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CHARACTERISTIC MESH GRID METHOD FOR TRANSIENT ANALYSIS OF NATURAL GAS FLOW IN PIPELINES NETWORKS

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Through this paper, transient flow of natural gas in pipeline network is numerically modeled. Transient analysis of gas flow in a single duct is performed reposing on the characteristic mesh grid method. The use of the latter method is also extended for analyzing transient behavior of gas flow in looped network. The used mathematical model considers the compressibility criterion of the gas. The efficiency of the proposed model is proved by comparing the obtained results to experimental and numerical ones issued from literature. Unlike previous research works on transient gas flow, the use of the numerical characteristic scheme has allowed following up the wave propagation path in the space-time plane by illustrating the characteristic curves.

Keywords: method of characteristics, natural gas, transient flow, pipelines network

1. Introduction

The analysis of gas flow through networks of pipelines is a major preoccupation for many workers. Nowadays, engineers and researchers become more and more interested in simulating gas flow in pipelines systems [1, 2]. In fact, controlling transient gas behaviors is of crucial importance to avoid pipelines destruction or any other damage caused by a sudden change of the gas flow characteristics (compressor failure, sudden closure of a valve, etc.). Mathematical modeling of transient gas flow reposes basically on a set of two non-linear first order hyperbolic Partial Differential Equations (PDEs), namely mass and momentum equations. Added to these governing PDEs, gas equation of state is commonly used to consider the real behavior of the gas molecules. To solve the latter equations, several numerical methods are encountered in literature. The reliability of each method is well related to the case study. Thus, resolution techniques of governing gas flow equations are still gaining the interest of many researchers. To simulate transient flow through pipes, the governing equations were solved using diverse numerical schemes.

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The method of characteristics was used to simulate transient behavior of compressible fluids [3, 4]. An adaptive method of lines was also utilized to simulate the transient gas flow in pipelines [5]. Furthermore, an analysis of the isothermal transient gas flow in pipelines networks was performed using an electrical analogy based model [6]. An implicit finite difference scheme was also used to investigate transient gas flow in pipelines [7]. Then, more recently, commercial CFD software have served for gas flow modeling. Finite element based solver of COMSOL Multiphysics software was employed to analyze transient flow of hydrogen-natural gas mixture in steel pipeline networks and to emphasize its effect on the structural integrity of the pipeline material [8]. The compressibility criterion of the gas makes many conventional numerical schemes time consuming. Thus, in literature there are encountered workers who analyzed transient isothermal gas flow assuming a constant compressibility factor [9]. Also, transient flow simulation of gas in pipelines networks was performed using semi implicit finite volume method and basing on Dranchuk-Purvis-Robinson correlation for compressibility factor [10]. Reposing on the transfer functions approach, MATLAB-Simulink library was also used for gas flow modeling [11].

Although being a classical method, the method of characteristics is still giving accurate results in predicting transient gas behavior in pipelines. Indeed, the latter method permits following the wave propagation through the duct and so, once boundary conditions are properly adjusted, it allows a full description of the physical phenomenon leading to accurate results mainly when analyzing gas flow through one straight duct. Whereas, using the characteristics grid method to simulate gas flow systems that involve pipes junctions (a gas network or a junction between two pipes) is still not well developed and such problems were usually treated using characteristics method with specified time intervals. The latter technique permits to overcome the problem of wave propagation in the junction by assuming a constant celerity for pressure waves and then proceeding by correction of the result using, usually, linear interpolations. Indeed, specified characteristic grid method was used to simulate a binary gas mixture flow in a pipeline with presence of a leak assumed to be a junction between two pipes [12]. Unfortunately, the required use of the interpolation technique for the correction step is likely to affect the accuracy of obtained results mainly when simulating looped networks in which junction may connect two or more pipes.

