AVAILABILITY ASSESSMENT WITH MONTE-CARLO SIMULATION OF MAINTENANCE PROCESS MODEL

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Nowadays, the maintenance is one of the most important territories of practical engineering. From the mathematical point of view, the operation of technical systems and equipment is a discrete state space stochastic process without after-effects, so it can be approximated with a Markov-chain. After setting up the transition probability matrix, matrix-algebraic tools can be used for investigating these processes with systems approach analysis. This paper is aimed to discuss the possibilities of the use of Markov matrix-based Monte-Carlo Simulation of maintenance processes. The proposed simulation method can be used for the assessment of requested number for spare part, availability, maintenance cost of a technical system operation depending on required estimating uncertainty.

Keywords: Maintenance, Availability, Monte-Carlo Simulation, Markov-chain.

1. Introduction

A maintenance system can be characterized by the availability of equipment. Availability may be generically be defined as the percentage of time that a repairable system is in an operating condition.

By Ushakov [1]: “Availability is the capability of a system to be ready to perform its functions when required. Failure is the total or partial loss of the capability of a system. Repair is restoration of an object. In many analyses, the »repair« means restoration to an operable condition.”

In recent years, there are several papers that discuss new methods from different aspects to help decision making in maintenance management. For example, the aim of Dodu’s article is to analyze the causes which conducted to the lack of availability of helicopters while the rate of cannibalization and the number of not available spare parts increased [2].

Duer presented a modeling method of the operation process of repairable technical objects of various classes. A particular attention was paid to the model of the process which includes a service expert system with an artificial neural network. Duer’s paper also included theoretical grounds of the modeling process of the operation of objects in the form of the following models: mathematical (analytical), graphical and descriptive ones [3].

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From mathematical point of view, the operation of technical systems and equipment is a discrete state space stochastic process without after-effects, so it can be approximated with a Markov-chain [4].

Lin & Asplund used a Weibull frailty model to analyze the locomotive wheels' degradation [5]. The proposed framework can deal with small and incomplete datasets; it can also simultaneously consider the influence of various covariates. The Markov Chain Monte Carlo (MCMC) technique is used to integrate high-dimensional probability distributions to make inferences and predictions about model parameters. Finally, they compared the statistics on re-profiling work orders, the performance of re-profiling, and wear rates.

Monte-Carlo (MC) is one of the classical simulation techniques. MC is only one particular application of another general method, which is applicable in both deterministic and probabilistic settings. At the heart of MC there is a computational procedure in which a performance measure is estimated using samples drawn randomly from a population with appropriate statistical properties. The selection of samples, in turn, requires an appropriate random number generator. Ideally, the generated "random" sequence is a completely faithful software counterpart of the non-determinism underlying the actual process.

The idea of the MC calculation is much older than the computer. The name "Monte-Carlo" is relatively recent – it was coined by Nicolas Metropolis in 1949 – but under the older name of "statistical sampling" the method has a history which goes back well into the last century, when numerical calculations were performed by using pencil and paper and perhaps a slide rule. An early example was a MC calculation of the motion and collision of the molecules in gas was described by William Thomson (another name for Lord Kelvin) in 1901 [6]. Kelvins's calculations were aimed at demonstrating the truth of the equipartition theorem for the internal energy of a classical system. The exponential growth in computer power since those early days is a familiar story to us all by now, and with this increase – in computational resources MC techniques have looked deeper and deeper into the subject of statistical physics. The Monte-Carlo Simulations (MCS) have also become more accurate as a result of the invention of a new algorithm.

The essence of the MCS is the invention of games of chance whose behavior and outcome can be used for studying some interesting phenomena. While there is no essential link to computers, the effectiveness of numerical or simulated gambling as a serious scientific pursuit is enormously enhanced by the availability of modern digital computers [7].

The term MC method is generally used to refer to any simulation techniques related to the use of random numbers [8]. Numerical experiments of MCS lead us to run the simulation on many sampled inputs before we can infer the values of the system performance measures of interest.

