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ESTIMATION OF RELATIVE X-RAY YIELDS OF EXCITED MUONIC DEUTERIUM ATOMS

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In this investigation, transition rates and relative X-ray yields during muonic cascade in pure deuterium were calculated. Radiative transition rate and all collisional processes were estimated at different initial states. In addition, the dependence density of the relative X-ray yields in excited muonic deuterium was calculated in the density range (0.05−1.15) (in units of Liquid Hydrogen Density, LHD = 4.25×10^{22} atoms/cm^3). All estimations were carried out using the standard cascade model and the recent X-ray yield equation presented earlier. Our calculated results are in good agreement with experimental data available in literature.

Keywords: Transition rate, Muonic deuterium, De-excitation process, X-ray yield.

1. Introduction

The catalysis of nuclear fusion reactions by muons in the cold mixture of hydrogen isotopes is known as muon-catalyzed fusion (μCF) [1-4]. Muons can be created by the decay of pions, which are generated during the collision of intermediate-energy protons with the target nuclei. The muons entering the deuterium target are slowed down to an energy of *~10 eV*, where they are captured by a deuterium molecule resulting a μd atom in the excited state [5, 6]. In this process, muon leaves the highest state of initial energy with principal quantum number, $n=14$. It is the starting point of radiative and collisional processes [7]. The known muonic deuterium collisional de-excitation processes are external Auger, Coulomb de-excitation, elastic scattering, molecular dissociation and Stark mixing. These processes continue until muon is captured by the deuterium or it decays. Muon, in contrast to the pion, kaon and antiproton is not affected by the strong interaction [8]. Therefore, muonic deuterium is the most appropriate exotic atom in the study of cascade processes [9, 10]. The de-excitation processes cause the transition of excited state of the muonic deuterium to the ground state. This is why, in this paper we have calculated the yield of X-ray resulted in these transitions [11]. Although the transition rate of muonic deuterium and their relative X-ray yield had been theoretically studied [12, 13], in

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this investigation we have improved the equation presented of X-ray yield [14] by considering the efficiency of the molecular dissociation process and also using the best quantity for the fixed parameters. In this study, the transition rates of all muonic deuterium cascade processes are calculated at different initial states. Subsequently, we have estimated the density dependence of X-ray yield for the muonic deuterium k_α and k_β Lines. Furthermore, we have calculated the density dependence of ratio, Y_{k_α}/Y_{k_β}. In this paper, we have mainly focused on the investigation of the standard cascade model.

2. DE-excitation processes

After capturing muon by deuterium, the cascade processes develop. These basic cascade de-excitation processes are listed in Table 1. Radiation transition is an uncollisional process which behaves independently of the kinetic energy and the density. In this process, transition energy is carried by photon. External Auger effect is a process that behaves independently of the kinetic energy. When an excitation energy of excited muonic deuterium is approximately equal to an ionization energy of deuterium *(=15.46 eV)* the external Auger processes can ionize deuterium molecule. Then, the excitation energy of the muonic deuterium might be captured by deuterium electron. Thus, the electron Auger continues to move by a momentum equal to $[2(\Delta E_{n_i n_f} - 15.46)]^{1/2}$, where $\Delta E_{n_i n_f}$ represents the transition energy. In this process critical state n_c is the greatest value of n that is allowed for transitions with $\Delta n = 1$.

Table 1

Basic cascade processes in muonic deuterium

Cascade mechanism	Processes	Refs.
Radiative transition	$(\mu^- d)_{n_i l_i} \rightarrow (\mu^- d)_{n_f l_f} + \gamma$	[15]
External Auger	$(\mu^- d)_{n_i l_i} + D_2 \rightarrow (\mu^- d)_{n_f l_f} + e^- + D_2^+$	[16]
Coulomb de-excitation	$(\mu^- d)_{n_i l_i} + D \rightarrow (\mu^- d)_{n_i - 1, l_i} + D$	[17]
Elastic scattering	$(\mu^- d)_{n_i l_i} + D \rightarrow (\mu^- d)_{n_i l_i} + D$	[18]
Stark mixing	$(\mu^- d)_{n_i l_i} + D \rightarrow (\mu^- d)_{n_i l_f} + D$	[19]
Molecular dissociation	$(\mu^- d)_{n_i l_i} + D_2 \rightarrow (\mu^- d)_{n_f l_i} + D + D$	[20]
Weak decay	$\mu^- \rightarrow \upsilon_\mu + \overline{\upsilon_e} + e^-$	[21]