In this paper, considering a linear regression describing the evolution with pressure of the compressibility factor under isothermal conditions, the transient behavior of natural gas flow in a single pipe and then in a looped network is analyzed. The characteristic mesh grid method is used to solve the governing equations. For the gas network, the characteristic mesh is built by imposing a common pressure and time value in each junction between two pipes. Results obtained through a Matlab code are validated by comparison to experimental and theoretical ones. Also, the mesh grid is illustrated and the wave propagation path in the space-time plane is emphasized. The used approach to adapt the characteristic mesh grid method to analyze flow in networks proves its efficiency in the studied case. Nevertheless, a generalized model has to be built.

2. Mathematical formulations

Under isothermal conditions, a one-dimensional compressible gas flow through an horizontal pipeline of diameter D is described by a set of two hyperbolic PDEs (mass and momentum equations) [8, 13]. To take into consideration the compressibility effect, gas equation of state $\rho=pM/(ZRT)$ is required, where ρ is the gas density, T is the temperature, M is the mole mass of the gas, R is the gas constant and Z is the gas compressibility factor. Assuming that gas velocity V is not close to the velocity of sound [10] and by writing the mass flow rate $m=\rho VA$ where $A=\pi D^2/4$ is the pipe section, governing equations of gas flow written versus pressure p and mass flow rate m are reduced to

$$\frac{1}{A}\frac{\partial m}{\partial x}+\frac{M}{RT}\frac{\partial}{\partial t}\left(\frac{p}{Z}\right)=0 \tag{1}$$

$$\frac{1}{A}\frac{\partial m}{\partial t}+\frac{\partial p}{\partial x}+\frac{8\lambda ZRTm|m|}{\pi^2 D^5 pM}=0 \tag{2}$$

where λ stands for the friction factor between the gas and the pipeline wall.
By assuming $Z=\alpha p+\beta$ for pressure values up to 100 bars where α and β depend on the temperature and the gas composition [14], equations (1) and (2) yield

$$\frac{\partial p}{\partial t}+\left[\frac{RT}{MA\beta}(\alpha p+\beta)^2\right]\frac{\partial m}{\partial x}=0 \tag{3}$$

$$\frac{\partial m}{\partial t}+A\frac{\partial p}{\partial x}+\frac{\lambda RT}{2DAM}\frac{(\alpha p+\beta)}{p}m|m|=0 \tag{4}$$

3. Transient model

3.1. Characteristic grid method formulation

In this section a matrix approach is adopted to convert the two PDEs (3) and (4) into four ordinary differential equations using the method of characteristics [15]. The time derivatives of the two variables m and p are written

$$\frac{dm}{dt}=\frac{\partial m}{\partial t}+\frac{\partial m}{\partial x}\frac{dx}{dt} \tag{5}$$

$$\frac{dp}{dt}=\frac{\partial p}{\partial t}+\frac{\partial p}{\partial x}\frac{dx}{dt} \tag{6}$$

Denoting $\sigma=RT(\alpha p+\beta)^2/(MA\beta)$ and $J=\lambda RT(\alpha p+\beta)m|m|/(2DAMp)$, under matrix form, equations (3), (4), (5) and (6) are written

$$\begin{pmatrix}1 & 0 & 0 & \sigma\\ 0 & A & 1 & 0\\ 1 & dx/dt & 0 & 0\\ 0 & 0 & 1 & dx/dt\end{pmatrix}\begin{pmatrix}\partial p/\partial t\\ \partial p/\partial x\\ \partial m/\partial t\\ \partial m/\partial x\end{pmatrix}=\begin{pmatrix}0\\ -J\\ dp/dt\\ dm/dt\end{pmatrix} \tag{7}$$

The characteristics method permits transforming the PDEs into ordinary differential equations that are valid only across the characteristic curves called C^+ and C^-. These curves are defined through the characteristic directions *dt/dx*, solutions of the equation

$$\det\left(\begin{pmatrix}1 & 0 & 0 & \sigma\\ 0 & A & 1 & 0\\ 1 & dx/dt & 0 & 0\\ 0 & 0 & 1 & dx/dt\end{pmatrix}\right)=0 \tag{8}$$

