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MHD OSCILLATORY SLIP FLOW AND HEAT TRANSFER IN A CHANNEL FILLED WITH POROUS MEDIA

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This paper deals with the effect of slip on the hydromagnetic pulsatile flow through a channel filled with saturated porous medium with time dependent boundary condition on the heated wall. Based on the pulsatile flow nature, the dimensionless flow governing equations are resolved to harmonic and non-harmonic parts. Exact solutions are obtained for the temperature and velocity fields. Parametric study of the solutions are conducted and discussed.

Keywords: Pulsatile flow, Navier slip, porous medium, heat transfer, magnetic field, thermal radiation

1. Introduction

Studies related to the oscillatory fluid flow are increasingly important in recent times due to its numerous applications in many real life problems. Some of these include, Makinde [1] studied the combined effects of radiative heat transfer and magnetohydrodynamics on oscillatory flow in a channel filled with porous medium. Mahmoud and Ali [2] investigated the effect of Navier slip imposed on the lower wall on the unsteady hydromagnetic oscillatory flow of an incompressible viscous fluid in a planer channel filled with porous medium. In addition, Abdul-Hakeem and Sathiyanathan [3] presented analytical solution for two-dimensional oscillatory flow of an incompressible viscous fluid, through a highly porous medium bounded by an infinite vertical plate. Jha and Ajibade [4-6] reported some interesting results on the free convective oscillatory flows induced by time dependent boundary conditions. While Umavathi et al [7] studied the unsteady oscillatory flow and heat transfer in a horizontal composite channel.

All the studies mentioned above ignored the pulsatile nature of the pressure gradient. However, the study of pulsatile flow is of considerable importance in many engineering and physiological problems like the fuel pump, water pumps, and pumping of blood through large arteries. Pulsatile flow has attracted several researches in the literature. For example, Sankar and Lee [8]

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analysed the pulsatile flow of blood through mild stenosed narrow arteries by treating the blood in the core region as a Casson fluid and the plasma in the peripheral layer as a Newtonian fluid. Massoudi and Phuoc [9] studied the unsteady pulsatile flow of blood in an artery taken the effects of body acceleration into consideration. El-Shahed [10] considered the pulsatile flow of blood through a stenosed porous medium under the influence of body acceleration. Mistra and Pal [11] considered the laminar pulsatile flow of blood under the influence of externally imposed body accelerations. Recently, Adesanya and Makinde [12] studied the heat transfer to pulsatile flow of hydromagnetic couple stress fluid through a porous channel. Other relevant articles on the pulsatile flow are not limited to Eldabe et al [13], Zuecco and Beg [14], Rathod and Shakera Tanveer [15], Adesanya and Ayeni [16].

In view of the studies [8-16], the pulsatile slip flow of viscous, incompressible hydromagmetic fluid flow through channel of non-uniform wall temperature filled with a porous medium is studied. It is assumed that the slip effect depends on shear stress at both walls as applied by Eegunjobi and Makinde [17]. Based on the oscillatory nature of the flow, the problem is formulated, non-dimensionalized and exact solution of the problem is obtained. The rest of the paper is organized as follows; section two presents the mathematical analysis. Section three describes the method of solution and results are discussed in section four, and finally section five provides a conclusion of the paper.

2. Mathematical Analysis

Consider the unsteady laminar slip flow of an incompressible, viscous and electrically conducting fluid through a channel with non-uniform wall temperature. The fluid is assumed to be under the influence of an external magnetic field applied across the channel. We choose a Cartesian coordinate system (x', y') where x' lies along the centre of the channel, y' is the distance measured in the normal section such that $y' = a$ is the channel's half width as shown in the figure below.

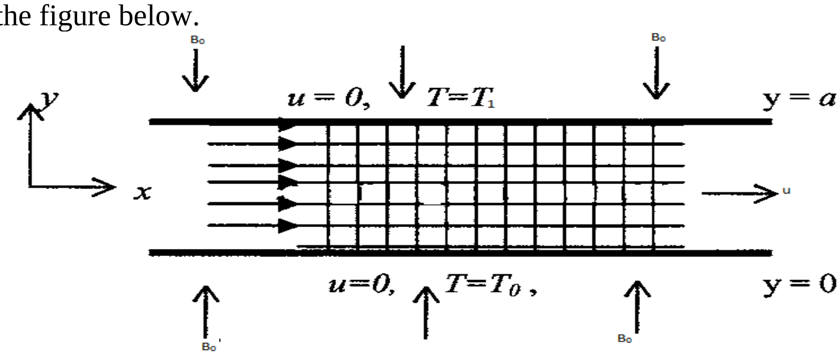

Fig. 1: Flow geometry

Under the usual Bousinesq approximation, the equations governing the flow are [1, 2]:

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{dP'}{dx'} + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{K}u' - \frac{\sigma_e B_0^2}{\rho}u' + g\beta(T' - T_0), \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k_f}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{4\alpha^2}{\rho C_p}(T' - T_0), \tag{2}$$

with the boundary conditions [17];

$$u' = \phi_1 \frac{du'}{dy'}, \quad T' = T_0, \qquad y' = 0 \tag{3}$$

$$u' = \phi_2 \frac{du'}{dy'}, \quad T' = T_1, \qquad y' = a. \tag{4}$$

Where t'-time, u' - axial velocity, ρ - fluid density, P'- fluid pressure, ν - kinematic viscosity, K -porous permeability, σ_e -electrical conductivity, B_0 - magnetic field intensity, g -gravitational acceleration, β -volumetric expansion, C_p -is the specific heat at constant pressure, α -is the term due to thermal radiation, k represents the thermal conductivity, T' fluid temperature, T_0 referenced fluid temperature, ϕ_1 and ϕ_2 are the slip parameters due to the porous medium. Introducing the dimensionless parameters and variables given below

$$(x, y) = \frac{(x', y')}{h}, u = \frac{hu'}{\nu}, t = \frac{\nu t'}{h^2}, p = \frac{h^2 p'}{\rho \nu^2}, Gr = \frac{g\beta(T_1 - T_0)h^3}{\nu^2}, \Pr = \frac{\rho C_p \nu}{k}, \gamma = \frac{\phi_1}{h}$$

$$\theta = \frac{T' - T_0}{T_1 - T_0}, \delta = \frac{4\alpha^2 h^2}{\rho C_p \nu}, \quad \sigma = \frac{\phi_2}{h}, H^2 = \frac{\sigma_e B_0^2 h^2}{\rho \nu}, Da = \frac{K}{h^2}, s^2 = \frac{1}{Da}, \lambda = -\frac{dP}{dx}.$$

We obtain the dimensionless equations

$$\mathrm{Re}\frac{\partial u}{\partial t} = \lambda + \frac{\partial^2 u}{\partial y^2} - (H^2 + s^2)u + Gr\theta, \tag{5}$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta, \tag{6}$$

with the appropriate boundary conditions

$$u = \gamma\frac{du}{dy}, \quad \theta = 0 \quad y = 0, \tag{7}$$

$$u = \sigma\frac{du}{dy}, \quad \theta = 1 \quad y = 1 \tag{8}$$

where Re is the Reynolds' number, s is the thermal conductivity parameter, H^2 is Hartmann's number, Gr is the Grashof number, Pe is the Peclet number, N is the thermal radiation parameter, γ is the cold wall slip parameter and σ is the heated wall slip parameter.

3. Method of Solution

For pulsatile flow, we assume a solution of the form,

$$\lambda = \lambda_0 + \lambda_1 e^{i\omega t},\ u(t,y) = A(y) + B(y)e^{i\omega t},\ \theta(t,y) = E(y) + F(y)e^{i\omega t}. \tag{9}$$

Substituting (9) in (5) to (8), we obtain the following ordinary differential equations

$$A''(y) - \left(H^2 + s^2\right)A(y) = -GrE(y) - \lambda_0, \tag{10}$$

$$B''(y) - \left(H^2 + s^2 + \mathrm{Re}\, i\omega\right)B(y) = -GrF(y) - \lambda_1, \tag{11}$$

$$E''(y) + N^2 E(y) = 0, \tag{12}$$

$$F''(y) + \left(N^2 - i\omega Pe\right)F(y) = 0, \tag{13}$$

subject to the following boundary conditions

$$A(0) = \gamma A'(0),\ A(1) = \sigma A'(1), \tag{14}$$

$$B(0) = \gamma B'(0),\ B(1) = \sigma B'(1), \tag{15}$$

$$F(0) = 0,\ F(1) = 1, \tag{16}$$

$$E(0) = 0,\ E(1) = 1, \tag{17}$$

Solving (10)-(12) together with (14)-(17), we get

$$E(y) = \frac{Sin(Ny)}{Sin(N)}, \tag{18}$$

$$F(y) = \frac{Sin\ m_1 y}{Sin\ m_1}, \tag{19}$$

$$A(y) = C_3 e^{m_3 y} + C_4 e^{m_3 y} + n_4 + n_3 Sin(Ny), \tag{20}$$

$$B(y) = C_1 e^{m_3 y} + C_2 e^{m_5 y} + n_1 + n_2 Sin(m_1 y). \tag{21}$$

The dimensionless heat transfer rate at the channel walls is given by

$$Nu = -\frac{\partial\theta}{\partial y} = -\frac{NCos\ Ny}{Sin\ N} - \frac{m_1 Cos\ m_1 y}{Sin\ m_1} e^{i\omega t} \tag{22}$$