There are several books and papers that state theory of the MCS and its applications. Rubinstein depicted detailed treatment of the theoretical backgrounds and the statistical aspects of these methods in his book [9]. Dagpunar provided an introduction to the theory and practice of MC and Simulation methods [10]. Newman & Barkema applied the MCS to investigate several statistical problems in physics [6]. Fang et al. proposed a calculation method for evaluating thermal performance of the solar cavity receivers [11]. The MC method was employed to calculate radiation inside the receiver.

By Ispas and Lungu simulations are usually performed with the Monte Carlo method which is capable of analyzing multidimensional situations, better said, the outcome depends on many variables or risk factors [12]. Their results show how the duration of the simulation process affects the total medium duration and the sensitivity value. The sensitivity depends on the range in which the medium duration take values and not on the medium duration values..

Kozelj et al. demonstrated the benefits of including prior information to improve the identifiability of estimated parameters [13]. They investigated the effect of different sampling strategies of pipe roughness coefficients in the inverse solution of hydraulic models of water distribution systems.

The paper of Madić & Radovanović has three objectives [14]:

(i) to investigate the MC method applicability for solving single-objective machining optimization problems;
(ii) to develop a framework for solving machining optimization problems by using the MC method;
(iii) to analyze efficiency of the MC method for solving machining optimization problems by comparing the optimization solutions to those obtained by the past researchers using meta-heuristic algorithms.

The aim of research Gál et al. was to develop a dairy farm technology planning system which models the material flow and value chain of a sample dairy farm [15].

In paper of Pengfei et al. a double-loop MCS method was been developed for investigating the so called delta indices [16]. Their method is purely based on the model evaluation and univariant density estimation.

The study of Yeelyong et al. proposes a new methodology that combines Dynamic Process Simulation (DPS) and MCS to determine the design pressure of fuel storage tanks on liquefied natural gas (LNG)-fueled ships [17]. This approach provides a realistic distribution of the operating pressure, which the conventional process simulation cannot provide. It should be noted that the conventional DPS does not account for the failure of equipment and predicts that the peak pressure will always be equal to the maximum working pressure in the failure-free mode.

Morariu & Zaharia presented a calculation methodology of the testing duration of the products' reliability, using the Weibull distribution, which allows the estimation of the mean duration of a censored and/or complete test, as well as the confidence intervals for this duration [18]. By using these values they improved the adequate planning and allocation of material and human resources for the specific testing activities. Their proposed methodology and the results' accuracy were verified using the MC data simulation method.

Pokorádi showed the possibilities of the use of Markov matrix in the case of stationary maintenance processes [4]. A well-algorithmizable method for mathematical modeling of stationary stochastic industrial process was presented by Pokorádi [19]. This modeling method can be used to estimate maintenance cost and the time of availability of equipment.

The aims of this investigation are the followings:

- apply the method proposed by Pokorádi [4] [19] to depict Markovian model of the investigated maintenance process;
- use MCS of the investigated maintenance process based on its stochastic model;
- propose a method determining availability and Required Number for Spare Part (RNSP) depending on required estimating uncertainty;
- propose a method estimating the Numbers of Failures (NoF) depending on required estimating uncertainty.

The outline of the paper is as follows: Section 2 presents the stochastic model of investigated maintenance process. Section 3 shows the proposed simulation method to determinate RNSP and NoF depending on required estimating uncertainty. Section 4 summarizes the paper, outlines the prospective scientific work of the Author.

2. The Model of Investigated Process

During the operation of the equipment used in a large number four different (A, B, C, D) types component-related failures have been distinguished. The feature of the repairs of equipment (except C type failure) is a long – approximately 45 day (1080 hours) – period because of logistical matters. This investigation is done from point of view of end-user; therefore the repairs are characterized by Mean Repair Turnaround Times (MRTT). Additionally it can be established that the time of replacement of faulty equipment is negligible. So these times are not taken into account during simulation modeling.