The resolution of equation (8), yields two characteristic directions

$$\lambda_1=\frac{1}{a}=\frac{1}{\alpha p+\beta}\sqrt{\frac{M\beta}{RT}} \tag{9}$$

$$\lambda_2=-\frac{1}{a}=-\frac{1}{\alpha p+\beta}\sqrt{\frac{M\beta}{RT}} \tag{10}$$

λ_1 and λ_2 are functions of the pressure. Non-constant characteristic directions are explained by the fact that for compressible fluids flow, the characteristic lines are curved due to the variation of the wave speed *a* with pressure. In fact, a characteristic mesh grid is obtained in the (*x*, *t*) plane as shown in Fig. 1

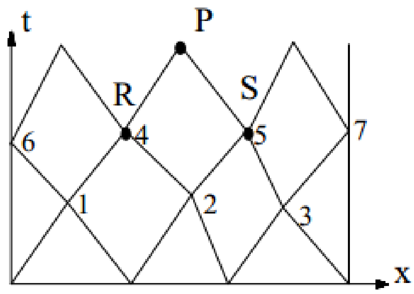

Fig. 1. Characteristics grid

The compatibility equations, describing the pressure and the mass flow rate variations along the characteristic curves are obtained by solving and then integrating the equation

$$\det\left(\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A & 1 & -J \\ 1 & dx/dt & 0 & dp/dt \\ 0 & 0 & 1 & dm/dt \end{pmatrix}\right) = 0 \tag{11}$$

Equation (11) is then written

$$Adp\frac{dt}{dx} + dm + Jdt = 0 \tag{12}$$

The integration of equation (12) along the characteristic curves C^+ ($dt/dx=\lambda_1$) and C^- ($dt/dx=\lambda_2$) has been usually performed through a first order finite difference approximation. Under the study assumption (linear variation of Z factor versus pressure), this approximation is used only to integrate the 3rd term of equation (12) since an analytical integration of the other terms is possible. Thus, the dependent variables to consider are the mass flow rate m and the compressibility factor Z (instead of the pressure p). Equation (12) is then rewritten

$$\pm\frac{A}{\alpha}\sqrt{\frac{M\beta}{RT}}\frac{dZ}{Z} + dm + Jdt = 0 \tag{13}$$

In fact, the unknown values of (m, Z, x, t), at any point P can be determined by knowing their values at the points R and S lying on the two characteristics passing through P. It's then obtained after integrating equation (13)

$$C^+\begin{cases} \dfrac{A}{\alpha}\sqrt{\dfrac{M\beta}{RT}}\ln\left(\dfrac{Z_P}{Z_R}\right) + m_P - m_R + J_R\left(t_P - t_R\right) = 0 \\ \left(t_P - t_R\right) = \lambda_{1R}\left(x_P - x_R\right) \end{cases}, \tag{14}$$

$$C^-\begin{cases} -\dfrac{A}{\alpha}\sqrt{\dfrac{M\beta}{RT}}\ln\left(\dfrac{Z_P}{Z_S}\right) + m_P - m_S + J_S\left(t_P - t_S\right) = 0 \\ \left(t_P - t_S\right) = \lambda_{2S}\left(x_P - x_S\right) \end{cases}, \tag{15}$$