while the sear stress at both wall is given by

$$\tau = -\mu\frac{\partial u}{\partial y} = -A'(y) - B'(y)e^{i\omega t} \tag{23}$$

where

$$m_1^2 = N^2 - i\omega Pe\,,\ C_1 = \frac{n_2 m_1 \gamma - m_2 c_2 \gamma - c_2 - n_1}{1 - m_2 \gamma}\,,\ n_1 = \frac{\lambda_1}{m_2^2}\,, n_2 = \frac{Gr}{\left(m_1^2 + m_2^2\right) Sin\ m_1}$$

$$m_2^2 = H^2 + s^2 + i\omega \mathrm{Re}\,,\ C_2 = \frac{m_2 \gamma C_1 + m_1 n_2 \gamma Cos\ m_1 - C_1 e^{m_2} - n_1 - n_2 Sin\ m_1}{\left(1 - m_2 \gamma\right) e^{-m_2}}$$

4. Results and Discussion

We obtain the exact solutions of the equations (10) - (13) subject to (14-17). As an accuracy check, setting $\sigma = 0$ and neglecting the steady state in (9), the work of Mahmood and Ali [2] is fully recovered. Therefore, study in [2] is a special form of the present paper. Figure 2 shows the velocity profile variations with cold wall slip parameter. The result shows that fluid velocity increases with an increase in the cold wall slip parameter thus enhancing fluid flow. The cold wall slip did not cause any appreciable effect on the heated wall. Figure 3 shows the variation of velocity with the magnetic field intensity. From the graph, it is noticed that an increase in Hartmann's number decreases the fluid velocity due to the resistive action of the Lorenz forces except at the heated wall where the reversed flow induced by wall slip caused increase in the fluid velocity. Figure 4 shows the variation of fluid velocity with the inverse of the Darcy parameter. The graph shows that an increase in s decreases the fluid velocity. Therefore, an increase in the Darcy parameter increases the fluid flow except at the flow reversal point at the heated wall. It is observed in figure 5 that $\sigma = 0$ corresponds to the pulsatile case with no slip at the heated wall. Further, increase in the heated wall slip decreases the fluid velocity minimally at the cold wall and the decrease is more pronounced at the heated wall leading to flow reversal.

Table 1

Computation showing the effect of Peclet number on the rate of heat transfer.

Y	N	Pe	Nu	Pe	Nu	Pe	Nu
0	1	0.044	-2.37627	0.71	-2.25028	1	-2.1403
0.25	1	0.044	-2.30252	0.71	-2.20992	1	-2.12871
0.5	1	0.044	-2.0858	0.71	-2.0793	1	-2.07293
0.75	1	0.044	-1.72945	0.71	-1.83128	1	-1.91173
1	1	0.044	-1.28476	0.71	-1.42551	1	-1.54997

Table 2

Computation showing the effect of cold wall slip on skin friction.

Y	σ	γ	τ	γ	τ	γ	τ
0	0.1	0	-0.985648	0.2	-0.719525	0.3	-0.632461
0.25	0.1	0	-0.482901	0.2	-0.294962	0.3	-0.234651
0.5	0.1	0	0.0757685	0.2	0.214156	0.3	0.25777
0.75	0.1	0	0.777629	0.2	0.891277	0.3	0.926637
1	0.1	0	1.72108	0.2	1.83256	0.3	1.8672

Table 3

Computation showing the effect of heated wall slip on skin friction.

Y	γ	σ	τ	σ	τ	σ	τ
0	0.1	0	-0.93067	0.1	-0.832963	0.2	-0.680045
0.25	0.1	0	-0.489438	0.1	-0.374386	0.2	-0.197833
0.5	0.1	0	0.00304197	0.1	0.156143	0.2	0.383768
0.75	0.1	0	0.629214	0.1	0.843908	0.2	1.15205
1	0.1	0	1.4829	0.1	1.78612	0.2	2.20552

Table 1 shows the effect of Peclet number on the rate of heat transfer within the channel. The result shows that an increase in the Peclet number increases the heat transfer rate at the cold wall and decreases the Nusselt number at the heated wall. Tables 2 and 3 shows the effects of wall slip on the skin friction. From the results, it is observed that an increase in both the cold and heated wall slip parameter enhances the skin friction at both walls.

