UNCERTAIN MULTI-OBJECTIVE RESTRICTED SOLID TRANSPORTATION PROBLEM WITH BUDGET AND VEHICLE COST

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In this paper, we investigate six new transportation models with breakability and vehicle cost under some restriction on transported amount. An extra constraint on the total budget at each destination is imposed. Here six models are formulated under different environments such as crisp, stochastic and fuzzy. Using expected value of fuzzy number and chance constraint programming technique, we convert the respective fuzzy and random Models into its crisp equivalent. To get the preference of the objective function, we apply weighted sum method and a gradient based optimisation technique-generalised reduced gradient (GRG) method are applied and using LINGO-13 software to get the optimal solutions.

Keywords: Solid Transportation Problem (STP), Budget Constraint, Interval Type-2 Fuzzy Number, Stochastic Variable, Weighted Sum Method

1. Introduction

Hitchcock [14] originally developed the transportation problem in 1941 with his research paper. This extra constraint is mainly due to modes of transportation (conveyance). The STP was stated by Shell [27]. Haley [12, 13] showed a comparison of the STP to the classical TP. Bit et al. [3] applied fuzzy programming technique to solve the MOSTP which is introduced by Zimmermann [35]. Zadeh [32] introduces the notion of fuzziness. Li et al. [20] improved genetic algorithm to solve the fuzzy multi-objective STP. The random STP was first described by Elmaghra [10] in 1960. The classic approach to the stochastic transportation problem is the application of the feasible direction method described by Cooper and Leblanc [8] and Cooper [9] in 1977 and 1978 respectively. Holmberg et al. [17] and Holmberg [16] applied several linearization and decomposition methods to solve the stochastic transportation problem. A

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transportation problem is said to be a chance constrained problem [4, 5] if its linear constraints are associated with a set of probability. Kataoka [19] proposed a stochastic programming model which considered the distribution of both objective function, probabilistic constraints and applied to a single objective transportation problem. Recently Baidya et al. [1, 2] solve two problems based on safety factors and uncertainty in transportation problem. Also, the distances between the origins and destinations are not taken into account in the network problems. Most of the transportation problems are unbalanced for breakable items. Few of these items are glass-goods, toys, ceramic goods, etc.

A type-2 fuzzy set was proposed by Zadeh [33]. Type-2 fuzzy sets are described by both primary and secondary membership to provide more degrees of freedom and flexibility. Type-2 fuzzy sets have the advantage of modeling uncertainty more accurately compared with type-1 fuzzy sets. However, when type-2 fuzzy sets are employed to solve problems, computational burden is heavy [17]. Hence, interval type-2 fuzzy sets are extensively utilized with some relative representations such as vertical slice representation, wavy-slice representation to reduce dimensions, which are extremely useful for computation and theoretical studies [23]. Interval type-2 fuzzy sets can be viewed as a special case of general type-2 fuzzy sets that all the values of secondary membership are equal to 1. Hence, it not only represents uncertainty better than type-1 fuzzy sets, also simplifies the computation compared with type-2 fuzzy sets. Research studies in this field can be categorized into two aspects. One aspect is the theoretic research. Mendel et al. [17] proposed some basic definitions of interval type-2 fuzzy sets. Mitchell [24] and Zeng and Li [34] designed methods to calculate the similarity among interval type-2 fuzzy sets. To reduce the limitations in these methods, Wu and Mendel [29] developed a new method named vector similarity method (VSM) to transform interval type-2 fuzzy sets. The other aspect is the application of interval type-2 fuzzy sets in real world. Ondrej and Milos [26] employed interval type-2 fuzzy sets to develop fuzzy voter design for fault tolerant systems. Shu and Liang [28] proposed a new approach based on interval type-2 fuzzy logic systems to analyze and estimate the network lifetime for wireless sensor networks. However, few studies have focused on the application of interval type-2 fuzzy sets in solving multi-criteria decision making problems. Wu and Mendel [30] defined linguistic weighted average and employed it to deal with hierarchical multi-criteria decision-making problems. Han and Mendel [15] employed interval type-2 fuzzy numbers in choosing logistics location and the results are more satisfactory. Chen and Lee [6] proposed the definition of possibility degree of trapezoidal interval type-2 fuzzy number and some arithmetic operations of it. Also Hu et al.[18] proposed a work as Multi-criteria decision making method based on possibility degree of interval type-2 fuzzy number.

