SOFTWARE FOR ANGULAR CORRELATION ANALYSIS

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*The spins of excited nuclear states and the mixing ratios of γ transitions represent valuable spectroscopic information about the nuclear structure of studied isotopes. In order to obtain these, angular analysis of γ-radiation emissions have to be used. In the case of completely unaligned initial states, these can be obtained using angular correlations between two successive transitions, but a number of practical problems make this process difficult. In order to solve these issues, we have developed a number of procedures, routines and programs to undertake a complete angular correlation analysis starting from the **GASPware** software suite and the ROOT analysis framework[2]. These handle creating a normalization, cutting the matrices to obtain the intensities at every angle, fitting the angular correlation plot to obtain the A_2 and A_4 parameters and obtaining the mixing ratios based on some spin hyphotesis.*

Keywords: spin, mixing ratio, neutron capture, ROOT, GASPware, fitting, angular correlation

1. Introduction

The emission directions of two successive γ-ray transitions are generally not independent. They depend on the spins of the three nuclear states involved and also on the mixing ratios of these transitions, generally defined as the ratio between the contribution of the next-lowest multipole order involved to the contribution of the lowest multipole order[1]:

$$\delta = \frac{I_\gamma(L_{min+1})}{I_\gamma(L_{min})} \tag{1}$$

The dependence of the γ-ray intensity on the relative emission angle can be described as

$$W(\theta) = \sum_{i=0,even}^{\infty} B_{ii} G_{ii}(t) A_{ii} P_i(cos\theta) \tag{2}$$

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where B_{ii} are the initial nuclear orientation parameters and $G_{ii}(t)$ represent the time dependent perturbation factors[3]. Considering a completely isotropic initial spin orientation and very short nuclear state lifetimes, both these terms can be reasonably approximated to unity and Eq.2 becomes

$$W(\theta) = A_0(1 + A_2P_2(cos\theta) + A_4P_4(cos\theta)) \tag{3}$$

where the A_2 and A_4 parameters are determined by the spins and mixing ratios of the nuclear states and transitions involved.

From an experimental point of view, obtaining the A_2 and A_4 values from the experimental data then allow comparisons with theory in order to determine the spins and mixing ratios, or at least place constraints on the possible values. Such a fit can be seen in Fig.1 for the 1680-1345 keV $0^+ \rightarrow 2^+ \rightarrow 0^+$ cascade from ^{64}Ni.

Investigating certain levels and transitions through multiple cascade combinations can be further used to determine experimental values.

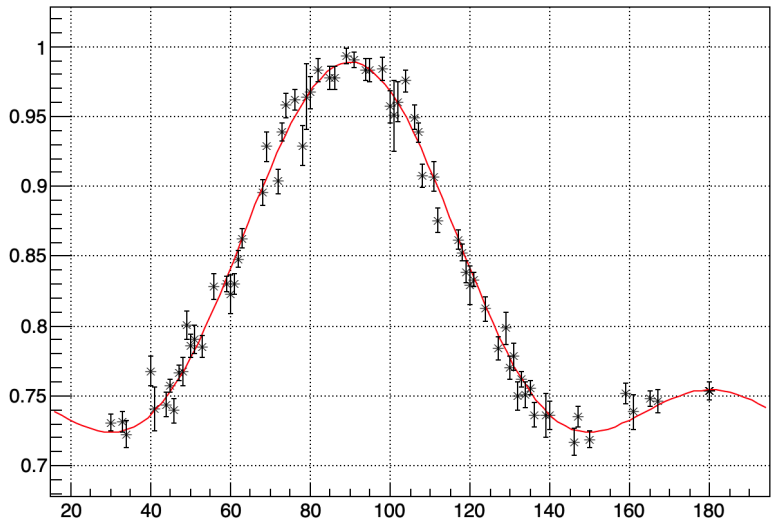

FIGURE 1. The angular correlation figure for the 1680-1345 keV $0^+ \rightarrow 2^+ \rightarrow 0^+$ cascade from ^{64}Ni, along with the fit allowing the extraction of the A_2 and A_4 values.