For $n > n_c$ the transitions with $\Delta n > 1$ that their released energy become greater than the ionization energy of the deuterium are allowed. Coulomb de-excitation process is the collisional process that it's released colliding energy is shared between colliding particles. Therefore, kinetic energy of the muonic deuterium increases. The transition rate of the Coulomb de-excitation process is approximately dependent on the kinetic energy with ratio $1/\sqrt{T_i}$. In this process, transitions with $\Delta l = 0$ and $\Delta n = 1$ are allowed. Another kind of processes can be considered is the elastic scattering. This process has an unchangeable principal quantum number and an unchangeable angular quantum number that leads to a deceleration of muonic deuterium. Since the muonic deuterium passes from a coulomb barrier of the deuterium, the stark mixing process occurs. This is the fastest collisional process which can mix atomic sublevels with $\Delta l = \pm 1$ and $\Delta m = 0$. Molecular dissociation process occurs when the transition energy of the muonic deuterium is greater than the dissociation energy $(\Delta E_{n_i n_f} > 4.55 eV)$. Therefore, the muonic deuterium can dissociate the muon of the deuterium. Weak decay is another kind of collisional processes that its rate inversely relates to a muon lifetime. The details of de-excitation processes and their transition rate can be found in Table 1. We have shown the results of the transition rates of the muonic deuterium atom at different initial states in Figure 1. In this estimation, we have used standard cascade model. We have also assumed the kinetic energy equal to *T=1 (eV)* and the density equal to *N=1 (LHD)* through the whole cascade process.

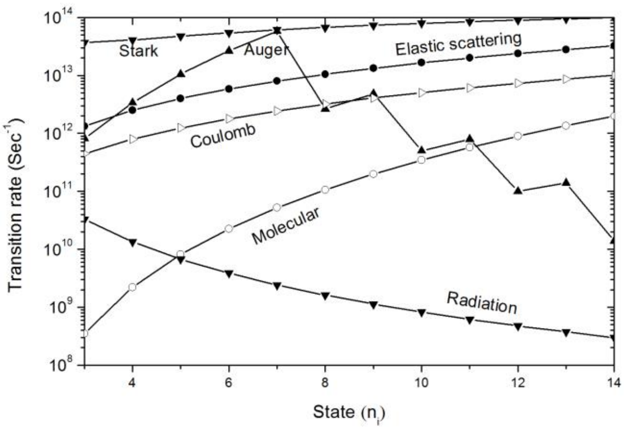

Fig. 1. The rates of various cascade processes vs. initial state n_i for the excited muonic deuterium in *N=1(LHD)*. The density is in units of Liquid Hydrogen Density, *LHD=4.25×10^{22} atoms/cm^{3}*.

The obtained results reveal that the greatest radiative transition rate between $nP \rightarrow 1S$ is occurred in states with low *n*. Also, we have calculated minimum transition rate of this process in the highest initial state, that is equal to $\lambda = 2.988 \times 10^{8}\ (Sec^{-1})$. In the external Auger process of the muonic deuterium, $n_c=7$ is known as a critical state that we have calculated its maximum transition rate equal to $\lambda = 5.805 \times 10^{13} (Sec^{-1})$. It should be mentioned that transition rates of the elastic scattering and the stark mixing are calculated by considering $k_c = k_{el} = 1$ and $k_{stk} = 1.5$. In this study, we have also extended Borie-Leon model in the calculation of the Stark mixing. The obtained results show the least transition rate of molecular dissociation in contrast to the transition rate of other processes in initial states with low *n*.

