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STRONG AMPLIFICATION OF SCATTERED NEUTRINO AVERAGE ENERGY IN ELASTIC RELIC NEUTRINO-POSITRON SCATTERING NEAR WHITE DWARFS AND NEUTRON STARS

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We have substantiated that (i) the elastic scattering of the lightest relic neutrinos by positrons passing through the close vicinity of white dwarfs and neutron stars is the alternative mechanism responsible for the production of high energy ($\sim 10^2\ GeV - \sim PeV$) astrophysical neutrinos, (ii) the close vicinity of white dwarfs and neutron stars is a unique natural intermediate laboratory for amplification of the energy of the lightest relic neutrinos and (iii) the detection of the scattered lightest relic neutrinos coming from the close vicinity of white dwarfs and neutron stars is quite possible due to the $\nu_l n \to p l^-$ reaction.

Keywords: neutrino, positron, flavour, white dwarf, neutron star, MF.

1. Introduction

Production mechanism of high and ultra-high energy neutrinos in cosmic accelerators is one of the most important problems in high-energy astrophysics and cosmology. Here, we discuss the questions connected with the amplification of the energy of a relic neutrino whose energy is extremely low ($\sim 10^{-4}\ eV$ [1])

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and production mechanism of high energy ($\sim 10^2\ GeV - \sim PeV$) cosmic neutrinos. When relic neutrinos scatter by ultra-relativistic cosmic positrons of sufficiently high energy (PAMELA [2, 3], AMS-02 [4, 5] and the Fermi Large Area Telescope [6] experiments), the energy transfer from the high-energy positrons to the relic neutrinos is more plausible than the energy transfer from the relic neutrinos to the high-energy positrons. This unique possibility leads to the increase of the energy of a scattered relic neutrino and therefore, it opens a gate to the solution of the problem connected with the production mechanism of high-energy neutrinos in cosmic accelerators. When the scattering of relic neutrinos by positrons takes place in a magnetic field (MF), the latter will affect the neutrino-positron scattering by deforming the energy spectrum of the positrons, polarizing the positron spins and changing the positron phase space. In general, the neutrino-electron scattering ($\nu e^{\pm} \to \nu' e^{\pm\prime}$, $\tilde{\nu} e^{\pm} \to \tilde{\nu}' e^{\pm\prime}$) processes in an external MF [7-25], which are similar to the elastic neutrino-positron scattering process, the energy-transfer in these processes and some aspects of the neutrino elastic scattering on polarized targets [26, 27] have been studied by numerous authors. V. P. Tsvetkov [7] calculated the neutrino (antineutrino) scattering on electron in the ground state in a homogeneous magnetostatic field, estimated the electron excitation cross section in the strong and weak MFs at $1\ KeV$ neutrino energy in the Weinberg-Salam model and $V - A$ theory and obtained the analytical expression for full probability of the neutrino (antineutrino)-electron scattering taking into account the MF effect. Buzardan and Vshivtsev [8] obtained an expression for the scattering cross-section of a neutrino by an electron in a MF using the exact solutions of the Dirac equation in a MF in the low-energy approximation. Lyul'ka [9] obtained the analytical expression for the differential probability of the neutrino-electron scattering processes in an external electromagnetic field. Zakhartsov, Loskutov and Parfenov [10] considered the anti-Stokes scattering of low energy neutrino on relativistic electrons in a MF and calculated the scattering cross-section, polarization and mean longitudinal momentum of recoil electrons after the scattering as functions of the initial polarization and structural constants of the electroweak interaction. Borisov, Nanaa and Ternov [11] calculated the cross section of the scattering of the muon neutrinos by electrons in a stationary external electromagnetic field with allowance for the effect of propagator of the intermediate vector boson within the Weinberg-Salam theory. Vilenkin [12] investigated the neutrino-electron scattering in a dense magnetized plasma and considered the questions connected with the parity non-conservation and neutrino transport in MFs of pulsars. Bezchastnov and Haensel [13] calculated the rate of neutrino-electron scattering in a strong MF by evaluating the matrix element for the related Feynman diagrams, using the exact wave-functions of the electrons in the strong field. Kuznetsov and Mikheev [14, 15], Mikheev and Narynskaya [16, 17] investigated the neutrino-electron (positron) scattering in a magnetized hot plasma in the framework of the Standard Model and obtained the expression for the probability and the mean values of the neutrino energy and momentum losses, as well the volume density of the neutrino energy and momentum losses. Gusseinov [18] calculated the cross section of the scattering of the muon antineutrinos by electrons in a constant MF with allowance for the Z-boson propagator. The scattering rate of the neutrino-electron (positron) scattering was considered by Hardy and Thoma [19] in a strong MF based on the technique of finite temperature field theory. Huseynov and co-workers [20-22] calculated the differential cross sections, energy and momentum losses for the neutrino (antineutrino)-electron scattering processes in a MF with allowance for the longitudinal and transverse polarizations of the electrons and the spin asymmetries arising in these processes. Guseinov, Jafarov and Gasimova [23, 24] obtained the analytical formulae for the differential cross sections of neutrino (antineutrino)-electron scattering in a MF with allowance for longitudinal polarizations of initial and final electrons and medium factors, analyzed polarization effects, calculated the asymmetry of the heating of electrons (electron gas) having a left-hand circular polarization and electrons (electron gas) having a right-hand circular polarization by neutrinos in a MF. Dobrynina, Moraru and Ognev [25] studied the neutrino-electron scattering ($\nu e^{\pm} \to \nu' e^{\pm\prime}$, $\tilde{\nu} e^{\pm} \to \tilde{\nu}' e^{\pm\prime}$) processes in a matter with an external MF of an arbitrary strength and integrated the probabilities of the processes over the transverse momenta of charged particles as well as the energy and momentum transferred in them from the medium to neutrinos.

