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BISTABLE-TYPE INTERACTIONS OF COMPLEX SYSTEMS IN A FRACTAL PARADIGM OF MOTION

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In the present paper the "interface" dynamics in the case of two complex systems interaction, assimilated to fractal-type mathematical objects, are analyzed. In such context, fractal bistable-type behaviors as transitions in the scale space are obtained. Our findings can be applied to natural bistable behaviors, such as temperature inversion in the planetary boundary layer.

Keywords: bistable behaviors, complex systems, fractal-type objects

1. Introduction

In complex systems dynamics, non-linearity and chaoticity represent both the structural and functional nature of turbulence and instabilities. Interactions between the constitutive entities of any complex systems give rise to mutual

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constraints and coupling local-global behavior types. In such a conjecture, the universality of dynamics laws for turbulence becomes natural and is reflected in its associated mathematical procedures, in the form of theoretical models that could describe their dynamics [1-3]. Regarding these models, they are usually founded on the supposition that variables describing the dynamics are differentiable. Thus, the success of the above-mentioned models should be understood as gradual, or on domains in which differentiability and integrability are still valid. However, the differentiable and integrable mathematical procedures prove themselves to be inadequate when these dynamics must be solved, because they imply both non-linearity and chaoticity. To describe such dynamics, while employing differential mathematical procedures, it is necessary to explicitly introduce the notion of scale resolution into the expression of variables associated with complex systems dynamics, and implicitly into the expression of fundamental equations that govern these dynamics.

The result is that, in the framework of non-integrability and non-differentiability, any variable classically dependent on space-time coordinates will also depend on scale resolution. Therefore, instead of operating with variables described through non-differentiable functions, approximations of these mathematical functions will be utilized, which are obtained by their averaging at various scale resolutions. Thus, any physical variable used in the description of complex system dynamics will instead be a limit of a family of mathematical functions, which are non-differentiable for null scale resolutions and differentiable for non-zero scale resolutions [1-3]. The main fundamental assumption of this theoretical model is that the dynamics of any and all entities of a complex system will be described by continuous but non-differentiable motion curves, which represent multifractal curves [1-3]. These multifractal motion curves will then exhibit self-similarity at every point, which is a property of holography – i.e., every part reflects the whole and vice versa. Thus, we are discussing "holographic implementations" of complex systems dynamics either through Schrödinger-type multifractal "regimes", (using Schrödinger-type equations at various scale resolutions), or through Madelung-type multifractal regimes (using the hydrodynamic equation system at various scale resolutions) [3].

In the present paper fractal bistable-type behaviors as transitions in the scale space are obtained. Our theoretical model is validated in the case of temperature inversion in the planetary boundary layer.

2. Mathematical model

Let us admit the functionality of the differential equation Eq. (1) "interface" dynamics in the case of two complex systems interaction (these systems will be assimilated to fractal-type mathematical objects).

$$\frac{dQ_t}{dt} = Q_i - Q_t\left[1 + \frac{A}{1 + Q_t^{2}}\right] \tag{1}$$

Such a result can be obtained through the general differential equation in the space of scale resolutions [2, 3]:

$$\frac{dQ_t}{dt} \approx \overline{A} + \overline{B}Q_t = \frac{A_1 Q_t^{3} + A_2 Q_t^{2} + A_3 Q_t + A_4}{\overline{A_3}Q_t^{2} + \overline{A_2}Q_t + \overline{A_1}} \tag{2}$$

by operating with the identities:

$$A_1 = -1, A_2 = Q_i, A_3 = -(1 + A), A_4 = Q_i, \overline{A_1} = 1, \overline{A_2} = 0, \overline{A_3} = 1 \tag{3}$$

where Q_t is the incident fractal field variable, τ is the temporal resolution scale with the role of affine parameter of the movement curves of complex system entities in the space of scale resolutions, and A is a parameter independent of the fractality degree in the space of the resolution scales through which it is possible to vary the different self-structuring modes of complex systems entities.