The main data of the failures and their repairs are included in Table 1. The Fig. 1 shows weighted directed graph of the process. In the graph, the weights of the edges show probability densities (failure or turnaround rates) of changes of operational states.

The failure rate λ_i is equal to the probability of the i^{th} failure in a unit time interval given that no failure has occurred before it [19]. The turnaround rate μ_j can be interpreted analogically.

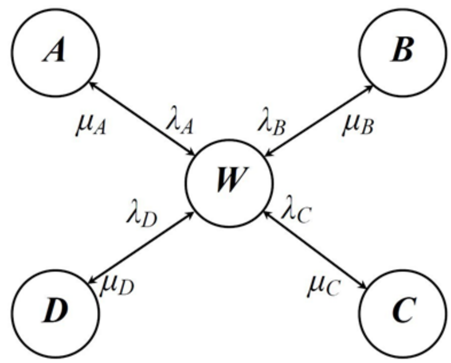

Fig. 1. The Graph Model of Investigated Maintenance Process

The system of differential equations of this process that describes the changes in time of the probability of staying in different states can be determined as

$$\begin{aligned} \frac{dP_W}{d\tau} &= -(\lambda_A + \lambda_B + \lambda_C + \lambda_D)P_W + \mu_A P_A + \mu_B P_B + \mu_C P_C + \mu_D P_D \\ \frac{dP_A}{d\tau} &= \lambda_A P_W - \mu_A P_A \\ \frac{dP_B}{d\tau} &= \lambda_B P_W - \mu_B P_B \\ \frac{dP_C}{d\tau} &= \lambda_C P_1 - \mu_C P_C \\ \frac{dP_D}{d\tau} &= \lambda_C P_1 - \mu_D P_D \end{aligned} \quad . \qquad (1)$$

Because of the investigated process is stationary, the differential coefficients of eq. (1) are:

$$\frac{dP_W}{d\tau} = \frac{dP_A}{d\tau} = \frac{dP_B}{d\tau} = \frac{dP_C}{d\tau} = \frac{dP_D}{d\tau} = 0 \quad . \qquad (2)$$

A further condition of the solution is the

$$\sum_{i=W}^{D} P_i(\tau) = 1 \qquad (3)$$

probability of event space ($i \in L$, where L set of Latin letters $W\ A\ B\ C\ D$) This equation expresses that the object of operation has to stay only in one of six states (in the present case, the state space consists of them). Then on the basis of equations (1) – (3) stochastic model of the investigated stationary operation process can be depicted as the following matrix formula:

$$\begin{bmatrix} -(\lambda_A+\lambda_B+\lambda_C+\lambda_D) & \mu_A & \mu_B & \mu_C & \mu_D & 1 \\ \lambda_A & -\mu_A & 0 & 0 & 0 & 1 \\ \lambda_B & 0 & -\mu_B & 0 & 0 & 1 \\ \lambda_C & 0 & 0 & -\mu_C & 0 & 1 \\ \lambda_D & 0 & 0 & 0 & -\mu_A & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_W \\ P_A \\ P_B \\ P_C \\ P_D \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad (4)$$

Table 1

Essential (Nominal) Data of Statistical Analysis

Failures	***A***	***B***	***C***	***D***
Mean Time Between Failures *MTBF* [hour]	183627	162059	152800	179789
Failure Rate λ [hour^{-1}]	5.446 10^{-6}	6.171 10^{-6}	6.545 10^{-6}	5.562 10^{-6}
Mean Repair Turnaround Time *MRTT* [hour]	1080.8	1081.1	167.13	1079.8
Turnaround Rate μ [hour^{-1}]	9.252 10^{-4}	9.250 0^{-4}	5.983 10^{-3}	9.261 10^{-4}

The methodology of setting up of the model mentioned above can be known profoundly by publications of Pokorádi [4].

Table 2 consists of results of equation (4) that is stochastic model of maintenance process using nominal values of Table 1.