Two types of uncertainities (stochastic and fuzzy) are used to build this manuscript. An extra constraint on the total budget at each destination is imposed. To derive the crisp equivalences of the stochastic and fuzzy model we apply chance-constrained programming and expected value model are applied respectively. To formulate the fuzzy models we consider unit transportation costs, supplies, demand, capacity of the conveyances and budget at each destination as interval type-2 fuzzy number. To convert the multi-objective into a single-objective we apply weighted sum method. We have presented two types of constraints one deterministic, another uncertain both fuzzy and stochastic senses. So our technique is highly fruitful in the sense of real life problems of practical importance. Practical numerical examples are provided to demonstrate the feasibility of all decision variables of the proposed methods.

2. Trapezoidal Interval Type-2 Fuzzy Number

Definition 1 (Defuzzification of Trapezoidal Interval Type-2 Fuzzy Number) [6, 17, 22]:
A trapezoidal interval type-2 fuzzy number, denoted by A, is expressed as follows:
$$A = (A^U, A^L) = \left(\left(a_1^U, a_2^U, a_3^U, a_4^U; H_1(A^U), H_2(A^U) \right), \left(a_1^L, a_2^L, a_3^L, a_4^L; H_1(A^L), H_2(A^L) \right) \right),$$
then the expected value of A defined as follows:

$$E(A) = \frac{1}{2} \left(\frac{1}{4} \sum_{i=1}^{4} (a_i^L + a_i^U) \right) \times \frac{1}{4} \left(\sum_{i=1}^{2} (H_i(A^L) + H_i(A^U)) \right) \tag{1}$$

3. Method used to convert constraints involving stochastic variables into its deterministic form (Chance Constraint Programming)

This technique was originally developed by Charnes and Cooper [4, 5, 8, 9] and as follows:
(i) If ε are the probabilities of non-violation of the constraint $\delta \leq \hat{\tau}$ then the constraint can be written as

$$\text{Prob}[\delta \leq \hat{\tau}] \geq \varepsilon \tag{2}$$
$$\Rightarrow \delta \leq m(\hat{\tau}) + \phi * Var(\hat{\tau}) \tag{3}$$

(ii) If ε are the probabilities of non-violation of the constraint $\hat{a} \geq \tilde{b}_1$ then the constraint can be written as

$$Prob[\hat{a} \geq \tilde{b}_1] \geq \varepsilon \tag{4}$$
$$\Rightarrow m_{\hat{q}} \geq \lambda \sigma_{\hat{q}}^2 \tag{5}$$
where $\hat{q} = \hat{a} - \tilde{b}_1$ and λ be the real number such that $Prob[\hat{T} \geq \lambda]$.

The objective function Z will also be the random variable, since $\hat{C}_{ijk}$ are random variables. The mean and variance of Z are given by $\bar{Z} = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \bar{C}_{ijk} x_{ijk}$. If

the random variable $\hat{C}_{ijk}$ are independent, the object function reduce to $Z(X) =$

$$\sum_{i=1}^{M} \theta_{1i} \sum_{j=1}^{N} \sum_{k=1}^{K} \bar{C}_{ijk} x_{ijk} + \sum_{i=1}^{M} \theta_{2i} \sum_{j=1}^{N} \sum_{k=1}^{K} \sqrt{Var(\hat{C}_{ijk})} \qquad (6)$$

and subject to given constraints.

4. Method used to reduce the respective multi-objective transportation models into single objective transportation models (Weighted Sum Method)

The weights of an objective are usually chosen in proportion to the objective's relative importance in the problem. A composite objective function F can be formed by summing the weighted normalized objectives and the MOSTP is then converted to a single-objective optimization problem as follows:

$$\text{Minimize } F = \sum_{l=1}^{L} \omega_l f_l , \omega_l \in [0,1].$$

Here, ω_l is the weight of the *l*-th objective function. Since the minimum of the above problem does not change if all the weights are multiplied by a constant, it is the usual practice to choose weights such that their sum is one, i.e., $\sum_{l=1}^{L} \omega_l = 1$. The objective functions of the MOSTP conflict with each other, a complete optimal solution (11, 25) does not always exist and so non-dominated optimality concept is introduced.

5. Notations and Assumptions

The following notation and assumption are used throughout the model.

(i) $C_{ijk}, \tilde{C}_{ijk}, \hat{C}_{ijk}$: Crisp, fuzzy, random unit transportation cost to transport the commodity from i-th plant to j-th destination by k-th conveyances respectively.

(ii) $t_{ijk}, \tilde{t}_{ijk}, \hat{t}_{ijk}$: Crisp, fuzzy, random transportation time to transport the commodity from i-th plant to j-th destination by k-th conveyances respectively.

(iii) $a_i, \tilde{a}_i, \hat{a}_i$: Crisp, fuzzy, random amount of homogeneous product available at i-th plant respectively.