However, the analyses of the papers [7-27] show that the average energy of low-energy neutrinos (including cosmic relic neutrinos) in the final state in the neutrino-positron scattering process and the cross section of the indicated process in a MF in the framework of the Weinberg-Salam electroweak interactions theory with allowance for the transverse polarizations of the positrons in both the initial and final states and the related terrestrial applications connected with the amplification of the energies of the scattered relic neutrinos have not been investigated. At the same time, it should be noted that the cross-sections (or the related differential probabilities per unit time) of the neutrino-positron scattering processes in a MF in the framework of the Weinberg-Salam electroweak interactions theory with simultaneous allowance for the transverse polarizations of the positrons in both the initial and final states, propagator effects and medium factors (chemical potential and temperature) have not been calculated. It is worth noting that the results obtained for the cross sections of the elastic neutrino-electron scattering processes in the presence of a MF in some works differ from each other (e.g., in the very low energy region in the weak field limiting case the cross section (22) obtained in the work [11] is two times more than the cross section (4a) obtained in [10], or the sign before the spin term in the formula (4a) of the work [10] is not true).

The purpose of the presented article is to identify the alternative mechanism responsible for the production of high energy ($\sim 10^2\ GeV - \sim PeV$) astrophysical neutrinos and amplification of the energy of the lightest relic neutrinos, to determine the realistic physical conditions under which the lightest relic neutrino energy is amplified to energies $\sim 10^2\ GeV - \sim PeV$ and to point the related astrophysical sources capable to produce $\sim 10^2\ GeV - \sim PeV$ astrophysical neutrinos. For this purpose, we calculate the differential probability of the elastic scattering of the lightest relic neutrinos by positrons

$$\nu_i + e^+ \to \nu_i' + e^{+\prime} \qquad (1)$$

in a constant homogeneous MF of the strength

$$B \ll B_0 = m_e^2/e = 4.414 \times 10^{13}\ G \qquad (2)$$

in the tree-level approximation of perturbation theory and in the low-energy approximation of the Weinberg-Salam electroweak interaction theory, derive the analytical formula for the average energy of the neutrino in the final state and clarify the role of the neutrino flavour in the elastic scattering of the lightest relic neutrino by a transversely polarized ultra-relativistic positron where B_0 is the critical Schwinger field strength, e is the elementary electric charge, m_e is the electron or positron mass. We assume that both the initial and final state positrons are ultra-relativistic ($\varepsilon^2 \gg m_e^2$, $\varepsilon'^2 \gg m_e^2$) and they possess large transverse momenta ($p_\perp = (2eBn)^{1/2} \gg m_e$, $p'_\perp = (2eBn')^{1/2} \gg m_e$ where n (n') is the number of the Landau energy level of the positron in the initial (final) state). If both the initial and final state ultra-relativistic positrons possessing large transverse momenta are situated in a MF which is directed along the z-axis and possesses the strength satisfying the condition (2), then the main contribution to the differential probability of the processes (1) comes from the positron states occupying very high Landau levels ($n,\ n' \gg 1$). We consider the case when the incident lightest relic neutrino flies along the z-axis (along the MF direction) and the longitudinal momentum of the positron in the initial state is zero: $p_z = 0$.

According to the neutrino oscillations, one of the three known mass eigenstates of a neutrino is expected to be massless [28]. The other two neutrino mass eigenstates are non-relativistic today. It is not excluded that the lightest neutrino is still relativistic today assuming that its mass is much less than the temperature of the cosmic relic neutrino background [1, 28-31]. Since the problem associated with the extraction of the neutrino mass ordering (normal versus inverted) has not been resolved, yet ([32] and the related references therein), it is difficult to predict which of the existing neutrino (antineutrino) flavors is the lightest one. Hereafter, in this paper we will only discuss the lightest relic neutrino (it can be, eg., an electron neutrino or a tauon neutrino depending on that which type of the mass ordering is realized in nature) assuming that it is ultra-relativistic and can be considered as a massless particle. Therefore, in the presented work, we have dealings with the massless neutrino model [1, 33] according to that there are only a left-handed polarized neutrino and a right-handed polarized antineutrino.