3. Relative X-ray yields

The yield estimation of X-ray depends on the study of the atomic transition by de-excitation processes. In k_α emission lines the transition of $2P \rightarrow 1S$ to be considered [22]. But, since the transitions of the muonic deuterium have been comforted at *2S*, the investigation of $2S \rightarrow 2P$ is important [23, 24]. In order to prevent of this evolution, collisional energy must be greater than $2S \rightarrow 2P$ transition energy ($\varepsilon = 0.2(eV)$) of the muonic deuterium. In k_β emission lines the transition of $3P \rightarrow 1S$ to be considered. In this paper, we have calculated the relative X-ray yield for the muonic deuterium on the deuterium density between *0.05* and *1.15* in units of liquid hydrogen density. In this estimation, we have improved the presented earlier formulas (in Ref. [14]) for X-ray yield of k_α and k_β by adding the transition rate of the molecular dissociation and other de-excitation processes on them. Therefore, we rewrite them as follows:

$$Y_{k_\alpha} \approx \tilde{P}_3 \frac{\lambda_{3\rightarrow 2}^{Rad} + N(\lambda_{3\rightarrow 2}^{Aug} + \lambda_{3\rightarrow 2}^{Coul} + \lambda_{3\rightarrow 2}^{Mol})}{\lambda_{3\rightarrow 1}^{Rad} + \lambda_{3\rightarrow 2}^{Rad} + N(\lambda_{3\rightarrow 2}^{Aug} + \lambda_{3\rightarrow 2}^{Coul} + \lambda_{3\rightarrow 2}^{Mol})} \tag{1}$$

$$Y_{k_\beta} = \tilde{P}_3 \frac{\lambda_{3\rightarrow 1}^{Rad}}{\lambda_{3\rightarrow 1}^{Rad} + \lambda_{3\rightarrow 2}^{Rad} + N(\lambda_{3\rightarrow 2}^{Aug} + \lambda_{3\rightarrow 2}^{Coul} + \lambda_{3\rightarrow 2}^{Mol})} \tag{2}$$

N refers to the density of the deuterium. $\lambda_{n_i l_i}^{Rad}$, $\lambda_{n_i l_i}^{Coul}$ and $\lambda_{n_i l_i}^{Aug}$ are the transition rates of the radiation, the Coulomb de-excitation and the external Auger, respectively. $\tilde{P}_{n,l}$ is an arrival probability of (*n* , *l*) levels which are resulted by the Stark mixing. In order to calculate $\tilde{P}_{n,l}$ the starting population of (*n* , *l*) levels before the Stark mixing, $P_{n,l}$, should be define. This starting distribution is estimated as follow [21]:

$$P_{n_i,l_i}(t=0)=\tilde{P}_{n_i,l_i}+\tilde{P}_{n_i,l_i+1}\frac{\lambda^{St}_{n_i,l_i+1}}{\lambda^{Total}_{n_i,l_i+1}}+\ldots+\tilde{P}_{n_i,l_i=n-1}\frac{\lambda^{St}_{n_i,l_i=n-1}}{\lambda^{Total}_{n_i,l_i=n-1}} \quad (3)$$

The above equation can rewrite as the matrix form as follow:

$$\begin{pmatrix} P_{n_il_i=0}(t=0) \\ P_{n_il_i=1}(t=0) \\ \vdots \\ \vdots \\ P_{n_il_i=n_i-1}(t=0) \end{pmatrix} =$$