(v) $b_j, \tilde{b}_j, \hat{b}_j$: Crisp, fuzzy, random demand at the j-th destination respectively.

(vi) $e_k, \tilde{e}_k, \hat{e}_k$: Crisp, fuzzy, random amount of product which can be carried by the k-th conveyance respectively.

(vi) $B_j, \tilde{B}_j, \hat{B}_j$: Crisp, fuzzy, random available budget at j-th destination respectively.

(vii) x_{ijk}: Unknown quantity which is to transport the commodity from i-th plant to j-th destination by k-th conveyances (decision variable).

(viii) If the unknown quantity which is to be transported from i-th source to j-th destination by k-th conveyances is $x_{ijk} > 0$ then for the convenience of modeling we define y_{ijk} as follows:

$$y_{ijk} = \begin{cases} 1 & for \quad x_{ijk} > 0 \\ 0 & otherwise \end{cases}$$

(ix) If in a particular destination the negligible amount of quantity (p, say) is transported then the decision maker (DM) can't deliver commodities in the particular destination. This means, if $x_{ijk} \geq p$, a desired real number, then we consider the restriction for this route as a part of the transportation. Thus for the expediency of modeling, the following notation is introduced:

$$z_{ijk} = \begin{cases} 1 & for \quad x_{ijk} \geq p \\ 0 & otherwise \end{cases}$$

6. Model Formulation

Model-1: Multi-Objective Solid Transportation Problem (MOSTP) with budget constraint and vehicle cost in Crisp Environment:

To transport the commodity from plant to customer by k-th conveyances, the budget at customer plays a vital role in transportation problem. Here we formulate a MOSTP with M plants, N customers and K conveyances as follows:

$$Min\, f_1 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} C_{ijk}x_{ijk} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} F(x_{ijk}),$$

$$Min\, f_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} t_{ijk}y_{ijk}$$

subject to the constraints,

$$\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq a_i \qquad (7)$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq b_j \qquad (8)$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq e_k \qquad (9)$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} C_{ijk}x_{ijk} \leq B_j \qquad (10)$$

$x_{ijk} \geq 0, \forall\, i, j, k$, where $F(x_{ijk})$ is the vehicle carrying cost for the quality x_{ijk} from i-th source O_i to j-th destination D_j via k-th conveyance is defined as: $F(x_{ijk}) = \begin{cases} m.v\ if\ m.v_c = x_{ijk} \\ (m+1).v\ otherwise \end{cases}$, $m = [x_{ijk}/v_c]$, v_c =vehicle capacity and v =vehicle cost.

Model-2: Restricted MOSTP with budget constraint and vehicle cost in Crisp Environment:

Here DM put a restriction on the transported amount p such that the DM consider those rout where the transported amount is greater than or equal to the restricted amount p, otherwise DM cannot transport the amount through the rout. Taking the above concept we formulate the following MOSTP:

$$Min\ f_1 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} C_{ijk}x_{ijk}z_{ijk} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} F(x_{ijk}),$$
$$Min\ f_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} t_{ijk}z_{ijk}$$

Subject to the constraints (7), (8), (9) and

$$\sum_{i=1}^{M}\sum_{k=1}^{K} C_{ijk}x_{ijk}z_{ijk} \leq B_j, \tag{11}$$

$x_{ijk} \geq p, \forall i,j,k$, where p is any desired real value.

Model-3: MOSTP with trapezoidal interval type-2 fuzzy number, budget constraint and vehicle cost:

We formulate a MOSTP with M plants, N customers and K conveyances and all supplies, demands, conveyances capacities, unit transportation cost, time and budget at each customer as trapezoidal interval type-2 fuzzy number as follows:

$$Min\ f_1 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\tilde{C}_{ijk}x_{ijk} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} F(x_{ijk})\ \text{and}$$
$$Min\ f_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\tilde{t}_{ijk}y_{ijk}$$

subject to the constraints,

$$\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \tilde{a}_i \tag{12}$$
$$\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \tilde{b}_j \tag{13}$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \tilde{e}_k, \tag{14}$$
$$\sum_{i=1}^{M}\sum_{k=1}^{K} \tilde{C}_{ijk}x_{ijk} \leq \tilde{B}_j \tag{15}$$

$x_{ijk} \geq 0, \forall i,j,k.$

Crisp Transformation of the above fuzzy model:

The deterministic objective functions of the above model are as follows:

$$Min\ f_1 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(\frac{1}{8}(C_1) \times \frac{1}{4}(C_2))x_{ijk} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} F(x_{ijk})$$

and $Min\ f_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(\frac{1}{8}(T_1) \times \frac{1}{4}(T_2))y_{ijk}$ respectively,

where, $C_1 = C_{ijk1}^{U} + C_{ijk2}^{U} + C_{ijk3}^{U} + C_{ijk4}^{U} + C_{ijk1}^{L} + C_{ijk2}^{L} + C_{ijk3}^{L} + C_{ijk4}^{L}$, $C_2 = H_1(C_{ijk}^{U}) + H_2(C_{ijk}^{U}) + H_1(C_{ijk}^{L}) + H_2(C_{ijk}^{L})$, $T_1 = t_{ijk1}^{U} + t_{ijk2}^{U} + t_{ijk3}^{U} + t_{ijk4}^{U} + t_{ijk1}^{L} + t_{ijk2}^{L} + t_{ijk3}^{L} + t_{ijk4}^{L}$, $T_2 = H_1(t_{ijk}^{U}) + H_2(t_{ijk}^{U}) + H_1(t_{ijk}^{L}) + H_2(t_{ijk}^{L})$.

The reduced Constraints of the above model are as follows:
$$\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \frac{1}{8}(A_1) \times \frac{1}{4}(A_2), \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \frac{1}{8}(B_1) \times \frac{1}{4}(B_2),$$
$$\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \frac{1}{8}(E_1) \times$$
$\frac{1}{4}(E_2)$ and $\sum_{i=1}^{M}\sum_{k=1}^{K}\frac{1}{8}(C_1).\frac{1}{4}(C_2)x_{ijk} \leq \frac{1}{8}(BC_1).\frac{1}{4}(BC_2)$ respectively.
where,
$A_1 = a_{i1}^{U} + a_{i1}^{U} + a_{i1}^{U} + a_{i1}^{U} + a_{i1}^{L} + a_{i1}^{L} + a_{i1}^{L} + a_{i1}^{L},$
$A_2 = H_1(a_i^{U}) + H_2(a_i^{U}) + H_1(a_i^{L}) + H_2(a_i^{L}),$
$B_1 = b_{j1}^{U} + b_{j1}^{U} + b_{j1}^{U} + b_{j1}^{U} + b_{j1}^{L} + b_{j1}^{L} + b_{j1}^{L} + b_{j1}^{L},$
$B_2 = H_1(b_j^{U}) + H_2(b_j^{U}) + H_1(b_j^{L}) + H_2(b_j^{L}),$
$E_1 = e_{k1}^{U} + e_{k1}^{U} + e_{k1}^{U} + e_{k1}^{U} + e_{k1}^{L} + e_{k1}^{L} + e_{k1}^{L} + e_{k1}^{L},$
$E_2 = H_1(e_k^{U}) + H_2(e_k^{U}) + H_1(e_k^{L}) + H_2(e_k^{L}),$
$BC_1 = B_{j1}^{U} + B_{j2}^{U} + B_{j3}^{U} + B_{j4}^{U} + B_{j1}^{L} + B_{j2}^{L} + B_{j3}^{L} + B_{j4}^{L},$
$BC_2 = H_1(B_j^{U}) + H_2(B_j^{U}) + H_1(B_j^{L}) + H_2(B_j^{L}).$

Model-4: Restricted MOSTP trapezoidal interval type-2 fuzzy number, budget constraint and vehicle cost:

Here we formulate a Restricted MOSTP with M plants, N customers and K conveyances and all supplies, demands, conveyances capacities, unit transportation cost, time and budget at each customer as trapezoidal interval type-2 fuzzy number as follows:
$$Min\, f_1 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \tilde{C}_{ijk}x_{ijk}z_{ijk} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} F(x_{ijk})$$
$$\text{and } Min\, f_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \tilde{t}_{ijk}z_{ijk}$$
subject to the constraints (12), (13), (14) and
$$\sum_{i=1}^{M}\sum_{k=1}^{K} \tilde{C}_{ijk}x_{ijk}z_{ijk} \leq \tilde{B}_j, \tag{16}$$
$x_{ijk} > p$, where p is any desired real value.