It should be noted that the relic neutrino energy ($\omega_r = 1.68 \times 10^{-4}\ eV$) satisfies the condition

$$\omega_{min} \ll \omega_r \ll m_e \tag{3}$$

where $\omega_{min} = eB/p_{\perp}$.

We use the system of units $c = \hbar = k_B = 1$ and the pseudo-Euclidean metric with the signature $(+\ ---)$.

2. Matrix element, amplitude and squared amplitude of processes

According to the Weinberg-Salam electroweak interactions theory, the $\nu_\mu e^+ \to \nu'_\mu e^{+'}$ process (or the $\nu_\tau e^+ \to \nu'_\tau e^{+'}$ process) only proceeds at the expense of a purely weak neutral current (Z-boson contribution). If an electron neutrino participates in the process (i.e., the $\nu_e e^+ \to \nu'_e e^{+\prime}$ process), both a weak charged current (W-boson) and a weak neutral current (Z-boson) contribute to the elastic electron neutrino-positron scattering.

In the low-energetic approximation of the Weinberg-Salam electroweak interactions theory, the terms of $O(q^2/m_W^2)$ and $O(q^2/m_Z^2)$ can be neglected where q^2 is the squared four-momentum transfer, m_W and m_Z are the W-boson mass and the Z-boson mass, respectively. In the case of relic neutrinos and ultra-relativistic positrons of energies below the $\sim PeV$ region, the conditions $|q^2| \ll m_W^2$ and $|q^2| \ll m_W^2, m_Z^2$ are satisfied, and the low-energetic approximation of the Weinberg-Salam electroweak interactions theory for the elastic relic neutrino-positron scattering is justified.

Using the standard Feynman rules, the matrix element of the $\nu_i e^+ \to \nu'_i e^{+\prime}$ processes in an external MF in the low-energetic approximation of the Weinberg-Salam electroweak interactions theory is written in the form

$$M = \frac{G_F}{\sqrt{2}} \int d^4x \mathrm{N}_\alpha(x) \Lambda^\alpha(x) \tag{4}$$

where

$$\mathrm{N}_\alpha(x) = \overline{\psi}_{\nu'_i}(x) \gamma_\alpha (1 + \gamma^5) \psi_{\nu_i}(x)\,, \tag{5}$$

$$\Lambda^\alpha(x) = \overline{\psi}_{e'}(x) \gamma^\alpha \left(g_V^i + g_A^i \gamma^5\right) \psi_e(x), \tag{6}$$

G_F is the Fermi constant, γ^α are the Dirac matrices, $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$, $g_V^i = g_V^{\mu(\tau)} = -0.5 + 2\sin^2\theta_W$ and $g_A^i = g_A^{\mu(\tau)} = -0.5$ are for $i = \mu, \tau$, i.e. for the $\nu_\mu e^+ (\nu_\tau e^+)$-scattering, $\sin^2\theta_W \cong 0.23$, θ_W is the Weinberg angle, $g_V^i = g_V^e = g_V^{\mu(\tau)} + 1 = 0.5 + 2\sin^2\theta_W$ and $g_A^i = g_A^e = g_A^{\mu(\tau)} + 1 = 0.5$ are for

$i = e$, i.e. for the $\nu_e e^+$-scattering, $\psi_\nu(x) = (2\omega V)^{-1/2} u(k) exp(-ikx)$ is the wave function of the incident neutrino possessing the four-momentum k and the energy ω, $\overline{\psi}_{\nu'}(x) = \psi^+_{\nu'}(x)\gamma^0$, $\psi_{\nu'}(x) = (2\omega' V)^{-1/2} u(k') exp(-ik'x)$ is the wave function of the scattered neutrino possessing the four-momentum k' and energy ω', V is the normalization volume, $u(k)$ and $u(k')$ are the Dirac bispinors of the incident and scattered neutrinos, respectively, $\psi_e(x)$ and $\psi_{e'}(x)$ are the exact wave functions of the positron (negative frequency electron) in the initial and final states, respectively, that are the solutions of the Dirac equation in a constant homogeneous external MF for a positron [34], $\overline{\psi}_{e'}(x) = \psi^+_{e'}(x)\gamma^0$. The primed quantities belong to the positrons in the final state. The explicit form of the exact wave function of the positron (negative frequency electron) in a constant homogenous MF can be found in [34].