In order to undertake this analysis, a set of scripts and programs have been developed to handle all necessary steps, as will be outlined in the following chapters.

2. **Normalization**

In order to obtain the angular correlation figure, the angles between all the pairs of detectors of the set-up must first be determined by any means available (theoretical positions, on-site measurements, etc). However, due to

the unequal number of detector pairs at each available angle, different efficiency responses and any differences in the thickness and compositions of the materials separating the target and detectors, a normalization is required in order to extract the angular correlations.

There are multiple ways of doing this outlined in Ref.[4], including using a cascade with both transitions emitted isotropically (such as a cascade with the middle level having spin 0^+, for which the angular correlation is independent of the angle) or using the extracted efficiencies of each detector. However, both of these methods have significant issues. The first one is only viable when studying other cascades with similar energy combinations, breaking down as the energies of the studied cascade diverge from those of the normalization cascade. Furthermore, this can only be applied when such a cascade can be identified in the first place and is sufficiently intense, which is not always the case.

In the case of applying efficiency corrections, while this method should be viable across the entire energy range, it requires the existence of calibration source runs with sufficient statistics and covering the entire energy range. While it is easy to visualize the principle of how these efficiency curves should be applied, their technical implementation is more complicated, as outlined in Ref.[4]. Furthermore, this cannot account for the changes of the detector or DAQ parameters and behavior during the experimental run.

A third method is presented in Ref.[3], involving the mixing of the data itself. In this way, the angular correlation data from the detectors can be wiped out while preserving the efficiency information. In the end, following this procedure, a normalization matrix can be obtained which can then be used to obtain the angular correlations. The procedure developed by us applies to data written in the ***GASPware*** format.

In our case, the data was taken using the FIPPS array[5], which consisted of 8 HPGe clover detectors with their respective BGO anti-Compton shields at the time of data taking. Thus, the first steps were to apply Compton rejection, calibration (both a general one and a run-dependent correction), addback and time gates, in this order. The data is then rewritten to file in a 'cut' format, which includes just the detected energy and the crystal index. The setup file used for this can be found as 'original.setup' in the provided files[6].

The 'cut' files are then fed into a specially created a C++/Qt5 program developed to mix the data in order to obtain the normalization matrices, which has received the name ***Jumbler***, which is available in the provided files[6]. The ***Jumbler*** reads the data from the 'cut' ***GASPware*** format and creates lists of 100,000 detector signals. These are randomly mixed into new 'fake' multiplicity-two coincidences. Each signal is mixed multiple times, depending on a user-chosen multiplication factor and the original multiplicity of the event. While introducing a larger multiplication factor can result in a higher-statistics normalization matrix, this is also quite space-intensive. This procedure wipes

out the angular-correlation information contained in the real coincidences, but preserves the efficiency information for each detector, which is independent of the coincidences.

In case the detectors that are mixed are clovers, the program does not mix signals from inside the same clover (considering 4 crystals per clover, in order). The simple interface of the program is written in Qt5 and easily allows the user to select the files to be mixed, the output folder and if the detectors are clovers or not.

The end result of the ***Jumbler*** program are a series of 'Mixed' data files in the ***GASPware*** 'cut' format. The ***Jumbler*** program can be found in the provided files.

Following the mixing procedure, the 'Mixed' and 'cut' data files are then sorted into two angle-γ energy-γ energy three-dimensional matrices using the ***GASPware*** software, one for the original data and one for normalization.