$$\begin{pmatrix} 1 & \frac{-\lambda^{St}_{n_il_i=1\to n_fl_f=0}}{\lambda^{Total}_{n_il_i=1}} & \cdots & \cdots & \frac{-\lambda^{St}_{n_il_i=n_i-1\to n_fl_f=0}}{\lambda^{Total}_{n_il_i=n_i-1}} \\ \vdots & 1 & \vdots & \vdots & \vdots \\ \vdots & \vdots & 1 & \vdots & \vdots \\ \vdots & \vdots & \vdots & 1 & \frac{-\lambda^{St}_{n_il_i=n_i-1\to n_fl_f=n_i-2}}{\lambda^{Total}_{n_il_i}} \\ \frac{-\lambda^{St}_{n_il_i\to n_fl_f=n_i-1}}{\lambda^{Total}_{n_il_i=0}} & \cdots & \cdots & \frac{-\lambda^{St}_{n_il_i=n_i-2\to n_fl_f=n_i-1}}{\lambda^{Total}_{n_il_i=0}} & 1 \end{pmatrix} \begin{pmatrix} \tilde{P}_{n_il_i=0} \\ \tilde{P}_{n_i,l_i=1} \\ \vdots \\ \vdots \\ \tilde{P}_{n_il_i=n_i-1} \end{pmatrix}$$

(4)

where $\lambda^{Total}_{n_il_i}$ is estimated as follow:

$$\lambda^{Total}_{n_i,l_i}=\sum_{n_f=1}^{n_i-1}\lambda^{Mol}_{n_i,l_i\to n_f,l_f}+\sum_{n_f=1}^{n_i-1}\sum_{l_f=0}^{n_f-1}\lambda^{Rad}_{n_i,l_i\to n_f,l_f}+\sum_{n_f=1}^{n_i-1}\sum_{l_f=0}^{n_f-1}\lambda^{Aug}_{n_i,l_i\to n_f,l_f}+\sum_{l_f\neq l_i}^{n_i-1}\lambda^{St}_{n_i,l_i\to n_f,l_f}+$$
$$\sum_{n_f=1}^{n_i-1}\lambda^{Coul}_{n_i,l_i\to n_f,l_f}+\lambda^{Sc}_{n_i,l_i\to n_f,l_f}+\lambda^{Weak}$$

(5)

where $\lambda^{Mol}_{n_il_i}$, $\lambda^{St}_{n_il_i}$, $\lambda^{Sc}_{n_il_i}$ and $\lambda^{Weak}_{n_il_i}$ are the transition rates of the molecular dissociation, the stark mixing, the elastic scattering and the transition of the weak decay, respectively. Our relative X-ray yield calculation of the muonic deuterium *K* lines on different densities between *0.05* and *1.15* in units of liquid hydrogen densities are shown in Figure 2. It is clear from Figure 1, the relative X-ray yield of the muonic deuterium k_α line increases non-linearly by increasing density. This increase explains the most effective role of collisional processes in contrast to the radiative transition. The effects of the molecular dissociation have caused the relative X-ray yield to some extent closer to the experimental data at densities higher than *N=0.18(LHD)*.

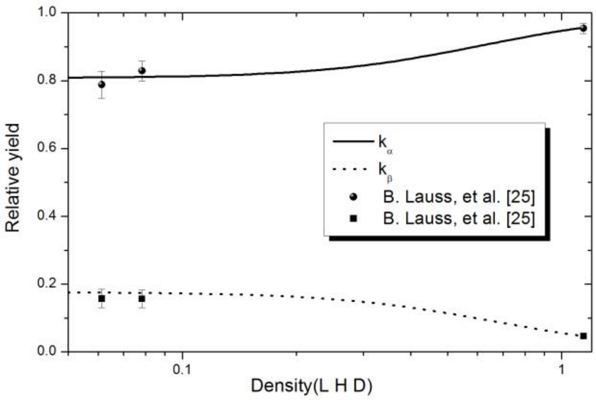

Fig. 2. The density dependence of the X-ray yield of muonic *K* lines on deuterium. Our theoretical results are compared with the measurements of B. Lauss *et al.* [25].