Let us also note that Q_i, Q_t, τ and A are dimensionless variables. Eq. (1) specifies that, at any scale resolution, the temporal variation of the transmitted fractal field variable, $\frac{dQ_t}{dt}$, is conditioned both by the difference between the transmitted and incident variables, $(Q_i - Q_t)$, and by a saturation component, $\frac{Q_t A}{1+Q_t^{2}}$.

In such a context, fractal dynamics systems are described in the space of scale resolutions through the fractal differential equation:

$$\frac{dQ_t}{d\tau} = -\frac{dV(Q_t)}{dQ_t} \tag{4}$$

where:

$$\begin{aligned} V(Q_t) &= -\int_0^{Q_t}\left(Q_i - Q_t' - \frac{AQ_t'}{1 + Q_t'^{2}}\right)dQ_t' \\ &= -Q_i Q_t + \frac{Q_t^{2}}{2} + \frac{A}{2}\ln\left(1 + Q_t^{2}\right) \end{aligned} \tag{5}$$

is the fractal potential function, which describes an important class of fractal dynamics systems which will be named "gradient fractal systems" [4-6].

Eq. (5) is represented in Fig. 1 for 4 values of the parameter A and it specifies the fact that $V(Q_t)$ presents a variation with two potential wells. This means that, from the perspective of an evolution towards equilibrium and towards the stability of equilibrium states, the fractal system given by Eq. (5) behaves regarding $V(Q_t)$ in an analogue manner to the behavior of the fractal oscillator regarding $V(Q_t)$, thus the quantities of $V(Q_t)$ will be equilibrium states while the maxima will be unstable equilibrium states.

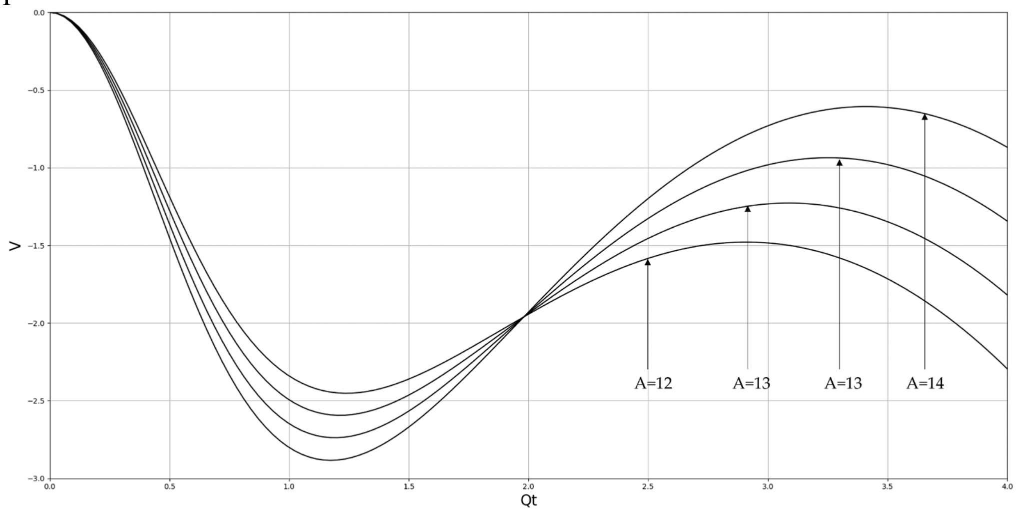

Fig. 1. Fractal potential $V(Q_t)$ for specified values of A.

Now, at all scale resolutions, the stationary behavior of the fractal system described by Eq. (1) is analyzed, which implies the functionality of:

$$Q_i = Q_t\left[1 + \frac{A}{1 + {Q_t}^2}\right] \tag{6}$$

This equation may yield 3 real roots, which is to say that for a value of the incident field variable Q_i there can be 3 different values of the transmitted field variable Q_t.