3. Angular Correlator

Because of the large number of angles available (70 distinct crystal-crystal angles) and the large number of cascades that are to be analysed, extracting the intensities at each angle for each cascade by hand is prohibitively time-consuming. In order to automate the process, a special set of programs was developed to handle the data, fit the angular correlation and determine the compatibility with various spin hypothesis and mixing ratios. These consist of a bash script that handles the matrices and spectra using the ***cmat*** and ***sadd*** programs from the ***GASPware*** suite, a C++ program that calculates the intensities and uncertainties and a ROOT script that fits the angular correlation and compares the results with various spin hypothesis.

The first step is selecting the gates and backgrounds to be cut from the matrices. These can be input either in the script on a case-by-case basis, or multiple entries can be compiled into a single text file that can be read based on a tag (usually the energy of the γ transition). In the case of reading from file, two separate files are used, one for the first gates to be applied, the other for the second gates. The reason for this is that some transitions are very weak or have significant background around them, and they have to be analyzed after the gate is place on the other, more intense transition. This mostly implies different background, but can also have different peak extensions.

The format for both files starts with a tag, which is usually the energy of the transition. Then, separated by a forward slash, are the left and right limits of the γ-peak, separated by a comma and a space. These are followed by the left and right limits for the first and second backgrounds (usually placed to the left and right of the γ-peak).

The next section is the only one that differs between the two files. For the 'first gates', three values are given, representing the multiplication factors for the three matrices that are subtracted. This is usually of the type '100, X, X'.

The X value is the ratio of the width of the peak gate with the summed width of the background gates and multiplied by 100. The purpose of the multiplication with this ratio is to adjust the background intensity to the width of the gates. The multiplication by 100 has the purpose of preventing the use of non-integer type values, which the ***cmat*** program doesn't handle well.

For the 'second gates', there is only one entry in this part, which is just the ratio of the widths, without any multiplication by 100, because the ***sadd*** software used in this section is better able to handle floating point values.

The last entry in each section are the limits of the peak that is used for the normalization. For this, an intense γ-peak from the immediate energy-vicinity of the one studied has to be chosen. This normalization peak does not have to be from the reaction of interest, any peak coming from any reaction or radiation source within the target is acceptable. These peaks are chosen from the ungated 2D matrix, as the mixing procedure eliminates all real coincidences and gating on the mixed matrix is pointless.

There is no limit to how many entries can be made in each of the 'gate' files. The script will give out error messages if the peaks that are selected are not found within the 'gate' files.

After the script reads the necessary gates, it removes any files it created beforehand and creates a new folder for the two transitions to be analyzed, using their tags. The script calls the ***cmat*** program of the ***GASPware*** suite, and uses the 'first gate' to cut the angle-energy-energy data matrix into three angle-energy matrices, corresponding to the gated peak and the two backgrounds. These are subtracted to obtain the background-subtracted angle-energy matrix. The same program then applies the 'second gate' to cut this matrix into three unidimensional matrices that give the intensity of the coincidence at each angle and the two backgrounds. These are subtracted using the ***sadd*** program to obtain the final background-subtracted intensities at each available angle.

The 'cmat' program is used to also cut the normalization matrix and obtain the normalization intensities. Both of these are placed in separate files in the analysis folder and also in the folder created especially for the analyzed cascade. These will be used further on by a C++ program.

After, the software also extracts 9 sets of angle intensities that will be used in the calculation of the uncertainties. These are obtained by applying the 'second gates' on the three angle-energy matrices obtained from the 'first gates', with no background subtraction, and roughly correspond to image 2.

After these eleven one-dimensional matrices are obtained, they are fed into a simple C++ program. For each angle, this program reads the background-subtracted intensities at every angle and divides them by the intensities from the normalization matrix, yielding the final angular correlation intensities. For the uncertainties, the square roots of the integrated intensities of the nine