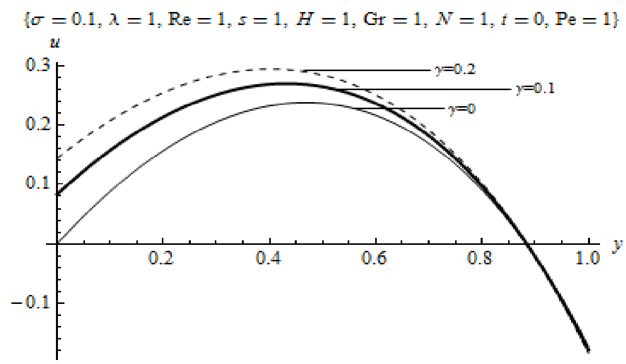

Fig. 2: Velocity profile against position for variations in Navier slip parameter

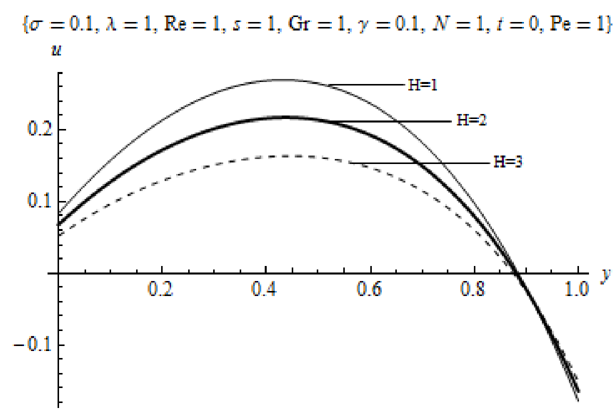

Fig. 3: Velocity profile against position for variations in Hartmann's number

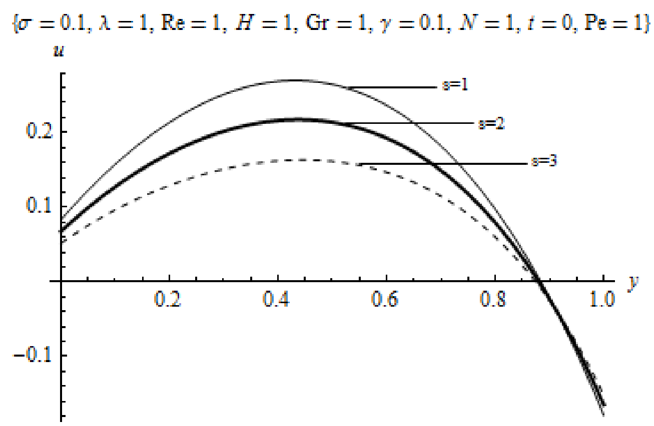

Fig. 4: Velocity profile against position for variations in porous permeability parameter

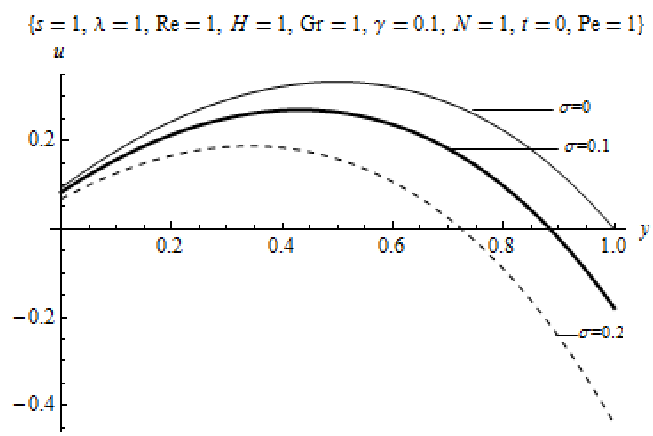

Fig. 5: Velocity profile against time for variations in Navier slip parameter at the heated wall

5. Conclusion

The goal of this paper is to investigate the effect of wall slip on the pulsatile flow through a channel with non-uniform wall temperature that is filled with a porous medium. Exact solutions of the velocity and temperature fields are obtained. There is a good agreement between the limiting case of the present result and the purely oscillatory case obtained by Makinde and Mhone [1] when $\sigma = 0 = \gamma$ and that obtained by Mahmood and Ali [2] when $\sigma = 0$. However, for the pulsatile case our result showed that; Nusselt number increases at the cold plate but decreases at the heated plate with an increase in Peclet number. Increasing heated wall slip causes a flow reversal towards the heated plate while both slip parameters encourages skin friction at both plates.

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