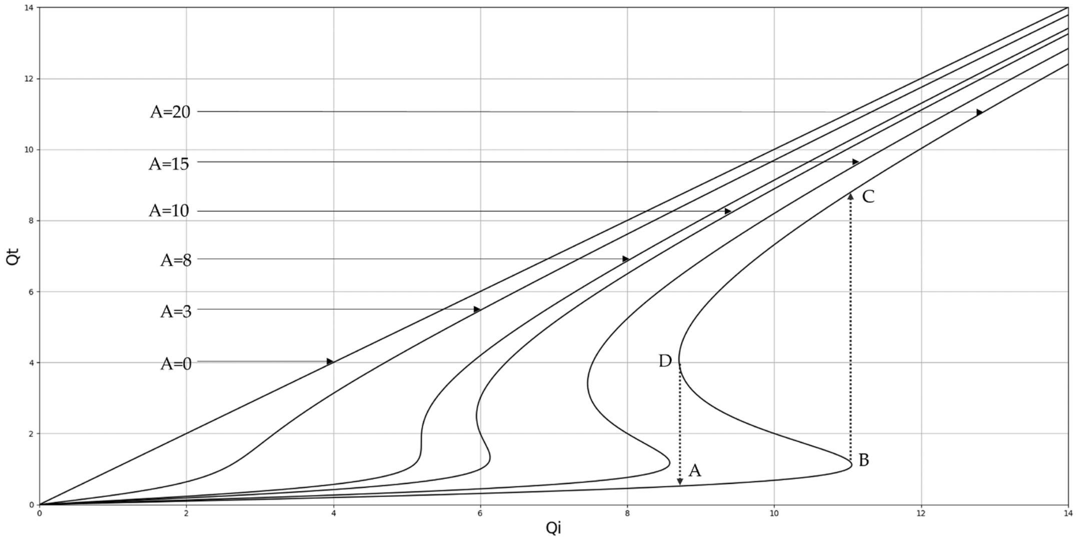

Fig. 2. Dependency of the transmitted fractal field variable to the incident fractal field variable in fractal bistability.

The curves $Q_t = F(Q_i)$ in Fig. 2 could show a maximum and a minimum when A attains certain values. These can be found by cancelling the derivative of Eq. (6).

The restriction:

$$\frac{dQ_i}{dQ_t} = 0 \tag{7}$$

implies:

$$1 + \frac{A\left(1 - 2{Q_t}^2\right)}{\left(1 + {Q_t}^2\right)} = 0 \tag{8}$$

This is a biquadratic equation which admits the solutions:

$${Q_t}^2 = \frac{1}{2}\left[(A - 2) \pm \sqrt{A(A - 8)}\right] \tag{9}$$

Eq. (9) should have only real (positive) solutions for A ≥ 8. What is indeed found is that no inversion takes place for all cases in which A ≥ 8.

For such values of A, 2 extremes are shown, so the system presents fractal bistability at all scale resolutions. The situation can be more easily perceived graphically, for example in the case of the $A = 20$ curve in Fig. 2.

Where Q_i increases slowly from $Q_i = 0$, Q_t increases until B. A continual increase of Q_i will have as a result a sudden increase of Q_t to C point, since the BD area of the curve represents unstable fractal states. When Q_i decreases from values superior to those found in C, Q_t decreases along the curve until the D point. Through continual decrease, Q_t will perform a sudden increase to the A point following the curve towards the origin.

Thus, for values of the incident fractal field variable in the AB interval, the transmitted fractal field variable can have two different stable values. This behavior points to fractal bistability [5, 6].

In the three-dimensional space (Q_i, Q_t, A) the surface $Q_t = Q_t(Q_i, A)$ is a fold catastrophe-type fractal surface (Fig. 3). For more details on the standard case, see [5, 7].

Let us note that the inversion curves presented in Figs. 1 and 2 can be mimed as transitions in the scale space.