Table 2

Nominal Results of Model

State of operation i	***W***	***A***	***B***	***C***	***D***
Probability of Staying in State P_i	9.8072 10^{-1}	5.7724 10^{-3}	6.5424 10^{-3}	1.0727 10^{-3}	5.8901 10^{-3}

3. The Situational Process

In this Chapter the MCS of operational process modeled in Chapter 2 will be done to investigate effects of parametric uncertainties of probability densities (failure and turnaround rates) of changes of operational states. Based on the simulation results theoretical and practical conclusions will be deduced for maintenance management decision.

3.1. Creation of Initial Data

Firstly, the available failure and turnaround data were analyzed statistically. The standard deviation, minimum, maximum and expected values of times between failures, and repair turnaround times were determined. These expected values are the MTBF (Mean Time Between Failures) and MRTT (Mean Repair Turnaround Time) parameters that commonly used to characterize the operational processes (see Table 1). Table 3 shows the statistical data.

Due to the relatively small number of available data, the goodness-of-fit tests have been left out. According to general engineering practice it is assumed that the measured parameters have normal (Gauss) probability distribution.

Table 3

Results of Statistical Analysis of (Measured) Maintenance Data

Failure	***A***	***B***	***C***	***D***
Number of samples	23	24	25	21
Mean Time Between Failures *MTBF* [hour]	183627	162059	152800	179789
Minimum of Times between Failures [hour]	176800	156460	147786	172643
Maximum of Times between Failures [hour]	190305	168287	157968	186602
Standard deviation of Times between Failures [hour]	2033	1881	1659	2198
Mean Repair Turnaround Time *MRTT* [hour]	1080.8	1081.1	167.13	1079.8
Minimum of Repair Turnaround Times [hour]	964.2	990	73.1	994.5
Maximum of Repair Turnaround Times [hour]	1161.2	1160.1	239.06	1164.6
Standard deviation of Repair Turnaround Times [hour]	23.9	23.7	23.16	24.3

3.2. The Simulation

During simulation to generate actual values of the times between failures, and repair turnaround times acceptance-rejection method shown in Chapter 2 was used. Table 4 shows results of statistical analysis of input data of simulation.

Using input data generated above stochastic model set up in Chapter 2 – in other words the equation (4) – was solved. On the basis of previous MCS experiences, the number of excitations was 10 000. This excitation number can provide sufficient statistical data, such as fair conclusions can be drawn from the results of simulation. The results can be seen on Table 5 and Figures 2. – 6. show the histograms of simulation results.

On the basis of the statistical fit tests it is stated that each probability of the staying in states has normal (Gauss) distribution.

Table 4

Results of Statistical Analysis of Input Data of Simulation

Failure	***A***	***B***	***C***	***D***
Mean Time Between Failures *MTBF* [hour]	183663	162129	152848	179820
Minimum of Times between Failures [hour]	179709	159714	149470	173679
Maximum of Times between Failures [hour]	187468	167897	155381	183656
Standard deviation of Times between Failures [hour]	2035	1873	1618	2247
Mean Repair Turnaround Time *MRTT* [hour]	1092,2	1081,8	161,86	1084,3
Minimum of Repair Turnaround Times [hour]	1062	1036,7	117,16	1043
Maximum of Repair Turnaround Times [hour]	1136,5	1142,9	196,94	1126,9
Standard deviation of Repair Turnaround Times [hour]	19,4	25,1	22,43	25,5