The two systems of equations (14) and (15) are rearranged to obtain the expression of the unknown variables following a characteristic mesh grid i.e. x_P and t_P are not assigned definite values but they are variables to compute in each iteration. It's obtained

$$t_P = \frac{Z_R t_R + Z_S t_S}{Z_R + Z_S} + \sqrt{\frac{M\beta}{RT}}\,\frac{x_S - x_R}{Z_R + Z_S} \tag{16}$$

$$x_P = x_R + \sqrt{\frac{RT}{M\beta}} Z_R (t_P - t_R) \tag{17}$$

$$Z_P = \exp\left(0.5\ln(Z_R Z_S) + \frac{\alpha}{2A}\sqrt{\frac{RT}{M\beta}}\left[(m_R - m_S) - J_R(t_P - t_R) + J_S(t_P - t_S)\right]\right) \tag{18}$$

$$m_P = m_R + \frac{A}{\alpha}\sqrt{\frac{M\beta}{RT}}\ln\left(\frac{Z_P}{Z_R}\right) - J_R(t_P - t_R) \tag{19}$$

3.2. Boundary conditions

For a regular mesh scheme using characteristics method with a specified time interval and since distance and time steps are assigned definite values, in the junction between two pipes or more the mesh scheme is continuous. Although, for a characteristic mesh grid in which time and distance variables (x_P and t_P) have to be calculated in each step, it may be encountered, in a junction, different time values (t_P) between the inlet and the outlet sides. A discontinuity in the numerical scheme is then observed. Physically, this difference has not to occur. It's due to numerical errors and it affects the convergence of the numerical scheme. The idea is to choose one value t_P to be the common time value in which the waves coming from the upstream and the downstream reach the junction section.

To determine the unknowns (m, Z, x, t) in a junction, the procedure to follow is:

i) assign a value for x_P: $x_P^u=L^u$ for the upstream side and $x_P^d=0$ for the downstream side, where L^u is the length of the upstream pipe and exponents u and d refer, respectively, to upstream and downstream sides.

ii) for both cases solve the characteristic equations (second equations of systems (14) and (15)) to determine t_P^u and t_P^d

iii) choose one time value t_P and calculate the mass flow rate at the inlet and the outlet of the junction. Referring to the compatibility equations of the systems (14) and (15), m_P^u and m_P^d are expressed as follows

$$m_P^u = m_R - \frac{A}{\alpha}\sqrt{\frac{M\beta}{RT}}\ln\left(\frac{Z_P^u}{Z_R}\right) - J_R(t_P - t_R) \tag{20}$$

$$m_P^d = m_S + \frac{A}{\alpha}\sqrt{\frac{M\beta}{RT}}\ln\left(\frac{Z_P^d}{Z_S}\right) - J_S(t_P - t_S) \tag{21}$$

iv) assuming a common Z factor value $Z_P^u=Z_P^d=Z_P$, write the continuity equation in the junction as function of Z_P

$$m_P^u - m_P^d \pm m_P^c = 0 \tag{22}$$

where m_P^c is the external supplied (negative sign) or demanded (positive sign) flow rate.

v) solve equation (22), determine Z_P and then using equations (20) and (21), calculate m_P^u and m_P^d

For the above procedure, the most important step is to fix the time value that permits the convergence of the iterative process and ensures the accuracy of results. This step is emphasized in the studied case.

4. Results and discussion

To approve the efficiency of characteristic mesh grid method in analyzing transient behavior of looped network, a Matlab code is established following the developed formulation and tested for two gas networks which have been analyzed firstly by Osiadacz [9] and then by several researchers [6, 8, 10, 11, 13].

4.1. Test case 1

The first test case is a straight gas pipeline with a periodic demand at the outlet as illustrated in Fig. 2. The pipeline length is *100 km* and it has a diameter of *0.6 m*. A constant upstream pressure value of *5 MPa* is maintained. An isothermal condition is assumed (*T=278 K*) and the gas density is $0.73\ kg/m^3$.