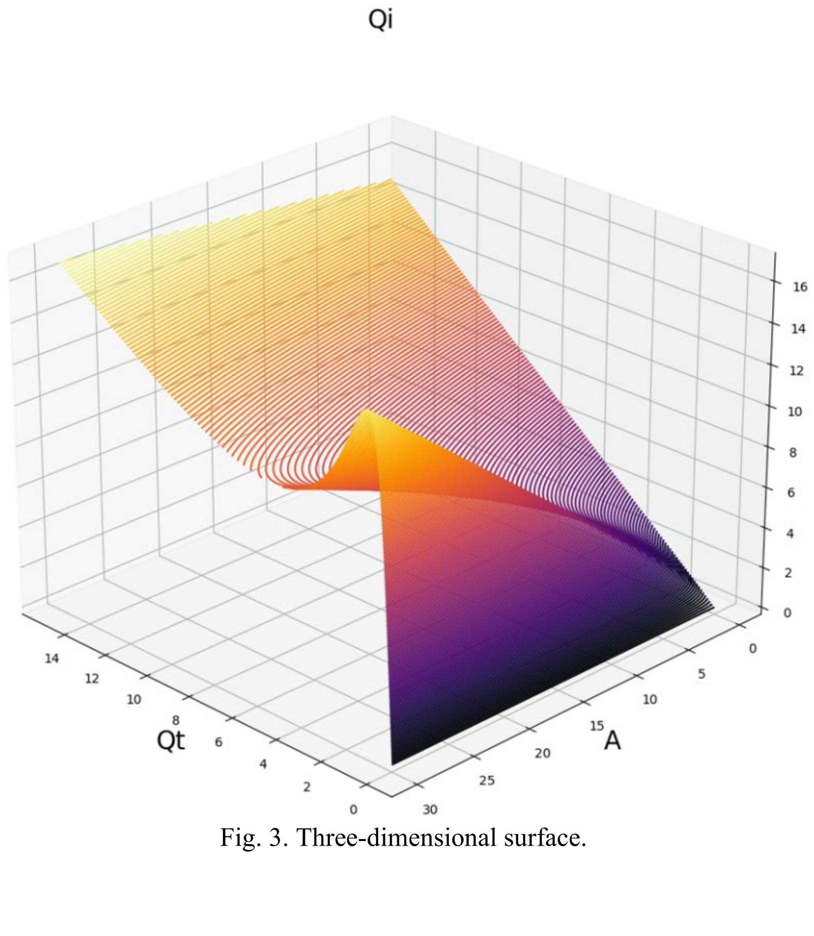

Fig. 3. Three-dimensional surface.

3. Application of the model

In the following, let us discuss a possible application of our model. Bistable behavior has been found throughout atmospheric profiles, and this has been shown through theory and through real radiometer data [8]. In the cited study, radiometer data has been obtained through a RPG-HATPRO radiometer platform positioned in Galați, Romania, at the UGAL – REXDAN facility found at coordinates 45.435125N, 28.036792E, 65 m ASL, which is a part of the "Dunărea de Jos" University of Galați [8]. This instrument has been chosen and set up so as to conform to the standards imposed by the ACTRIS community [9-12].

From this study, an instance is chosen: a data timeseries on the 14th of January 2022 (Fig. 4). A static profile is also shown as an example, extracted from the beginning of the timeseries (Fig. 5).