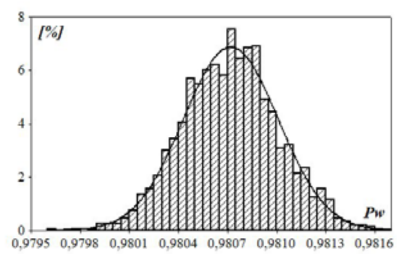

Fig. 2. Histogram of the Availabilities

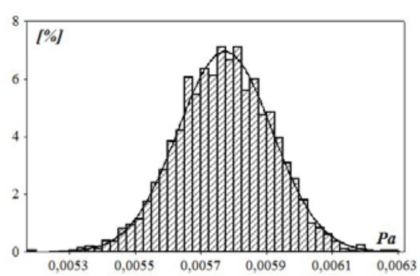

Fig. 3. Histogram of Probabilities of the Staying in State A-type Failure

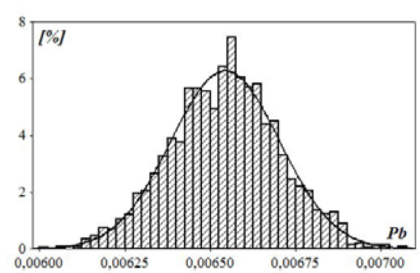

Fig. 4. Histogram of Probabilities of the Staying in State B-type Failure

Table 5

Results of Statistical Data Analysis of Probabilities of the Staying in States

State of operation i	***W***	***A***	***B***	***C***	***D***
Mean m_i	9.81 10^{-1}	5.77 10^{-3}	6.54 10^{-3}	1.07 10^{-3}	5.89 10^{-3}
Minimum	2.91 10^{-4}	1.44 10^{-4}	1.59 10^{-4}	1.49 10^{-4}	1.51 10^{-4}
Maximum	9.80 10^{-1}	5.19 10^{-3}	6.00 10^{-3}	4.70 10^{-4}	5.39 10^{-3}
Standard Deviation s_i	9.82 10^{-1}	6.30 10^{-3}	7.07 10^{-3}	1.51 10^{-3}	6.41 10^{-3}

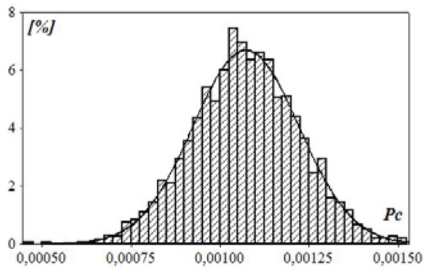

Fig. 5. Histogram of Probabilities of the Staying in State C-type Failure

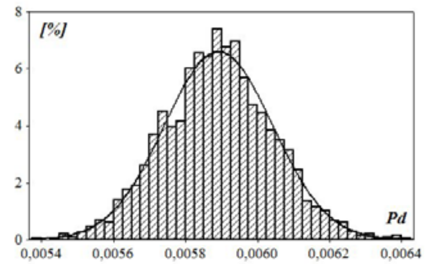

Fig. 6. Histogram of Probabilities of the Staying in State D-type Failure

3.3. Determination of Requested Number for Spare Part

This model simulation is done from point of view of end-user fundamentally. Thus, the most important question is the Required Number of Spare Part (RNSP). Knowing probability of the availability P_w, the RNSP can be determined by the following equation:

$$RNSP = \left\lceil \left(\frac{1}{P_W} - 1 \right) N \right\rceil \quad , \tag{5}$$

where N is the number of equipment in the system (in the present case: N=5000).

Applying expected (nominal) value of the availability (P_W=0.98072) shown by Table 2, in case of 5000 equipment the RNSP is 96. But, this result is “only” the expected value of RNSP. The probability that more equipment failure occurs at the same time is 0.5 (50%), which is unacceptable for end-users.

Table 6

Required Number for Spare Part Depending on Estimating Uncertainty

Estimating Uncertainty R	Availability P_{RNS}	Number for Spare Part RNSP
10 %	0.9803	101
5 %	0.9802	101
2 %	0.9801	102
1 %	0.9800	102
0.5 %	0.9800	103
0.2 %	0.9799	103
0.01 %	0.9798	103

Therefore using the probability distribution of simulation results (see

Table 5.), it should be determined, in case of which, value P_{RNS} the probability of availability P_w will be less than the acceptable estimating uncertainty R.

For example, – on the basis of the standard normal distribution – in case of 10% estimating uncertainty (the probability of not having a spare part in case the equipment failures is 0.1):

$$P_{RNS} = m_W - 1.29 s_W \quad . \tag{6}$$

The RNSPs were determined in cases of different assessing uncertainty values. These results are shown in Table 6.

