}$. From the definition of mean rupture degree, we have

$$mr(P_n) \geq \omega(P_n - S^*) - |S^*| - \overline{m}(P_n - S^*) = \frac{n}{2} - \left(\frac{n}{2}-1\right) - \frac{n+6}{n+2} = \frac{-4}{n+2}. \tag{5}$$

If $r \geq \frac{n}{2}$, then $\omega(P_n - S) \leq n-r$ and $\overline{m}(P_n - S) \geq 1$. Hence

$$\omega(P_n - S) - |S| - \overline{m}(P_n - S) \leq n-r-r-1 = n-2r-1$$

the function $f(r)$ takes its maximum value at $r = \frac{n}{2}$ and we get

$$\omega(P_n - S) - |S| - \overline{m}(P_n - S) \leq n-2.\frac{n}{2}-1$$

$$\omega(P_n - S) - |S| - \overline{m}(P_n - S) \leq -1 \tag{6}$$

By (4), (5) and (6) we have $mr(P_n) = \frac{-4}{n+2}$. ▌

Theorem 2.2. *For n≥4, then,*

$$mr(C_n) = \begin{cases} -\frac{n+5}{n+1}, & if\ n\ is\ odd, \\ -1, & if\ n\ is\ even. \end{cases}$$

Proof. The proof is very similar to that of Theorem 2.1. ▌

Theorem 2.3. *Let $K_{m,n}$ be a complete bipartite graph . Then,*

$$mr(K_{m,n}) = \begin{cases} m-n-1, if\ n < m, \\ n-m-1, if\ n \geq m. \end{cases}$$

Proof. Assume $n < m$. Let S be a cut set of $K_{m,n}$ and $|S| = r$.

If $r < n$, then we have $\omega(K_{m,n}-S)=1$. So it contradicts to the definition of mean rupture degree.

If $r \geq n$, then $\omega(K_{m,n}-S) \leq m+n-r$ and $\overline{m}(K_{m,n}-S) \geq 1$. Hence

$$\omega(K_{m,n}-S) - |S| - \overline{m}(K_{m,n}-S) \leq m+n-r-r-1 = m+n-2r-1$$

the function $f(r)$ takes its maximum value at r = n and we get

$$\omega(K_{m,n}-S) - |S| - \overline{m}(K_{m,n}-S) \leq m+n-2n-1$$

$$\omega(K_{m,n}-S) - |S| - \overline{m}(K_{m,n}-S) \leq m-n-1.$$

It is easy to see that there is a cut set S^* such that $|S^*| = n$ then $\omega(K_{m,n}-S^*) = m$ and $\overline{m}(K_{m,n}-S^*) = 1$. From the definition of mean rupture degree, we have

$$mr(K_{m,n}) \geq \omega(K_{m,n}-S) - |S| - \overline{m}(K_{m,n}-S) = m-n-1.$$

This implies that $mr(K_{m,n}) = m-n-1$.

By symmetry, when $n \geq m$, $mr(K_{m,n}) = n-m-1$.

Finally, we have

$$mr(K_{m,n}) = \begin{cases} m-n-1, & if\ n < m, \\ n-m-1, & if\ n \geq m. \end{cases} \qquad \blacksquare$$

It is obvious that we can give the following equlity for the mean rupture degree of $K_{1,n}$.

- The mean rupture degree of the star $K_{1,n}$ is $n-2$.

The wheel graph with n spokes, W_n, is the graph that consists of an *n-cycle* and one additional vertex, say u, that is adjacent to all the vertices of the cycle. In Figure 2, we display W_6.

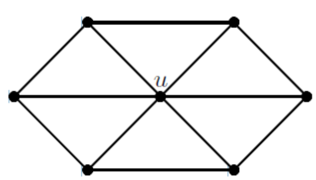

Fig. 2. The wheel graph W_6.

Theorem 2.4. *Let W_n be a wheel graph of order $n(\geq 4)$. Then*

$$mr(W_n) = \begin{cases} -\dfrac{2n+6}{n+1}, & if\ n\ is\ odd, \\ -2, & if\ n\ is\ even. \end{cases}$$

Proof. The graph W_n has a subgraph C_n and $K_{1,n}$. Let S be a cut set of W_n. If $u \notin S$, then $\omega(W_n-S)=1$. So it contradicts to definition of mean rupture degree. If $u \in S$, then we get $\omega(W_n-S) = C_n$. So we have

$$mr(W_n) = mr(C_n) - 1. \qquad \blacksquare$$

The *comet* $C_{t,r}$ to be the graph obtained by identifying one end of the path P_t with the center of the star $K_{1,r}$.