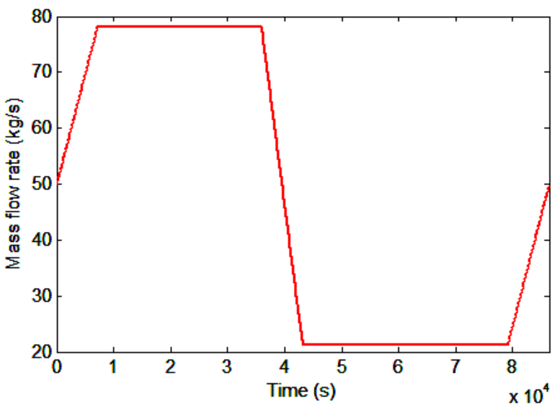

Fig. 2. Varying demand at the outlet

In the considered operational temperature, the values of α and β are calculated by solving gas equation of state and approximate linearly the *Z* factor evolution [14]. It's obtained $\alpha = -190.25 \times 10^{-5} bar^{-1}$ and $\beta = 0.9929$. For a simple pipeline, the characteristic mesh grid method can be easily applied since there are no constraints of time value correction at the extreme sides.

In Fig. 3 is depicted the obtained outlet pressure evolution versus time. The obtained result is compared to ones issued from different other models. The presented model reproduces the same shape of the pressure curve with the same period of evolution despite the difference in the minimum pressure value reached during transients. The observed discrepancy of the obtained pressure evolution with respect to previous works that are, in turn, not in good concordance with each other can be essentially justified by the different correlations used to express *Z* factor evolution with pressure when solving motion equations.

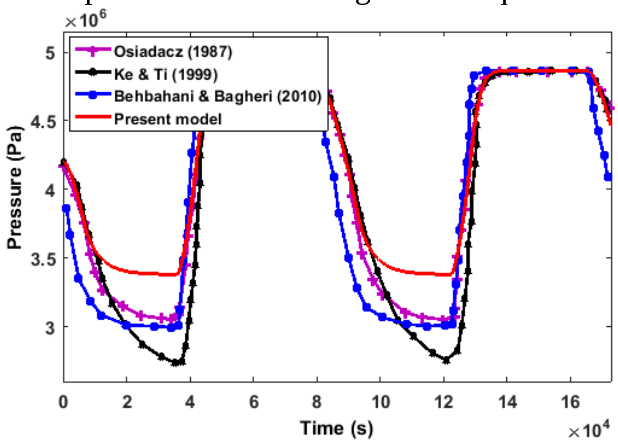

Fig. 3. Pressure evolution at the outlet

4.2. Test case 2

The second test case is a typical network containing a single mesh with three nodes and three branches as illustrated in Fig. 4. The geometrical data of the network are introduced in Table 1. The operational temperature is *278 K*, the mole mass of natural gas is *M=16.04 g/mol*, the friction factor is considered to be equal to 0.001 and $\alpha=-190.25\text{x}10^{-5}bar^{-1}$ and $\beta=0.9929$.

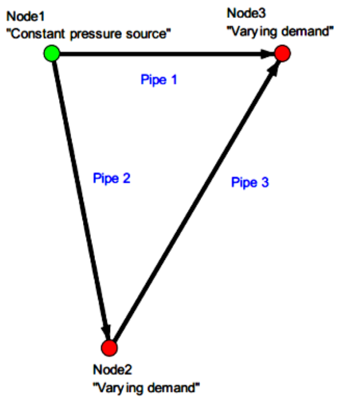

Fig. 4. Looped network topology

Table 1

Data of the single loop network

Pipe	Upstream node	Downstream node	Internal diameter [m]	Length [m]
1	1	3	0.6	80000
2	1	2	0.6	90000
3	2	3	0.6	100000

Node 1 is the pressure source with a constant pressure of *5 MPa*. The variations of the gas demand at the nodes 2 and 3 are depicted in Fig. 5.

Once supplied from node 2 and 3, the gas is assumed to be at a standard pressure value of *1.01325 bar*. The density of gas is then equal to *0.7096* kg/m^3. The steady state outlet discharges from node 2 and node 3 are respectively *14.192 kg/s* and *28.384 kg/s*.

The equilibrium of the network using Newton-Raphson method [16] leads to a steady state distribution of mass flow-rates as reported in Table 2.