Crisp Transformation of the above fuzzy model:
Using expected value model we have the reduced crisp objective functions are as follows:
$$Min\, f_1 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(\frac{1}{8}(C_1) \times \frac{1}{4}(C_2))x_{ijk}z_{ijk} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} F(x_{ijk}).$$
$$Min\, f_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}(\frac{1}{8}(T_1) \times \frac{1}{4}(T_2))z_{ijk} \text{ respectively.}$$
The crisp transformations of the constraints (12), (13), (14) are same as model-3 and crisp transformation of the constraint (16) is as follows:
$$\sum_{i=1}^{M}\sum_{k=1}^{K}\frac{1}{8}(C_1).\frac{1}{4}(C_2)x_{ijk}z_{ijk} \leq \frac{1}{8}(BC_1).\frac{1}{4}(BC_2)$$

Model-5: MOSTP with budget constraint and vehicle cost in Stochastic Environment:

Sometimes it may happen that the demand or any factor of a commodity in the society is uncertain, not precisely known, but some past data about it is available. For this purpose we consider the supplies, demands, conveyances capacities, unit transportation cost, time and budget at each customer as stochastic variable and we formulate a model as follows:

$$Min \; f_1 = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \hat{C}_{ijk} x_{ijk} + \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} F(x_{ijk}) \; and$$
$$Min \; f_2 = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} \hat{t}_{ijk} y_{ijk}$$

Subject to the constraints,

$$\sum_{j=1}^{N} \sum_{k=1}^{K} x_{ijk} \leq \hat{a}_i \qquad (17)$$

$$\sum_{i=1}^{M} \sum_{k=1}^{K} x_{ijk} \geq \hat{b}_j \qquad (18)$$
$$\sum_{i=1}^{M} \sum_{j=1}^{N} x_{ijk} \leq \hat{e}_k, \qquad (19)$$
$$\sum_{i=1}^{M} \sum_{k=1}^{K} \hat{C}_{ijk} x_{ijk} \leq \hat{B}_j \qquad (20)$$
$$x_{ijk} \geq 0, \forall \; i,j,k.$$

Crisp Transformation of the above stochastic model:

We convert the stochastic model-5 into its equivalence crisp model using the chance constraint programming technique.

Using Chance Constraint programming technique the objective functions are respectively reduced to,

$$Min \; f_1 = \theta_{11} \left(\sum_{j=1}^{N} \sum_{k=1}^{K} \bar{C}_{1jk} x_{1jk} \right) + \theta_{12} \left(\sum_{j=1}^{N} \sum_{k=1}^{K} \bar{C}_{2jk} x_{2jk} \right) + \theta_{21} \left(\sum_{j=1}^{N} \sum_{k=1}^{K} \sqrt{Var(\hat{C}_{1jk}) x_{ijk}^2} \right)$$

$$+ \; \theta_{22} \left(\sum_{j=1}^{N} \sum_{k=1}^{K} \sqrt{Var(\hat{C}_{1jk}) x_{ijk}^2} \right) + \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} F(x_{ijk}) \; and$$

$$Min \; f_2 = \theta_{31} \left(\sum_{j=1}^{N} \sum_{k=1}^{K} \bar{t}_{1jk} y_{1jk} \right) + \theta_{32} \left(\sum_{j=1}^{N} \sum_{k=1}^{K} t_{2jk} y_{2jk} \right)$$

$$+ \theta_{41} \left(\sum_{j=1}^{N} \sum_{k=1}^{K} \sqrt{Var(\hat{t}_{1jk}) y_{ijk}^2} \right) + \theta_{42} \left(\sum_{j=1}^{N} \sum_{k=1}^{K} \sqrt{Var(t_{1jk}) y_{ijk}^2} \right).$$

Also the constraints (17), (18), (19) and (20) reduced to, $\sum_{j=1}^{N} \sum_{k=1}^{K} x_{ijk} \leq \bar{a}_i + \phi_1 . var(\hat{a}_i)$, $\sum_{i=1}^{M} \sum_{k=1}^{K} x_{ijk} \geq \bar{b}_j + \phi_2 . var(\hat{b}_j)$, $\sum_{i=1}^{M} \sum_{j=1}^{N} x_{ijk} \leq \bar{e}_k + var(\hat{e}_k)$ and $\sum_{i=1}^{M} \sum_{k=1}^{K} \bar{C}_{ijk} x_{ijk} - \bar{B}_j \leq \lambda_j . \left(\sum_{i=1}^{M} \sum_{k=1}^{K} var(\hat{C}_{ijk}) x_{ijk} - var(\hat{B}_j) \right)$ respectively.

Model-6: Restricted MOSTP with budget constraint and vehicle cost in Stochastic Environment:

Here we formulate a Restricted MOSTP with M plants, N customers and K conveyances and all supplies, demands, conveyances capacities, unit

transportation cost, time and budget at each customer as stochastic variable as follows:

$Min\ f_1 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\hat{C}_{ijk}x_{ijk}z_{ijk} + \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}F(x_{ijk})$ and

$Min\ f_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\hat{t}_{ijk}z_{ijk}$

Subject to the constraints (17), (18), (19) and $\sum_{i=1}^{M}\sum_{k=1}^{K}\hat{C}_{ijk}x_{ijk}z_{ijk} \leq \hat{B}_j$, (21)

$x_{ijk} \geq p$, where p is any desired real value.