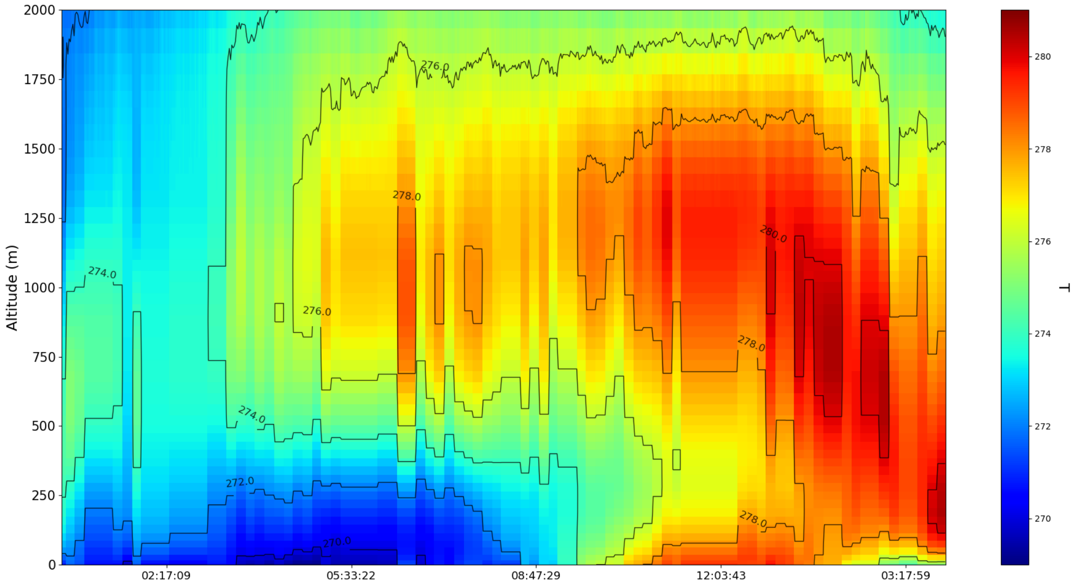

Fig. 4. Timeseries of atmospheric temperature profiles; Galati, Romania, 16/01/2022.

A small discussion regarding the nature of temperature profile is in order; it is known that for diurnal profiles there exists a slightly greater decrease in temperature in the SL, and for nocturnal profiles there is an inversion at the SL [9-12].

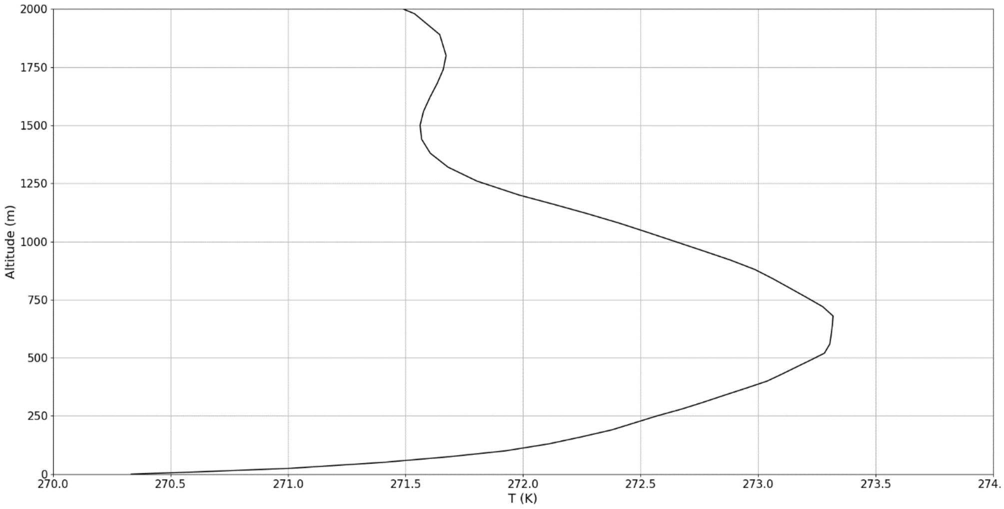

Fig. 5. Example of atmospheric temperature profile; Galati, Romania, 14/01/2022.

Otherwise, inversions also mark the occurrence of the PBLH [9-12]. Fig. 5 presents an inverse evolution characteristic of nocturnal conditions with a large SL and a higher inversion. Let us note that, in these atmospheric temperature profiles, the altitude corresponds to $\mathrm{Q_t}$ and the absolute temperature corresponds to Q_i.

4. Conclusions

By assimilating complex systems to fractal-type mathematical objects, the "interface" dynamics as a result of these systems interaction are analyzed. Let us note that a wide range of nonlinear behaviors [13-19] can be sequentially assimilated to bistable-type behaviors. One particular case is highlighted in the form of fractal bistable-type behaviors.

These fractal bistable-type behaviors are discussed in the case of planetary boundary layer bistability. The physical context for this behavior is the presence of water vapors and aerosols which provide a nonlinear propagation environment between the planetary boundary layer and ground level. Given inherent bistability, and given the connection between multifractal parameters and temperature, it is then suggested that such bistable behavior can explain the well-known boundary layer temperature inversion, and inversions of other parameters as well. Finally, radiometer data offers various examples of atmospheric temperature inversions, wherein theoretical data agrees with experimental data.

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