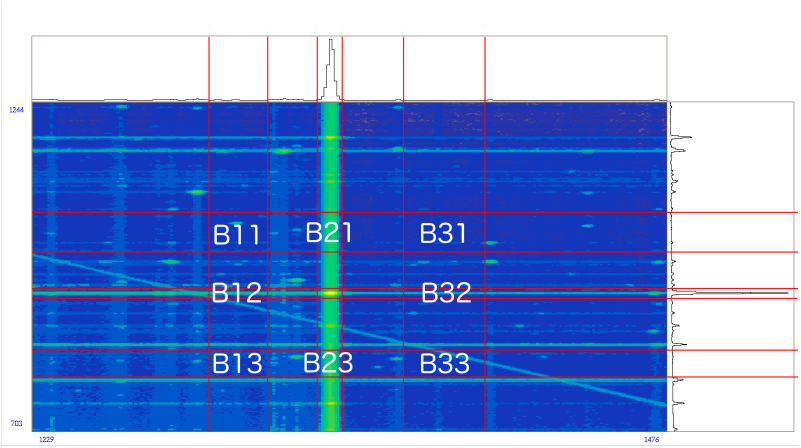

FIGURE 2. Representative illustration of the how the background intensities used for the uncertainty calculation process are obtained. The 9 squares labelled B_{11} to B_{33} give the angle-dependent no-background-substraction intensities that are used in calculating the uncertainties. What would be B_{22} is the actual peak, which is also used in the calculation.

squares shown in Fig.2 are added quadratically, taking into account the associated multiplication factors from the background subtraction. The uncertainties from the normalization gates are also added quadratically.

Considering B^i_{11} to B^i_{33} as the number of counts in the squares shown in Fig.2 for angle i, the uncertainty for each angle is calculated as

$$\sigma^i_{\gamma_1\gamma_2} = \sqrt{B^i_{22} + f_1(B^i_{12} + B^i_{32}) + f_2(B^i_{21} + B^i_{23}) + \overline{+f_1 * f_2(B^i_{11} + B^i_{31} + B^i_{13} + B^i_{33}) + N^i_{norm}}} \quad (4)$$

where f_1 and f_2 are multiplication terms that depend on the widths of the gates and their associated backgrounds as follows

$$f_1 = \frac{gate1}{bkg_{left} + bkg_{right}} \quad (5) \qquad f_2 = \frac{gate2}{bkg_{top} + bkg_{bottom}} \quad (6)$$

and, finally, N^i_{norm} is the intensity used for normalization, obtained as described before.

The final output file from this document represents the background-subtracted normalized angular correlation intensities with uncertainty calculations. This can be analyzed in order to extract the A_2 and A_4 values, and then compare these with theory in order to extract information about the possible spins and mixing ratios.

4. Fitting and Comparison to Theory

In order to apply fitting and carry out a comparison to theory, a special ROOT script has been created, which reads the final background-subtracted normalized angular correlation intensities and uncertainties. This program is called automatically by the bash script presented beforehand, but can also be called separately to try other hypotheses without redoing the rather lengthy cutting process.

The ROOT script, entitled ***fitcor***, reads the final output data from the previous steps. It then fits the data in order to obtain parameter A_0 from Eq.3, and then uses it to divide all the data points by this value. This serves to eliminate one of the fitting parameters and also brings all the angular correlation plots to the same scale, making their comparison far easier.

The next fit, with $A_0 = 1$, determines the values of the A_2 and A_4 parameters, and the minimum reduced χ^2, which we will call χ^2_{min}. In order to obtain the uncertainties of each parameter, the following procedure is applied. For example, for determining the uncertainty of the A_2 parameter, the A_4 value is fixed to its extracted value, but the A_2 parameter is varied until the reduced χ^2 exceeds $\chi^2_{min} + 1$. The lower and upper values of A_2 for which this condition is met give the uncertainty of the fit, as seen in Fig.3.