Crisp Transformation of the above stochastic model:

Applying Chance Constraint programming we have the reduced crisp objective function are as follows:

$$Min\ f_1 = \theta_{11}\left(\sum_{j=1}^{N}\sum_{k=1}^{K}\bar{C}_{1jk}x_{1jk}z_{1jk}\right) + \theta_{12}\left(\sum_{j=1}^{N}\sum_{k=1}^{K}\bar{C}_{2jk}x_{2jk}z_{2jk}\right) + \theta_{21}$$

$$\left(\sum_{j=1}^{N}\sum_{k=1}^{K}\sqrt{Var(\hat{C}_{1jk})x_{1jk}^2 z_{1jk}^2}\right) + \theta_{22}\left(\sum_{j=1}^{N}\sum_{k=1}^{K}\sqrt{Var(\hat{C}_{1jk})x_{2jk}^2 z_{2jk}^2}\right)$$

$$+ \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}F(x_{ijk}).$$

$$Min\ f_2 = \theta_{31}\left(\sum_{j=1}^{N}\sum_{k=1}^{K}\bar{t}_{1jk}z_{1jk}\right) + \theta_{32}\left(\sum_{j=1}^{N}\sum_{k=1}^{K}\bar{t}_{2jk}z_{2jk}\right)$$

$$+\theta_{41}\left(\sum_{j=1}^{N}\sum_{k=1}^{K}\sqrt{Var(\hat{t}_{1jk})z_{ijk}^2}\right) + \theta_{42}\left(\sum_{j=1}^{N}\sum_{k=1}^{K}\sqrt{Var(t_{1jk})z_{ijk}^2}\right)$$ respectively.

and the constraint (21) is reduced to

$$\sum_{i=1}^{M}\sum_{k=1}^{K}\bar{C}_{ijk}x_{ijk}z_{ijk} - \bar{B}_j \leq \lambda_j.\left(\sum_{i=1}^{M}\sum_{k=1}^{K}var(\hat{C}_{ijk})x_{ijk}z_{ijk} - var(\hat{B}_j)\right)$$

7. Solution Methodology

Sometime in transportation problem the transportation parameters are vague in nature. For this reason Model-3, 4 and 5, 6 are respectively formulated with fuzzy and random environment. Expected value method and chance constraint programming technique are used to reduce the uncertain STPs into its crisp equivalent and reduced crisp models are solved using GRG-technique (LINGO-13.0 optimization software).

8. Numerical Illustration

A company produces a product at the two warehouses and this item is then shipped to two customers by three different modes of transport with different vehicle cost. The transportation parameters for our respective models are as follows and also for model-2, 4, 6, we restricted the transported amount as 30 units.

Table-1

Crisp Unit Transportation Cost and Time

	Unit Transportation Cost					
i \ j	K=1		K=2		K=3	
	1	2	1	2	1	2
1	25	36	32	35	24	38
2	34	36	36	35	25	31
	Unit Transportation Time					
1	2	3	2	3	3	2
2	2	3	2	3	2	2

$a_1 = 60, a_2 = 90, b_1 = 80, b_2 = 70, e_1 = 50, e_2 = 40, e_3 = 60, B_1 = 3030, B_2 = 3040,$

$\tilde{a}_1 = \big((50,50,60,60; 0.98,0.99), (50,60,70,80; 0.97,0.98)\big),$

$\tilde{a}_2 = \big((70,70,90,100; 0.96,0.99), (80,90,110,120; 0.97,0.99)\big).$

$\tilde{b}_1 = \big((50,70,70,90; 0.95,0.98), (80,80,90,90; 0.97,0.99)\big),$

$\tilde{b}_2 = \big((60,70,70,100; 0.94,0.99), (50,60,70,80; 0.96,0.97)\big).$

$\tilde{e}_1 = \big((40,40,50,60; 0.92,0.93), (40,40,50,70; 0.91,0.99)\big),$

$\tilde{e}_2 = \big((30,40,40,50; 0.90,0.98), (30,40,40,50; 0.98,0.99)\big),$

$\tilde{e}_2 = \big((50,60,60,70; 0.95,0.99), (50,60,60,70; 0.94,0.99)\big).$

$\tilde{B}_1 = \big((3010,3020,3040,3050; 0.98,0.99), (3000,3030,3030,3060; 0.97,0.99)\big),$

$\tilde{B}_2 = \big((3020,3030,3050,3060; 0.96,0.98), (3030,3030,3040,3060; 0.98,0.99)\big).$