Theorem 2.5. *Let $C_{t,r}$ be a comet graph . Then,*

$$mr(C_{t,r}) = \begin{cases} \dfrac{2r^2+tr-r-t-5}{2r+t+1}, & if\ t\ is\ odd, \\ r-1, & if\ t\ is\ even. \end{cases}$$

Proof. Let S be a cut set of $C_{t,r}$ and $|S| = s$. We have the following two cases, depending on t.

Case 1. We consider the case when t is odd.

If $s \le \frac{t-1}{2}$, then we have $\omega(C_{t,r}-S) \le s+r$ and $\overline{m}(C_{t,r}-S) \ge \frac{r.1^2+(s-1)1^2+2^2}{t+r-s}$. So,

$$\omega(C_{t,r}-S) - |S| - \overline{m}(C_{t,r}-S) \le s + r - s - \frac{r.1^2+(s-1)1^2+2^2}{t+r-s}$$

$$\omega(C_{t,r}-S) - |S| - \overline{m}(C_{t,r}-S) \le r - \frac{r.1^2+(s-1)1^2+2^2}{t+r-s} \qquad (7)$$

It is easy to see that there is a cut set S^* such that $|S^*| = \frac{t-1}{2}$ then $\omega(C_{t,r}-S^*) = \frac{t-1}{2}+r$ and $\overline{m}(C_{t,r}-S^*) = \dfrac{r.1^2+(\frac{t-1}{2}-1)1^2+2^2}{t+r-\frac{t-1}{2}}$. From the definition of mean rupture degree, we have

$$mr(C_{t,r}) \ge \omega(C_{t,r}-S^*) - |S^*| - \overline{m}(C_{t,r}-S^*) = r - \frac{2r+t+5}{2r+t+1} \qquad (8)$$

If $s > \frac{t-1}{2}$, then $\omega(C_{t,r}-S) \le r + t - s$ and $\overline{m}(C_{t,r}- S) \ge 1$. Hence

$$\omega(C_{t,r}-S) - |S| - \overline{m}(C_{t,r}-S) \le r + t - s - s - 1 = r + t - 2s - 1$$

the function $f(r)$ takes its maximum value at $r = \frac{t-1}{2}+1$ and we get

$$\omega(C_{t,r}-S) - |S| - \overline{m}(C_{t,r}-S) \le r + t - 2.\left(\frac{t-1}{2}+1\right) - 1$$

$$\omega(C_{t,r}-S) - |S| - \overline{m}(C_{t,r}-S) \le r - 4 \qquad (9)$$

By (7), (8) and (9) we have $mr(C_{t,r}) = \frac{2r^2+tr-r-t-5}{2r+t+1}$.

Case 2. We consider the case when t is even.

If $s \le \frac{t}{2}$, then we have $\omega(C_{t,r} - S) \le s + r$ and $\overline{m}(C_{t,r} - S) \ge 1$. So,

$$\omega(C_{t,r} - S) - |S| - \overline{m}(C_{t,r} - S) \le s + r - s - 1$$
$$\omega(C_{t,r} - S) - |S| - \overline{m}(C_{t,r} - S) \le r - 1. \qquad (10)$$

It is easy to see that there is a cut set S^* such that $|S^*| = \frac{t}{2}$ then $\omega(C_{t,r} - S^*) = \frac{t}{2} + r$ and $\overline{m}(C_{t,r} - S^*) = 1$. From the definition of mean rupture degree, we have

$$mr(C_{t,r}) \ge \omega(C_{t,r} - S^*) - |S^*| - \overline{m}(C_{t,r} - S^*) = \frac{t}{2} + r - \frac{t}{2} - 1 = r - 1 \qquad (11)$$

If $s > \frac{t}{2}$, then $\omega(C_{t,r} - \mathrm{S}) \le r + t - s$ and $\overline{m}(C_{t,r} - S) \ge 1$. Hence

$$\omega(C_{t,r} - S) - |S| - \overline{m}(C_{t,r} - S) \le r + t - s - s - 1 = r + t - 2s - 1$$

the function $f(r)$ takes its maximum value at $r = \frac{t}{2} + 1$ and we get

$$\omega(C_{t,r} - S) - |S| - \overline{m}(C_{t,r} - S) \le r + t - 2.\left(\frac{t}{2} + 1\right) - 1$$
$$\omega(C_{t,r} - S) - |S| - \overline{m}(C_{t,r} - S) \le r - 3 \qquad (12)$$

By (10), (11) and (12) we have $mr(C_{t,r}) = r - 1$. ▮

3. Relationships between mean rupture degree and other graph parameters

In this section some lower and upper bounds are given for the mean rupture degree of a graph using different graph parameters.