After elementary calculations the matrix element of the processes (1) is given by

$$M = 2\pi A \delta(\varepsilon' - \varepsilon + q_0) \tag{7}$$

where

$$A = \frac{\sqrt{2} G_F}{4V(\omega\omega')^{1/2}} N_0 U_\alpha J^\alpha, \tag{8}$$

is the amplitude of the considered processes,

$$U_\alpha = [\overline{u}(k')\gamma_\alpha(1+\gamma^5)u(k)], \tag{9}$$

$$N_0 = \frac{4\pi^2}{L_y L_z} e^{i\alpha_0} e^{i(n-n')\varphi'} \delta(p_y - p'_y - q_y)\delta(p_z - p'_z - q_z), \tag{10}$$

$\alpha_0 = [q_x(p_y + p'_y)]/(2h)$, $h = eB$, $\varphi' = \varphi - (\pi/2)$, $\tan\varphi = q_y/q_x$,

$q_0 = \omega' - \omega$, $q_x = k'_x - k_x$, $q_y = k'_y - k_y$, $q_z = k'_z - k_z$, L_y (L_z) is the normalization length along the y-axis (z-axis), $p_y(p'_y)$ and $p_z(p'_z)$ are y- and z-components of the initial (final) state positron momentum, respectively,

$$J^\alpha = \begin{pmatrix} J^0 \\ J^1 \\ J^2 \\ J^3 \end{pmatrix} = \begin{pmatrix} t_1 I_4 + t_2 I_3 \\ t_3 e^{i\varphi} I_1 + t_4 e^{-i\varphi} I_2 \\ -t_3 i e^{i\varphi} I_1 + t_4 i e^{-i\varphi} I_2 \\ t_5 I_4 + t_6 I_3 \end{pmatrix} \tag{11}$$

are the components of the transition amplitude of the 4-current, $I_1 = I_{n,n'-1}$, $I_2 = I_{n-1,n'}$, $I_3 = I_{n-1,n'-1}$, $I_4 = I_{n,n'}$ are the Laguerre functions defined as

$$I_{nn'}(\rho) = \left(\frac{n'!}{n!}\right)^{1/2} e^{-\rho/2} \rho^{\frac{n-n'}{2}} L_{n'}^{n-n'}(\rho), \tag{12}$$

$L_{n'}^{n-n'}(\rho)$ are the Legendre polynomial that is given by the formula [35]

$$L_k^s(\rho) = \frac{1}{n!} e^{\rho} \rho^{-\alpha} \frac{d^n}{d\rho^n}\left(e^{-\rho}\rho^{k+s}\right) \qquad (13)$$

and depends on the variable

$$\rho = \frac{q_\perp^2}{2h}, \qquad (14)$$

$$q_\perp^2 = q_x^2 + q_y^2 = k_\perp^2 + k_\perp'^2 - 2k_\perp k_\perp' \cos(\alpha - \alpha'), \qquad (15)$$

$$k_\perp = \omega \sin\vartheta, \qquad (16)$$

$$k_\perp' = \omega' \sin\vartheta', \qquad (17)$$

$\vartheta(\vartheta')$ and $\alpha(\alpha')$ are the polar and azimuthal angles of the incident (scattered) neutrino momentum. The coefficients t_i $(i = 1,2,\dots,6)$ in the formula (11) are given by

$$t_1 = g_V^i a_2 - g_A^i f_2, \qquad t_2 = g_V^i a_1 - g_A^i f_1, \qquad (18)$$

$$t_3 = g_V^i b_1 - g_A^i e_1, \qquad t_4 = g_V^i b_2 - g_A^i e_2, \qquad (19)$$

$$t_5 = -\left(g_V^i f_2 - g_A^i a_2\right), \quad t_6 = g_V^i f_1 - g_A^i a_1, \qquad (20)$$

where

$$a_1 = c_1 c_1' + c_3 c_3', \qquad a_2 = c_2 c_2' + c_4 c_4', \qquad (21)$$

$$b_1 = c_1' c_4 + c_3' c_2, \qquad b_2 = c_1 c_4' + c_3 c_2', \qquad (22)$$

$$f_1 = c_1 c_3' + c_1' c_3, \qquad f_2 = c_2 c_4' + c_2' c_4, \qquad (23)$$

$$e_1 = c_1' c_2 + c_3' c_4, \qquad e_1 = c_1 c_2' + c_3 c_4', \qquad (24)$$

c_i (c_i') are the spin coefficients of the positron in the initial (final) state in the case of the transverse polarization of the spin of the positron and given by the formulae [34]

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} B_3(A_3 + A_4) \\ B_4(A_4 - A_3) \\ B_3(A_3 - A_4) \\ B_4(A_4 + A_3) \end{pmatrix}, \qquad (25)$$

$$A_3 = \sqrt{1 - (p_z/\varepsilon)}, \qquad A_4 = -\zeta\sqrt{1 + (p_z/\varepsilon)}, \qquad (26)$$

$$B_3 = \sqrt{1 + \zeta(m_e/\varepsilon_\perp)}, \qquad B_4 = \zeta\sqrt{1 - \zeta(m_e/\varepsilon_\perp)}, \qquad (27)$$

$\varepsilon = \varepsilon_{n,p_z} = \sqrt{m_e^2 + 2eBn + p_z^2}$ - energy of the positron in the initial state, m_e is the positron mass, $\varepsilon_\perp = \sqrt{\varepsilon^2 - p_z^2} = m_e\sqrt{1 + 2fn}$, $f = B/B_0$, $\zeta = \pm 1$ is the spin quantum number that determines the projection of the positron spin onto the direction of the MF vector **B** ($\zeta = +1$ corresponds to the case when the spin of the positron is oriented along the direction of the MF vector **B**, $\zeta = -1$ corresponds to the case when the spin of the positron is oriented opposite to the direction of the MF vector **B**).