$\tilde{C}_{111} = \big((1,3,3,4; .90,.91), (1,2,4,5; .92,.93)\big), \tilde{C}_{121} = \big((2,3,5,5; .91,.94), (2,3,6,8; .93,.95)\big),$

$\tilde{C}_{112} = \big((3,4,5,6; .96,.97), (2,4,4,5; .92,.97)\big), \tilde{C}_{122} = \big((4,5,6,6; .90,.91), (2,3,4,5; .90,.93)\big),$

$\tilde{C}_{113} = \big((2,3,4,5; .95,.99), (1,2,3,3; .92,.97)\big), \tilde{C}_{123} = \big((3,4,5,6; .96,.98), (1,2,3,3; .95,.96)\big),$

$\tilde{C}_{211} = \big((1,2,5,7; .98,.99), (3,3,6,7; .92,.97)\big), \tilde{C}_{221} = \big((3,4,6,7; .90,.91), (3,4,4,5; .92,.93)\big),$

$\tilde{C}_{212} = \big((3,3,4,5; .90,.91), (4,4,5,6; .92,.93)\big), \tilde{C}_{222} = \big((3,5,5,7; .90,.98), (2,4,4,5; .92,.97)\big),$

$\tilde{C}_{213} = \big((2,3,3,5; .95,.97), (2,2,3,4; .93,.99)\big), \tilde{C}_{223} = \big((2,4,4,5; .90,.91), (3,4,5,5; .92,.93)\big).$

$\tilde{t}_{111} = \big((.1,.1,.1,.5; .94,.99), (.2,.2,.2,.4; .92,.94)\big),$

$\tilde{t}_{121} = \big((.1,.2,.2,.3; .91,.94), (.1,.2,.3,.4; .94,.95)\big),$

$\tilde{t}_{112} = \big((.1,.2,.3,.5; .97,.98), (.1,.2,.2,.3; .92,.97)\big),$

$\tilde{t}_{122} = \big((.1,.2,.2.,3; .95,.96), (.1,.1,.2,.2; .91,.93)\big),$

$\tilde{t}_{113} = \big((.2,.2,.3,.4; .98,.99), (.1,.2,.2,.3; .95,.97)\big),$

$\tilde{t}_{123} = \big((.1,.1,.2,.3; .96,.98), (.1,.1,.1,.2; .94,.96)\big),$

$\tilde{t}_{211} = \big((.1,.2,.3,.3; .97,.99), (.1,.1,.2,.2; .93,.97)\big),$

$\tilde{t}_{221} = \big((.1,.1,.2,.3; .94,.96), (.2,.3,.4,.5; .92,.93)\big),$

$\tilde{t}_{212} = \big((.1,.1,.3,.4; .90,.91), (.1,.3,.4,.5; .92,.93)\big),$

$\tilde{t}_{222} = \big((.1,.1,.2,.2; .95,.98), (.1,.1,.2,.3; .92,.97)\big),$

$\tilde{t}_{213} = \big((.2,..3,.3,.3; .95,.97), (.1,.2,.2,.3; .95,.99)\big),$

$\tilde{t}_{223} = \big((.2.,3,.3,.3; .93,.98), (.2,.2,.3,.3; .92,.93)\big).$

$\theta_{11} = .9, \theta_{12} = 1, \theta_{21} = 1, \theta_{22} = .96, \theta_{31} = 1, \theta_{32} = .95, \theta_{41} = .97, \theta_{42} = 1, \phi_1 = .2, \phi_2 = .96, \phi_3 = .2, \phi_4 = .95, \phi_5 = .2, \phi_6 = .5, \phi_7 = .97, \lambda_1 = .99, \lambda_2 = .98, \omega_1 = 0.5, \omega_2 = 0.5, v = 12$ and $v_c = 6$.

Table-2

Mean and Variance of $\hat{a}_i$, $\hat{b}_j$, $\hat{e}_k$, $\hat{B}_j$

	$\hat{a}_1$	$\hat{a}_2$	$\hat{b}_1$	$\hat{b}_2$	$\hat{e}_1$	$\hat{e}_2$	$\hat{e}_3$	$\hat{B}_1$	$\hat{B}_2$
Mean	60	70	69	58	50	37	59	3010	3020
Variance	4	2	5	1	5	5	2	26	20

Table-3

Mean and variance of Transportation Cost ($\hat{C}_{ijk}$) and Time($\hat{t}_{ijk}$)