Theorem 3.1. *Let G be a q-connected graph of order n. Then,*

$$mr(G) \le n - 2q - 1.$$

Proof. Let S be a cut set of graph G. Then we have $|S| \ge q$, $\omega(G - S) \le n - q$ and $\overline{m}(G - S) \ge 1$. So

$$\omega(G - S) - |S| - \overline{m}(G - S) \le n - q - q - 1.$$
$$mr(G) \le n - 2q - 1.$$ ▮

Theorem 3.2. *Let G be a q-connected graph of order n and* $\beta(G)$ *is the covering number of G. If* $\beta(G) = q$, *then* $mr(G) = n - 2q - 1$.

Proof. Let S be a cut set of graph G. Then we have the following three cases, depending on S.

Case1. If $|S| < q$, then we have $\omega(G - S) = 1$. So it contradicts to definition of mean rupture degree.

Case2. If $|S| = q$, then we have $\omega(G-S) = n-\beta(G) = \alpha(G)$ and $\overline{m}(G-S) = 1$. Therefore,

$$\omega(G-S) - |S| - \overline{m}(G-S) = n-q-q-1$$
$$mr(G) = n-2q-1. \quad (13)$$

Case3. If $|S| > q$, then $\overline{m}(G-S) \geq 1$ and $\omega(G-S) \leq n - q$. So,

$$\omega(G-S) - |S| - \overline{m}(G-S) \leq n-q-q-1.$$
$$mr(G) \leq n-2q-1. \quad (14)$$

The proof is completed by (13) and (14). ∎

Theorem 3.3. *Let G be a connected graph of order n and $\delta(G)$ be the minimum degree of G.* Then,

$$mr(G) \leq n-2\delta(G) - 1.$$

Proof. Let S be a cut set of graph G. Then we have $|S| \geq \delta(G)$, $\omega(G-S) \leq n-\delta(G)$ and $\overline{m}(G-S) \geq 1$. So

$$\omega(G-S) - |S| - \overline{m}(G-S) \leq n-\delta(G) - \delta(G) - 1.$$
$$mr(G) \leq n-2\delta(G) - 1.$$ ∎

Theorem 3.4. *Let G be a connected graph of order n and $\beta(G)$, $\alpha(G)$ is the covering number and independent number of G respectively. Then,*

$$mr(G) \geq \alpha(G) - \beta(G) - 1.$$

Proof. It is easy to see that there is a cut set S^* such that $|S^*| = \beta(G)$ then $\omega(G-S^*) = n-\beta(G) = \alpha(G)$ and $\overline{m}(G-S^*) = 1$. From the definition of mean rupture degree, we have

$$mr(G) \geq \omega(G-S^*) - |S^*| - \overline{m}(G-S^*) \geq \alpha(G) - \beta(G) - 1.$$ ∎

Theorem 3.5. *Let G be a graph of order n. Then* $mr(G) = n-3$ iff $\beta(G) = 1$.

Proof. Let S be an mr-set of G. Since $mr(G) = \omega(G-S) - |S| - \overline{m}(G-S) = n-3$ by the hypotesis, we have $|S| = 1$, $\omega(G-S) = n-1$ and $\overline{m}(G-S) = 1$. (Otherwise, if $|S| = 2$, then $\omega(G-S) = n-2$ and $\overline{m}(G-S) = -1$. But $\overline{m}(G-S)$ must be at least 1 for every graph G – S). Then

$\overline{m}(G-S) = \frac{\sum_{i=1}^{k} p_i^2}{\sum_{i=1}^{k} p_i} = 1$ and so $|p_i| = 1$ for every i. That is, each one of components of

$G-S$ is isolated vertex. Hence S is a cover set and $|S| = \beta(G) = 1$.

On the other hand, If we remove only one vertex, then we have the isolated vertices by $\beta(G)=1$. Hence $\omega(G-S) = n-1$, $\overline{m}(G-S) = \frac{1^2+1^2+\ldots+1^2}{1+1+\ldots+1} = 1$ and $|S| = 1$. Therefore,

$$mr(G) = n-1-1-1 = n-3.$$ ∎

4. Conclusion

A network has often as considerable an impact on network's performance as the edges or vertices themselves. Performance measures for the networks are essential to guide the designer in choosing an appropriate topology. In order to measure the performance we are interested the following performance metrics:

1. The number of elements that are not functioning,
2. The number of the components of the remaining network,
3. The size of a largest remaining group within which mutual communication can still occur.

Many graph-theoretical parameters have been used in the past to describe the stability of communication networks. Most of these parameters do not take into account what remains after the graph is disconnected. We can say that the disruption is more successful if the disconnected network contains more components, and is much more successful if, in addition, the components are small. We can associate the cost with the number of vertices destroyed to obtain small components and associate the benefit with the number of components remaining after destruction.

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