We obtain the following expression for the squared amplitude per unit time

$$|A|^2 = \frac{(2\pi)^2 G_F^2}{V^2 L_y L_z \omega\omega'} R\delta_y\delta_z. \tag{28}$$

where

$$R = L^{\alpha\beta} J_\alpha J_\beta^*, \tag{29}$$

$$L^{\alpha\beta} = k^\alpha k'^\beta + k'^\alpha k^\beta - (kk')g^{\alpha\beta} + i\epsilon^{\alpha\beta\mu\nu}k_\mu k'_\nu, \tag{30}$$

$$\delta_y = \delta\left(p_y - p'_y - q_y\right), \tag{31}$$

$$\delta_z = \delta(p_z - p'_z - q_z), \tag{32}$$

$\epsilon^{\alpha\beta\mu\nu}$ is the completely antisymmetric unit tensor of the fourth rank ($\epsilon^{0123} = +1$), $g^{\alpha\beta}$ is the metric tensor with the signature (+ - - -).

3. Differential probability of processes

Knowing the squared amplitude per unit time, we can calculate the differential probability per unit time according to the formula

$$dw = 2\pi \sum_{n'} |A|^2 \delta_\varepsilon (1 - f'_e) \frac{L_y}{2\pi} dp'_y \frac{L_z}{2\pi} dp'_z \left(1 - f'_{\nu_i}\right) \frac{V d\mathbf{k}'}{(2\pi)^3} \tag{33}$$

where $f'_e = \{exp[(E' + \mu'_e)/T'_e] + 1\}^{-1}$ is the Fermi-Dirac distribution function for the positrons in the final state, $f'_{\nu_i} = \left\{exp\left[\left(\omega' - \mu'_{\nu_i}\right)/T'_{\nu_i}\right] + 1\right\}^{-1}$ is the Fermi-Dirac distribution function for the neutrinos in the final state, μ'_e (μ'_{ν_i}) and T'_e (T'_{ν_i}) are the chemical potential and the temperature of the positron (neutrino) gas after the elastic lightest relic neutrino-positron scattering, respectively, $\delta_\varepsilon = \delta(\varepsilon' - \varepsilon + q_0)$.

As we have already noted that the incident lightest relic neutrino is left-handed polarized and the longitudinal momentum of the positron in the initial state is zero: $p_z = 0$. It means that the initial positron and neutrino states are fixed. Therefore, we do not perform averaging over the initial neutrino and positron states when we calculate the differential probability of the processes (1).

We obtain the following analytical formula for the differential probability of the processes (1)

$$dw = \frac{G_F^2 m_e^2}{\pi^{3/2} V} \left\{ \left[\frac{1}{2} \left[\left(g_R^i\right)^2 (1+u)^2 + \left(g_L^i\right)^2 + 2 g_L^i g_R^i (1+u)\zeta\zeta' \right] \frac{\kappa}{u} - \right.\right.$$

$$-g_L^i g_R^i (1+u)(1+\zeta\zeta')\big]\Phi_1(z) -$$

$$-\left[\left(g_R^i\right)^2 (1+u)^2 + \left(g_L^i\right)^2 + 2g_L^i g_R^i (1+u)\zeta\zeta'\right]\left(\frac{\chi}{u}\right)^{2/3}\Phi'(z) -$$

$$-\left[\left(g_R^i\right)^2 \zeta'(1+u)^2 - \left(g_L^i\right)^2 \zeta +\right.$$

$$\left. + g_L^i g_R^i (\zeta - \zeta')(1+u)\right]\left(\frac{\chi}{u}\right)^{1/3}\Phi(z)\Big\} F_f \frac{u du}{(1+u)^4} \tag{34}$$

where $F_f = (1 - f_e')(1 - f_\nu')$ is the statistical factor formed from the Fermi-Dirac distribution functions,

$$u = \frac{\chi}{\chi'} - 1 = \frac{p_\perp}{p'_\perp} - 1 \simeq \frac{\omega'}{\varepsilon - \omega'} \tag{35}$$

is the the spectral variable,

$$\chi = \frac{e}{m_e^3}\left[-\left(F_{\mu\nu}p^\nu\right)^2\right]^{1/2} = \frac{B}{B_0}\frac{p_\perp}{m_e} \tag{36}$$