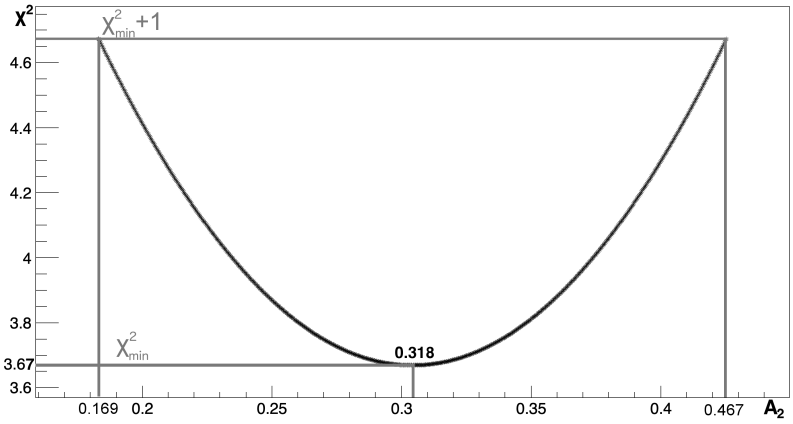

FIGURE 3. Representative illustration of how the best fits and uncertainties for the A_2 parameter are obtained for the 930-1345 keV cascade. The best fit is taken from the minimum χ^2 value, in this case, at $A_2 = 0.318$. The A_2 parameter is then varied while keeping A_4 fixed, until the χ^2 of the fits reaches $\chi^2_{min} + 1$, which gives the uncertainty. In this case, they are symmetrical, corresponding to ± 0.149.

After the A_2 and A_4 parameters and their uncertainties are determined, the script requests information from the user. This consists of spin hypothesis for the 3 levels involved and mixing ratios for the two transitions. For the spin hypothesis, the user can introduce a fixed value, or let the program vary

through all variants (from 0 to 9) by entering 'v' or nothing. However, do note that, if all three spins are left to vary, this will result in around 1000 spin hypotheses being checked, which will be quite computationally intensive.

For the mixing ratios, the user can input a value and its uncertainty, which will limit the values that the script will investigate, or allow the script to investigate the full range from -5 to $+5$.

Based on the spin hypothesis being checked, three distinct cases emerge. The first one is that in which the theoretical A_2 and A_4 parameters are uniquely determined, as the spin combinations mean that the mixing ratios are 0. These combinations are of the type $X \to 0 \to Y$ and $0 \to X \to 0$. In this case, the script will check if the theoretical A_2 and A_4 values are compatible with the experimental values within one or two σ, as seen in Fig.4.

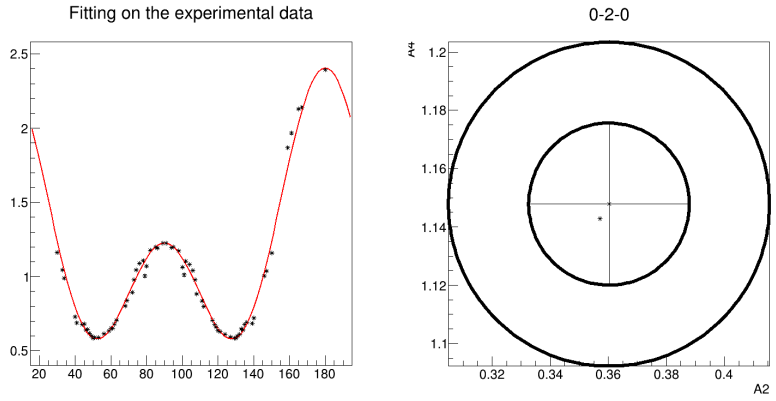

FIGURE 4. Illustration for the comparison between the experimental and theoretical values for the 1680-1345 keV $0^+ \to 2^+ \to 0^+$ cascade from ^{64}Ni. On the left is the fitting of the experimental data, used in order to obtain the experimental A_2 and A_4 values. On the right, the cross in the center of the two circles corresponds to the experimental value, with the two circles corresponding to one and two σ. The star that is slightly to the right and above is the theoretical value for this cascade, which matches the experimental value very well.