		$(\bar{C}_{ijk}, Var(\hat{C}_{ijk}))$					
	j	K=1		K=2		K=3	
i		1	2	1	2	1	2
1		(25,12)	(36,13)	(32,12)	(35,13)	(34,13)	(38,12)
2		(34,12)	(36,13)	(36,12)	(35,13)	(25,12)	(31,12)
		$(\bar{t}_{ijk}, Var(\hat{t}_{ijk}))$					
1		(2,.2)	(3,.1)	(2,.3)	(3,.3)	(4,.3)	(2,.2)
2		(2,.1)	(3,.3)	(1,.2)	(3,.3)	(2,.1)	(2,.3)

9. Optimal Result of Different Models

The optimal results for the different models with transporting amount are restricted to 30 units are as follows:

Table-4

Optimal Result of Models

	Model-1	Model-2	Model-3	Model-4	Model-5	Model-6
F_1	4620	5260	4462.5	5110	3982.95	4609.66
F_2	13.1	10.5	8.1	7.8	12.43	9.93
x_{111}	50	0	48.75	0	51	0
x_{211}	0	50	0	47	0	38.06
x_{121}	0	0	0	0	0	0
x_{221}	0	0	0	0	0	0
x_{112}	0	0	0	0	8.29	0
x_{212}	0	0	0	0	0	0
x_{122}	0	0	11.25	0	0	30
x_{222}	40	40	27.5	40	3.86	0
x_{113}	10	30	0	30	0	0
x_{213}	20	0	28.75	0	0	30.94
x_{123}	0	30	0	30	0	30
x_{223}	30	0	31.25	0	51.24	0

10. Analysis of the results

In this paper, we solved six solid transportation models where three models (models-1,-3 and -5) are with restriction and another three models(models-2, -4 and -6) are without restriction. To solve the restricted MOSTP, we neglate the small amount of quantity which is transported from plant

to customer by different modes of transport. If we impose the restriction in the respective multi-objective transportation models, then total transportation cost increase and total transportation time is decreased. If the decision maker (DM) imposed the restriction on transported amount, then the DM cannot transport the amount which is less than the restricted amount. For this reason, the transporting time of that particular type of rout cannot be added into the total time. Due to this reason the total time is less than the total time of unrestricted models. Also that restricted amount can be adjusted through the routes where the amounts are transported and for this reason, the total cost is increases compare to the total cost of unrestricted model i.e. the model-1,-2 and -3. This type of incident we observed in the real life problem.

11. Sensitivity Analysis

If we give more importance to cost function i.e. we increase the weight ω_1, then we notice that the composite objective function value Z will increase. Also if we give more importance to transporting time i.e. ω_2 will increases then the composite objective function Z will decreases. It is as per expectation in real life problem. Similarly, in the next table we presented the sensitivity analysis for the restricted crisp model-2.

Table-5

Sensitivity Analysis of model-1

ω_1	ω_2	$Z = \omega_1.f_1 + \omega_2.f_2$	ω_1	ω_2	$Z = \omega_1.f_1 + \omega_2.f_2$
0.1	0.9	477.07	0.6	0.4	2777.24
0.2	0.8	936.48	0.7	0.3	3237.93
0.3	0.7	1398.17	0.8	0.2	3698.62
0.4	0.6	1855.86	0.9	0.1	4159.31
0.5	0.5	2316.55			

Table-6

Sensitivity Analysis of model-2

ω_1	ω_2	$Z = \omega_1.f_1 + \omega_2.f_2$	ω_1	ω_2	$Z = \omega_1.f_1 + \omega_2.f_2$
0.1	0.9	535.45	0.6	0.4	3167.36
0.2	0.8	1072.72	0.7	0.3	3689.77
0.3	0.7	1595.00	0.8	0.2	4130.00
0.4	0.6	2117.74	0.9	0.1	4675.00
0.5	0.5	2635.25			

12. Conclusion

The main aim of this paper is to present the solution procedure of the with and without restriction multi-objective soild transportation problem. In model-3 &-4 for the first time, we consider the unit transportation penalties, demand source, capacity, budget at each destination as interval type-2 fuzzy number. The

respective model-3 & -4 can be converted into its equivalence crisp model using expected value operator, however to reduce the crisp equivalent of the models-5 &-6, we use chance constraint programming technique. In numerical example, all the models are solve using LINGO. 13.0 optimization software. Also if we follow the results of the different models, we observed that, it is as per our expectation because the transportation time and cost is decreases and increases respectively, if we introduce the restriction in to our models.

The models can be extended to include breakable/deteriorating items, space constraints, price discount, etc. The methods used for solution here are quite general in nature and these can be applied to other similar uncertain / impricise models in other areas such as inventory control, ecology, sustainable farm management, etc.

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