With present theory, we have calculated the relative X-ray yield of k_α and k_β lines equal to $Y_{k_\alpha}/Y_{k_{Total}} = 0.955$ and $Y_{k_\beta}/Y_{k_{Total}} = 0.045$ at *N=1.142(LHD)*, respectively. In Table 2, we have presented our relative X-ray yield calculation of k_α and k_β lines and experimental data at different density. We have calculated maximum relative X-ray yield of k_α line equal to $Y_{k_\alpha}/Y_{k_{Total}} = 0.956$ and minimum relative X-ray yield of k_β equal to $Y_{k_\beta}/Y_{k_{Total}} = 0.044$ at the highest density. Due to the theory and constant parameters we have used the relative X-ray yield of muonic *K* lines, in contrast to the X-ray yield of SCM model (in ref.[14]), are closed to the experimental data. Finally, we have calculated the ratio of Y_{k_α}/Y_{k_β} in various densities between *0.05* and *1.15* in units of liquid hydrogen densities.

Table 2

The relative X-ray yield of k_α and k_β lines for excited muonic deuterium atom in different densities

Density (LHD)	Relative X-ray yield			
	k_α		k_β	
	Present Theory	Exp. (Ref.[25])	Present Theory	Exp. (Ref.[25])
N=0.0613	0.810	0.788 ± 0.040	0.175	0.157 ± 0.027
N=0.0783	0.811	0.829 ± 0.029	0.174	0.156 ± 0.026
N=1.142	0.955	0.954 ± 0.015	0.045	0.046 ± 0.006

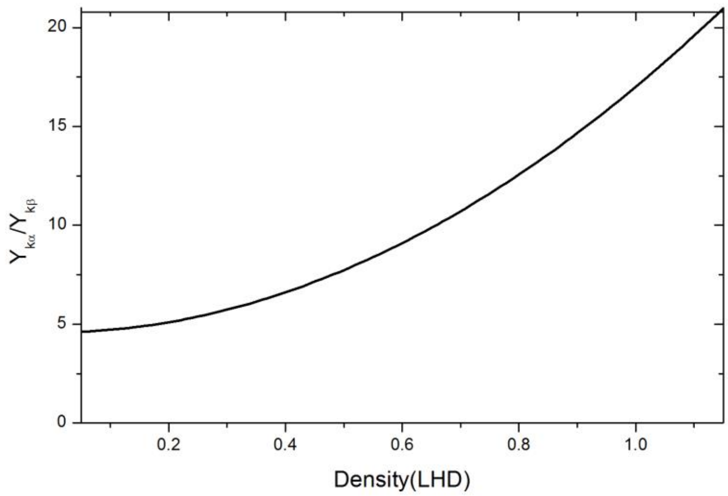

Fig. 3. The density dependence of the ratio of Y_{k_α}/Y_{k_β} in excited muonic deuterium atom.

The calculated ratio as function of density is shown in figure 4. Our estimations are shown minimum value of this ratio equal to $Y_{k_\alpha}/Y_{k_\beta} = 4.619$ at *N=0.05(LHD).*

4. Conclusions

In this investigation, all de-excitation processes of the excited muonic deuterium at different initial state have been presented (Figure 1). Although the obtained results are shown the least transition rate of the molecular dissociation at low-laying initial state, but their effects on the results of the relative X-ray yield are effected to some extent. The relative X-ray yield of k_α line increases and the relative X-ray yield of k_β line decreases with increasing the density (Figure 2). We have extended the equation presented earlier in Ref. [14] by considering all collisional processes. Eventually, obtained results of the relative X-ray yield of k_α and k_β lines are in good agreement with experimental data. Finally, we have calculated the ratio of Y_{k_α}/Y_{k_β}. Results based on our approach shown that the ratio of Y_{k_α}/Y_{k_β} increases non-linearly by increasing density.