Besides the graphical representation seen in Fig.4, the script also informs the user if the experimental and theoretical values agree within one σ, two σ or not at all.

The next case is that in which only one of the two mixing ratios is fixed to 0, which appears for spin combinations of the type $0 \to X \to Y$ or $X \to Y \to 0$. In this case, the list of theoretically possible A_2 and A_4 values form a line as a function of the remaining mixing ratio. The script shows the position of this line with respect to the experimental values, and gives the best fitting mixing ratio, and the intersection points with the one σ or two σ circles, as seen in Fig.5.

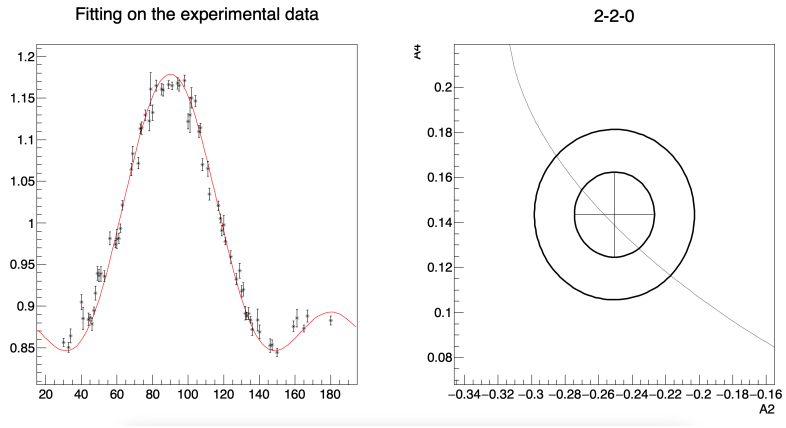

FIGURE 5. Illustration for the comparison between the experimental and theoretical values for the 930-1345 keV $2^+ \rightarrow 2^+ \rightarrow 0^+$ cascade from ^{64}Ni. On the left is the fitting of the experimental data, used in order to obtain the experimental A_2 and A_4 values. On the right, the center of the two circles corresponds to the experimental value, with the two circles corresponding to one and two σ. The thinner line running through the plot corresponds to the possible theoretical values. The script will output the corresponding mixing ratio for the point closest to the experimental values, and also the intersection points of the theoretical line with either the one σ or two σ circles.

The last case is that in which both of the two mixing ratios can have non-zero values, and they can both vary. This means that the comparison with the experimental data must be made at every combination of δ_1 and δ_2. This is done by taking the theoretical A_2 and A_4 values and checking whether they are inside the ellipse defined by the experimental values and their uncertainties. Values that are inside the ellipse are marked in the graphical plot as light blue, as can be seen in Fig.6, and the contour of the areas they define is saved in a file in the folder.

Ideally, if at least one mixing ratio is already known, this can be used to constrain the testing, yielding clearer results. For the example given before, if we constrain the mixing ratio of the 930 keV transition to 0.86(3), as determined from other cascades, then the results are far more restricted and narrow, as can be seen in Fig.7.

5. **Analysis Procedure**

Because the angular correlation figure depends on five distinct physical properties (the spins of the three states involved in the cascade and the mixing ratios of the two transitions), it is almost impossible to extract all five values from a single angular correlation. If no information is previously known, the