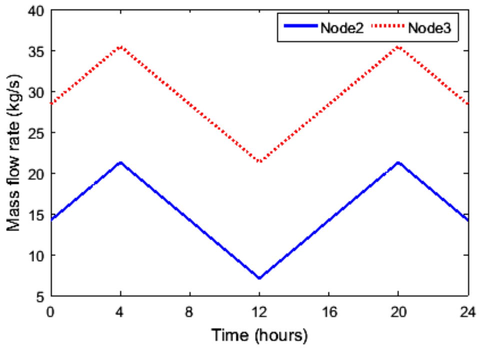

Fig.5. Varying demands in nodes 2 and 3

Table 2

Steady state discharges

Pipe	Steady state flow rate [kg/s]
1	22.4086
2	20.1665
3	5.9748

Prior to the simulation of the transient state, the important step of time value choice in junction points has to be achieved as mentioned in the previous section. For each junction between two pipes, one obtains two time values t_P^u and t_P^d and from which a judicious choice has to be done to adjust the characteristic scheme by fixing a common t_P value for both pipes. In fact, numerous trials were made considering as values for t_P the minimum, the maximum, the arithmetic mean and the geometric mean values between t_P^u and t_P^d. The judgment criterion was the steady state results. Indeed, the idea was to run the transient code under the steady state conditions i.e. without perturbations in node 2 and 3, and to compare obtained pressure evolutions in nodes 2 and 3 with those yet known through the steady state analysis (normally, a constant pressure values in nodes have to be obtained). Results have shown that considering the minimum time value in the junction gives pressure evolutions in agreement with steady state values although some minor fluctuations were observed.

The above approach has to be tested for more complex networks containing junctions between more than two pipes in order to be generalized or to give another alternative instead. In fact, for the studied case, the proposed model gives results that compare very favorably to experimental and theoretical ones.

Figs. 6 and 7 illustrate the outlet pressure evolutions in nodes 2 and 3 respectively compared to experimental results and also to those obtained using other models. A good concordance is observed between results issued from the proposed model and previous works. Indeed, the characteristic mesh grid method proves its efficiency in transient analysis of looped gas network. Additionally, the method allows following up the wave propagation path in the (x, t) plane by illustrating the evolution of characteristic curves.