is the dynamical parameter,

$$\kappa = \frac{2kp}{m_e^2} = \frac{2\omega\varepsilon}{m_e^2} \tag{37}$$

is the kinematical parameter (see: e.g., [36-38]), p is the four-momentum of the positron in the initial state, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the tensor of an external electromagnetic field of the magnetic type, $g_L^i = g_L^{\mu(\tau)} = (1/2)\left(g_V^{\mu(\tau)} + g_A^{\mu(\tau)}\right) = -0.5 + \sin^2\theta_W$ and $g_R^i = g_R^{\mu(\tau)} = (1/2)\left(g_V^{\mu(\tau)} - g_A^{\mu(\tau)}\right) = \sin^2\theta_W$ are for $i = \mu, \tau$, i.e. for the $\nu_\mu e^+(\nu_\tau e^+)$-scattering, $g_L^i = g_L^e = g_L^{\mu(\tau)} + 1 = 0.5 + \sin^2\theta_W$ and $g_R^i = g_R^e = g_R^{\mu(\tau)} = \sin^2\theta_W$ are for $i = e$, i.e. for the $\nu_e e^+$-scattering, ζ' is the spin quantum number of the positron in the final state: $\zeta' = \pm 1$; $\zeta' = +1$ corresponds to the case when the spin of the positron in the final state is oriented along the direction of the MF vector **B**, $\zeta' = -1$ corresponds to the case when the spin of the positron in the final state is oriented opposite to the direction of the MF vector **B**.

The behaviour of the differential probability is determined by the behavior of the Airy functions $\Phi(z)$, $\Phi'(z)$ and $\Phi_1(z)$. At the same time, the behavior of the Airy functions $\Phi(z)$, $\Phi'(z)$ and $\Phi_1(z)$ is determined by the behavior of the argument z that can be presented in a modified form

$$z = \left(\eta^2 \frac{u}{\kappa}\right)^{-1/3}\left(\frac{u}{\kappa} - 1\right) \tag{38}$$

where

$$\eta = \frac{\chi}{\kappa} \tag{39}$$

is a new parameter. The analyses of the argument z of the Airy functions show that the influence of the external MF on the neutrino-positron scattering is determined by the parameter η that can be written in another form (see: [23, 24, 36],) if we take the relations $\varepsilon^2 \gg m_e^2$, $\varepsilon'^2 \gg m_e^2$, $p_\perp = (2eBn)^{1/2} \gg m_e$, $p'_\perp = (2eBn')^{1/2} \gg m_e$, (36), (37) and the fact $p_z = 0$ into account in (39)

$$\eta = \frac{1}{2}\frac{B}{B_0}\frac{m_e}{\omega}. \tag{40}$$

The influence of the external MF on the elastic neutrino-positron scattering is essential when $\eta \gtrsim 1$. The condition $\eta \gg 1$ corresponds to the strong field case (see: eg., [37] devoted to the investigation of the inverse muon decay [39]) and the condition $\eta \ll 1$ corresponds to the weak field case. It should be noted that in the limiting case $\eta \gg 1$, the energy loss by the electrons due to the processes $\tilde{\nu}_i e^- \to \tilde{\nu}'_i e^{-\prime}$ which is related to the processes $\nu_i e^+ \to \nu'_i e^{+\prime}$ was calculated in [40].

4. Average energy of scattered neutrino

The average energy of the scattered relic neutrinos in the $\nu_i e^+ \to \nu'_i e^{+\prime}$ processes is calculated according to the following general formula:

$$\langle\omega'\rangle = \frac{\int_0^\infty \omega' \, dw}{\int_0^\infty dw}. \tag{41}$$

Hereafter, we investigate strong field case $\eta \gg 1$ ($\chi \gg 1$, $\kappa \ll 1$). In this limiting case, the argument z of the Airy functions becomes $|z| \ll 1$ (see the formula (41)). Main contribution to the integral $w = \int_0^\infty dw = \int_0^\infty (\ldots) du$ comes from the domain where $|z| \ll 1$.

If we perform summation over the spin polarization of the final state positron in the formula (34) for the differential probability, substitute the obtained expression for the differential probability after summation over the final state positron spin polarization and the relation $\omega' = u\varepsilon/(1+u)$ in the formula (41) and take into account the following value of $\Phi'(z)$ at $z \simeq 0$ [41]

$$\Phi'(z) \simeq \Phi'(0) = -3^{1/6}\frac{\Gamma(2/3)}{2\sqrt{\pi}}, \tag{42}$$

we obtain the analytical formula for the scattered neutrino average energy in the elastic relic neutrino-positron scattering in a MF

$$\langle\omega'\rangle_{\nu_i e^+} = \frac{54\left(g_R^i\right)^2 + 5\left(g_L^i\right)^2}{81\left(g_R^i\right)^2 + 15\left(g_L^i\right)^2}\varepsilon. \tag{43}$$

It is visible from the formula (43) that the average energy of the neutrino in the final state in the elastic relic neutrino-positron scattering in a MF depends on the energy of the positron in the initial state linearly: $\langle\omega'\rangle_{\nu_i e^+} \sim \varepsilon$. At the same time, the average energy of the relic neutrino in the final state depends on the structural constants g_L^i and g_R^i of the electroweak interactions.