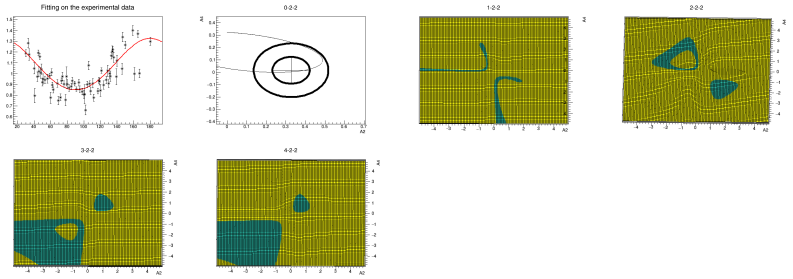

FIGURE 6. Illustration for the comparison between the experimental and theoretical values for the 695-930 keV $(1,2) \rightarrow 2^+ \rightarrow 2^+$ cascade from ^{64}Ni. On the top left is the fitting of the experimental data, used in order to obtain the experimental A_2 and A_4 values. After that, the five spin hypotheses (0,1,2,3,4) for the first excited state are checked with the data. The first one is a verification with using the second case presented before, as the first mixing ratio is 0. The next four hypotheses are checked using the third case just presented. The light blue areas represent $\delta_1 - \delta_2$ combinations that are compatible with the experimental values.

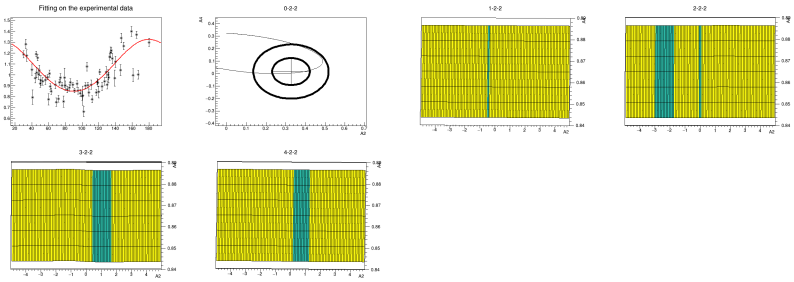

FIGURE 7. This represents the same analysis as the previous Fig.6, but with $\delta_2 = 0.86(3)$. The range of possible δ_1 values for each hypothesis is significantly lowered, making it far easier to compare the results with other cascades. Furthermore, this method excluded the 0 spin assignment within one σ.

number of spin hypotheses that can match a certain pair of A_2, A_4 parameters is very high.

Thus, it is paramount to start with excited states and transitions for which at least some information is already available from the literature or from other determinations. These can include information taken from literature or other study types, limits imposed by previous or subsequent decays to these states, lifetimes, the reaction type, etc.

If at least a few such 'grounding' values already exist, it is necessary to take advantage of them and then branch out from these 'known' states

towards lesser known ones. In the case of ^{64}Ni, for which the spins of the low-lying states are relatively well known, it was advantageous to start from these, and slowly build up information upwards in the scheme.

However, as can be seen in Fig.7, even with two spins and one mixing ratio extracted through other means, this method can be unable to uniquely determine the two remaining values (besides excluding the 0 spin assignment). Thus, it is important to check the same levels and transitions through multiple cascades, cross-checking the results for internal consistency. Thus, the acceptable spin hypotheses have to match up between different cascades involving the same level. For different cascades involving the same transition, the mixing ratios also have to agree for the same spin hypotheses, giving a powerful tool to narrow down the possible values.

Of course, the quality of this analysis depends on a number of issues, such as available statistics, the number of different detector angles available and contamination from other transitions.

6. **Conclusion**

This paper has presented an implementation of two programs made for angular correlation analysis. The first is used to obtain normalization matrices for angular correlation analysis starting from the data itself, with no need for calibration sources. The second one is used to handle the data manipulation, fit the angular correlation and investigate how well it fits with various spin hypotheses and mixing ratio values. While these two programs have been developed and are meant to be used together, there is no limitation on their application for other purposes. For example, the ***Jumbler*** can also be used for DCO analysis for nuclear reactions with accelerated beams, which require a different analysis and interpretation method. Similarly, the second program can be used with normalizations obtained in other ways, albeit with some changes to the code itself.

Taken together, they provide a powerful tool for extracting valuable and reliable nuclear spectroscopy information, which can then be used to further refine our understanding of nuclear structure.

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[6] Example files used for the program available on GitHub: https://github.com/standlucian/jumblerfiles.git