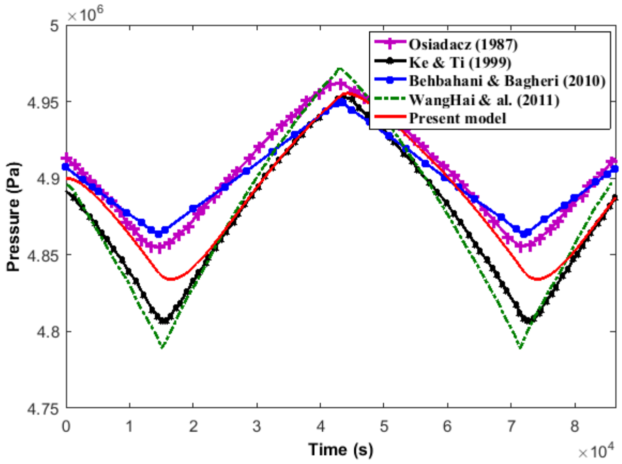

Fig. 6. The variation of pressure at node 2

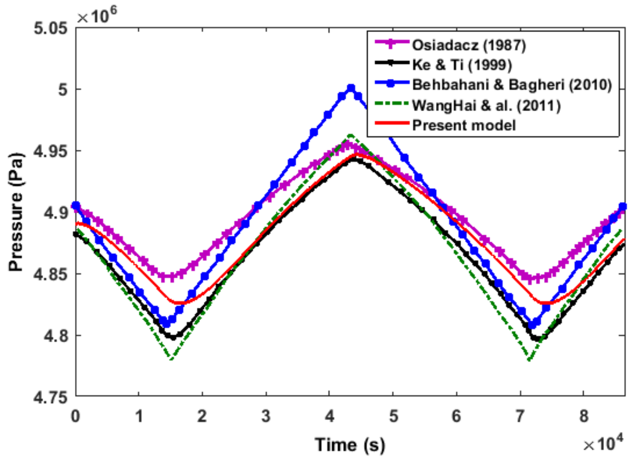

Fig. 7. The variation of pressure at node 3

4.3 .Characteristic mesh grid scheme

In this section, the numerical scheme is followed up. In Fig. 8 is plotted the characteristic mesh grid obtained numerically using the proposed model. In the latter figure, the mesh grid along pipes 2 and 3 is illustrated in the space-time plane and therefore the characteristic curves passing through the junction section (in node 2) are shown. Also, the irregularity as well as the continuity of the mesh grid in the junction is emphasized by a zoomed section of the junction zone.

It's worth mentioning that the characteristic grid is built reposing on the relation between x_P and t_P given by equation (17). The latter shows a linear relation between time and space which is, actually, not the case. In fact, this linearity is due to the approximation in the integration of the characteristic equations using the implicit method (considering the value of the wave speed in the previous step). This approximation may be corrected using trapezoid rule [16], but even without correction and while the characteristic mesh is already refined; such approximation doesn't affect significantly the accuracy of obtained results.

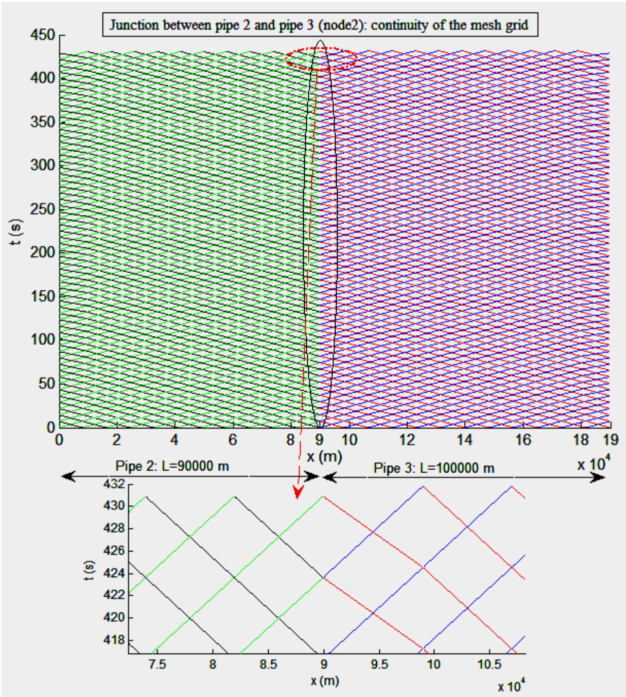

Fig.8. The characteristic mesh grid of pipes 2 and 3

5. Conclusions

The efficiency of the characteristic mesh grid method in modeling natural gas flow through a single pipe or a looped network was proved. The numerical model reposed on the resolution of governing gas flow equations taking in consideration the compressibility criterion of the gas. In this paper, an assumption of a linear isothermal evolution of the compressibility factor versus pressure was considered. The last part of this work was devoted to illustrate the evolution of characteristic curves in the (x, t) plane and hence emphasizing the transient wave propagation and the numerical mesh grid scheme.

Despite being time consuming, the characteristic grid technique with variable time and space step sizes is of major importance mainly when aiming to follow up the wave propagation path. Nevertheless, the use of the latter method is still not evident to simulate transient gas flow in networks. In fact, it's highly dependent on the studied case; initial values (steady state results) and boundary conditions as well as the used technique to ensure the continuity of the mesh grid in the junction points. A generalized model is, then, needed to be built to allow the use of the characteristic mesh grid method to simulate transient flow of highly compressible gas in which the wave speed (proportional to the Z factor) can reach significant values that may affect the stability of the mesh grid scheme.

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