REFERENCES

[1]. *M. Filipowicz, V. M. Bystritsky, V. V. Gerasimov, and J. Wozniak,* Eur. Phys. J. D, **vol. 47**, 2008, 157.
[2]. *K. Ishida, K. Nagamine, T. Matsuzaki and N. Kawamura,* J. Phys. G, **vol. 29**, 2003, 2043.
[3]. *A. Adamczak, M. P. Faifman,* Phys. Rev. A, **vol. 64**, 2001, 052705.
[4]. *S. M. Motevalli, M. R. Pahlavani,* Some Applications of Quantum Mechanics, InTech Press, 109, 2012.
[5]. *T. Matsuzaki, K. Nagamine, N. Kawamura, K. Ishida, Y. Matsuda, S. N. Nakamura, M. Kato, H. Sugai, M. Tanase, K. Kudo, N. Takeda and G. H. Eaton,* Phys. Lett. B, **vol. 557**, 2003, 176.
[6]. *S. M. Motevalli, O. N. Ghodsi, S. Sheikhian,* Indian J. Phys., **vol. 18**, 2013, 107.
[7]. *T. E. Jensen, V. E. Markushin,* Eur. Phys. J. D, **vol. 19**, 2002, 165.
[8]. *S. Weinberg,* Physica A, **vol. 96**, 1979, 327.
[9]. *J. Gasser, H. Leutwyler,* Nucl. Phys. B, **vol. 250**, 1985, 465.
[10]. *L. I. Menshikov,* Muon Catalyzed Fusion, **vol. 2**, 1988, 173.
[11]. *M. Bregant, D. Chatellard, J. –P. Egger, P. Hauser, E. Jeannet, K. Kirch, F. Kottmann, E. Milotti, C. Petitjean, L. M. Simons, D. Taqqu and E. Zavattini,* Phys. Lett. A, **vol. 241**, 1998, 344.
[12]. *E. C. Aschenauer, V. E. Markushin,* Hyperfine Interact., **vol. 101/102**, 1996, 97.
[13]. *E. C. Aschenauer, V. E. Markushin,* Z. Phys. D, **vol. 39**, 1997, 165.
[14]. *V. E. Markushin,* Hyperfine Interact., **vol. 199**, 1999, 11.
[15]. *B. Lauss, P. Ackerbauer, W. H. Breunlich, B. Gartner, M. Jeitler, P. Kammel, J. Marton, W. Prymas, J. Zmeskal, D. Chatellard, J.-P. Egger, E. Jeannet, H. Daniel, A. Kosak, F. J. Hartmann, and C. Petitjean,* Phys. Rev. Lett. **vol. 76**, 1996, 4693.
[16]. *T. Koike, T. Harada, Y. Akaishiet,* Phys. Rev. C, **vol. 53**, 1996, 79.
[17]. *W. Czaplinski, A. Gula, A. Kravtsov, A. Mikhailov, N. Popov,* Phys. Rev. A, **vol. 50**, 1994, 525.
[18]. *G. Ya. Korenman, V. N. Pomerantsev, V. P. Popov,* JETP Lett., **vol. 81**, 2005, 543.
[19]. *I. Levine, D. Koltick, B. Howell, E. Shibata, J. Fujimoto, T. Tauchi, K. Abe, T. Abe, I. Adachi, K. Adachi, et al.,* Phys. Rev. Lett., **vol. 78**, 1997, 424.
[20]. *E. Borie, M. Leon,* Phys. Rev. A, **vol. 21**, 1980, 1460.
[21]. *T. P. Terada, R. S. Hayano,* Phys. Rev. C, **vol. 55**, 1997, 73.
[22]. *M. P. Faifman, L. I. Menshikov,* Hyperfine Interact., **vol. 138**, 2001, 61.
[23]. *V. E. Markushin,* 1998, arXiv: 9812019v1 [nucl-th].
[24]. *V. E. Markushin,* Phys. Rev. A, **vol. 50**, 1994, 1137.
[25]. *B. Lauss, P. Ackerbauer, W. H. Breunlich, B. Gartner, M. Jeitler, P. Kammel, J. Marton, W. Prymas, J. Zmeskal, D. Chatellard, J. – P. Egger, H. Daniel, F. J. Hartmann, A. Kosak, C. Petitjean,* Phys. Rev. A, **vol. 60**, 1999, 209.