5. Discussion and numerical estimations

Using the inequality (3) we obtain the following relation

$$\omega_{min} \simeq m_e \frac{f}{\gamma} \ll \omega = \omega_r \tag{44}$$

where $\gamma = \varepsilon / m_e$. The requirement for simultaneous satisfaction of the conditions $\chi \gg 1$ and (44) leads to the relation $(1/\gamma) \ll f \ll (\omega_r/m_e)\gamma$ that determines the approximate lowest and uppermost boundaries of the range of change of the MF strength B or f:

$$(10/\gamma) \lesssim f \lesssim (\omega_r/10m_e)\gamma. \tag{45}$$

The possible minimum value of the energy of the positron in the initial state, at which the influence of the external MF on the elastic relic neutrino-positron scattering is significant, is determined by $\gamma_{min} = 10(m_e/\omega_r)^{1/2} \simeq 5.52 \times 10^5$ which is found from the equality

$$(10/\gamma_{min}) = f_0 = (\omega_r/10m_e)\gamma_{min}. \tag{46}$$

The relation (46) determines the minimum value of the energy of the positron in the initial state ($\varepsilon_{min} \simeq 282\ GeV$) and the corresponding MF strength ($B \simeq 7.98 \times 10^8\ G$). Using the condition (45), we can write $f_{max} \simeq (\omega_r/10m_e)\gamma$. At the same time, using the condition $f \ll 1$, we get $f_{max} \simeq 10^{-1}$. Comparing the last two relations obtained for f_{max}, we obtain the related uppermost boundaries for $\gamma_{max} \simeq m_e/\omega_r$ and $\varepsilon_{max} \simeq m_e^2/\omega_r \simeq 1.58\ PeV$. In this case, $f_{min} = 10/\gamma_{max} = 10\omega_r/m_e$ and $B_{min} \simeq 1.45 \times 10^5 G$. So, the influence of the external MF on the elastic relic neutrino-positron scattering is significant when the MF strength B and the initial state positron energy ε vary in the ranges $1.45 \times 10^5 G \lesssim B \lesssim 4.41 \times 10^{12}\ G$ and $282\ GeV \lesssim \varepsilon \lesssim 1.58\ PeV$, respectively. These two ranges obtained for the MF strength and initial state positron energy determine the realistic physical conditions under which the energy of a relic neutrino is amplified significantly. The MFs of the strengths $\sim 10^5 - 10^9 G$ and $\sim 10^{12}\ G$ exist near the white dwarfs and neutron stars, respectively.

So, when relic neutrinos scatter by the cosmic positrons possessing energy in the range $282\ GeV \lesssim \varepsilon \lesssim 1.58\ PeV$ and passing through the close vicinity to white dwarfs or neutron stars, the influence of the external MF on the elastic relic neutrino-positron scattering is significant and the energy of a relic neutrino is amplified essentially.

We have obtained the analytical formula (43) enabling us to calculate the average energy $\langle \omega' \rangle_{\nu_i e^+}$ of the scattered neutrino. The linear increase of the average energy of the neutrino in the final state means that after the scattering the energy of the positron in the final state becomes less than the energy of the positron in the initial state. This happens at the expense of the anti-Stokes transitions: the positrons make transitions from higher Landau levels to the lower Landau levels.

The presence of the terms containing the structural constant g_{l}^{i} in the formula (43) means that the average energy of the scattered neutrino is sensitive to the neutrino flavour. It is also visible from the Table 1.

Table 1

The sensitivity of the average energy of the neutrino in the final state to the neutrino flavour

Reaction	$\langle\omega'\rangle_{\nu_i e^+}$
$\nu_e e^+ \to \nu_e' e^{+\prime}$	$0.45\ \varepsilon$
$\nu_\mu e^+ \to \nu_\mu' e^{+\prime}$	$0.60\ \varepsilon$
$\nu_\tau e^+ \to \nu_\tau' e^{+\prime}$	$0.60\ \varepsilon$

Table 1 shows that strong amplification of the energy of the neutrino in the final state is more pronounced for the $\nu_\mu e^+ \to \nu_\mu' e^{+\prime}$ (or $\nu_\tau e^+ \to \nu_\tau' e^{+\prime}$) scattering than that for the $\nu_e e^+ \to \nu_e' e^{+\prime}$ scattering. For the purpose of numerical estimation of the average energy of the final state neutrino, we choose the minimum and maximum values of the energy of the initial state positron: $\varepsilon_{min} \simeq 282\ GeV$ and $\varepsilon_{max} \simeq 1.58\ PeV$, respectively. Using these values of the energy of the final state positron and Table 1, we obtain the following ranges for the average energy of the final state neutrino:

$$126.9\ GeV \lesssim \langle\omega\rangle_{\nu_e e^+} \lesssim 0.71\ PeV, \qquad (47)$$

$$169.2\ GeV \lesssim \langle\omega\rangle_{\nu_l e^+} \lesssim 0.95\ PeV, \qquad (48)$$

where ν_l is a taon (muon) neutrino.