3. 4. Determination of Numbers of Failures

The Number of i-th type Failure (NoF_i) can be determined by the following equation:

$$NoF_i = \left\lceil \frac{T \cdot P_i}{MRTT_i} N \right\rceil \quad , \tag{7}$$

where T is the length of investigational time (in the present case: T=1 year= 365·24=8760 hours).

Applying nominal (expected) values of failure probabilities shown by Table 2, results are “only” the expected values of NoFs. Therefore probability distributions of simulation results (see Table 5.) should be used too. But, in cases of failures, – on the basis of the standard normal distribution – if the estimating uncertainty is 10%:

$$P_{iE} = m_i + 1.29 s_i \quad . \tag{8}$$

The determined NoFs are shown in Table 7.

The model simulation data presented by Table 7 can be used to estimate the NoFs, maintenance cost and work expenditures of operated systems in investigated time interval depending on required estimating uncertainties.

Comparing data of Tables 6 and 7, it is easily remarked that sum of NoFs are more than RNSP. At first it may seem to be contradictory. However, it should also be taken into account that the NoFs were estimated by a time interval, but the RNSP is time-independent. Be it remembered that the repaired equipment will be returned to the end-user, where firstly they might be spare ones, and later they will replace the other failed ones.

Table 7

Required Number of Failures Depending on Estimating Uncertainty

i	Expected (R = 50 %)		R = 10%		R = 5%		R = 2%	
	P_i	NoF_i	P_{iE}	NoF_i	P_{iE}	NoF_i	P_{iE}	NoF_i
A	$5.7724\ 10^{-3}$	234	$5.959\ 10^{-3}$	241	$6.011\ 10^{-3}$	243	$6.070\ 10^{-3}$	246
B	$6.5424\ 10^{-3}$	265	$6.748\ 10^{-3}$	273	$6.805\ 10^{-3}$	275	$6.871\ 10^{-3}$	278
C	$1.0727\ 10^{-3}$	279	$1.265\ 10^{-3}$	329	$1.319\ 10^{-3}$	343	$1.380\ 10^{-3}$	359
D	$5.8901\ 10^{-3}$	238	6.086 10-3	246	$6.140\ 10^{-3}$	249	$6.202\ 10^{-3}$	251
	R = 1%		R = 0,50%		R = 0,20%		R = 0,01%	
	P_{iE}	NoF_i	P_{iE}	NoF_i	P_{iE}	NoF_i	P_{iE}	NoF_i
A	$6.110\ 10^{-3}$	247	$6.145\ 10^{-3}$	249	$6.188\ 10^{-3}$	250	$6.219\ 10^{-3}$	252
B	$6.915\ 10^{-3}$	280	$6.953\ 10^{-3}$	281	$7.001\ 10^{-3}$	283	$7.036\ 10^{-3}$	285
C	$1.422\ 10^{-3}$	370	$1.457\ 10^{-3}$	379	$1.502\ 10^{-3}$	391	$1.535\ 10^{-3}$	400
D	$6.244\ 10^{-3}$	253	$6.281\ 10^{-3}$	254	$6.326\ 10^{-3}$	256	$6.359\ 10^{-3}$	257

4. Conclusions

This paper discussed a Monte-Carlo Simulation-based method of maintenance processes analysis. Its possibility of use was demonstrated by a case study. The following conclusions can be deduced from the results of modeling and analysis: The proposed method can be used:

- for analyzing of maintenance processes;
- for supporting decision making in maintenance management;
- for estimating the availability and Required Number for Spare Part depending on required estimating uncertainty;
- for assessing the Numbers of Failures depending on required estimating uncertainty.

The Author's planned prospective scientific research related to this field of applied mathematics and maintenance management decision making includes the study of methodologies of mathematical tools for analysis of maintenance systems and processes for example stochastic model and simulation-based sensitivity analysis of maintenance systems and processes.

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