These numerical estimations show that the average energy of the scattered relic neutrino can reach even the value about $1\,PeV$. Based on the above-performed numerical estimations on the average energy of the scattered neutrino and Table 1, we obtain $10^{14} \lesssim \langle\omega'\rangle_{\nu_i e^+}/\omega_r \lesssim 10^{18}$. It means that after the elastic scattering of a relic neutrino by the positron passing through the close vicinity to a white dwarf or a neutron star, the average energy of the scattered neutrino can increase at least about $\sim 10^{14}$ times and even $\sim 10^{18}$ times.

Let us estimate the detection possibility of the scattered relic neutrinos coming from the close vicinity of white dwarfs or neutron stars. When these scattered neutrinos reach the terrestrial detector (e.g., IceCube detector), they participate in the inelastic neutrino-neutron scattering $\nu_l n \to p l^-$ where $\nu_l = \nu_e, \nu_\mu, \nu_\tau$ and $l^- = e^-, \mu^-, \tau^-$. Using the single power law flux model [42]

$$\Phi_{astr}^{\nu+\tilde{\nu}} = C_0 \Phi_0 \left(\frac{\omega'}{\omega'_0}\right)^{-\gamma} \quad (49)$$

and knowing the energy of incoming neutrino, we can calculate the incoming neutrino flux at Earth as

$$\Phi_{astr}^{\nu} = \frac{1}{2} \Phi_{astr}^{\nu+\tilde{\nu}} \quad (50)$$

where $C_0 = 3 \times 10^{-18} GeV^{-1} \cdot cm^{-2} \cdot s^{-1} \cdot sr^{-1}$, $\Phi_0 = 1.66^{+0.25}_{-0.27}$, $\omega'_0 = 100\ TeV$ and $\gamma = 2.53^{+0.07}_{-0.07}$ [42]. If the energy of the incoming neutrino is, for instance, $\omega' = \langle\omega\rangle_{\nu_i} \simeq 16\ TeV$, the incoming neutrino flux at Earth is $\Phi_{astr}^{\nu} \sim 10^{-10} cm^{-2} \cdot s^{-1}$ and the cross-section for the $\nu_e n$-scattering is estimated as $\sigma_{\nu_e n} \sim 10^{-34}\ cm^2$ [43]. The number of the related $\nu_e n \rightarrow pe^-$ events per year at the IceCube detector ($V = 10^{15}\ cm^3$) is calculated according to the formula

$$N = \sigma_{\nu_e n} \Phi_{astr}^{\nu} n_0 V t \quad (51)$$

where n_0 is the neutron number density in an ice ($n_0 \sim 10^{22}\ cm^{-3}$). Numerical estimation yields $N = 4$ events pear. Despite the fact, in the current situation, the number of the $\nu_e n \rightarrow pe^-$ events per year at the IceCube detector is few, it is quite possible to detect the scattered lightest relic neutrinos coming from the close vicinity of white dwarfs and neutron stars. For the incoming neutrinos of the energy $\sim 10^2\ GeV$, the number of the $\nu_e n \rightarrow pe^-$ events per year is more than $N = 4$. By increasing the detector volume, it is quite possible to detect sufficiently high number of the scattered relic neutrinos due to the $\nu_e n \rightarrow pe^-$ reaction.

Thus, the obtained results in this work enable us to come to the conclusion that (i) the elastic scattering of the lightest relic neutrinos by positrons passing through the close vicinity of white dwarfs and neutron stars is the alternative mechanism responsible for the production of high energy ($\sim 10^2\ GeV - \sim PeV$) astrophysical neutrinos, (ii) the close vicinity of white dwarfs and neutron stars is a unique natural intermediate laboratory for amplification of the energy of the lightest relic neutrinos and (iii) the detection of the scattered lightest relic neutrinos coming from the close vicinity of white dwarfs and neutron stars is quite possible due to the $\nu_e n \rightarrow pe^-$ reaction.

Note. When an external MF is present and ultra-relativistic positrons of the energy in the range $282\ GeV \lesssim \varepsilon \lesssim 1.58\ PeV$ participate in the considered processes, the QED corrections should be taken into account. In the four-fermion approximation, the related QED corrections consist of the two types of photonic radiative corrections: the virtual corrections due to loop diagrams (virtual corrections) and the bremsstrahlung radiation (real photons) [44]. The virtual QED corrections are divided into two groups: QED vertex corrections involving virtual photons (Fig. 4 in [45]) and QED closed fermion loop contributions from leptons and heavy quarks (Fig. 5 in [45]). The numerical estimations show that in the considered energy region of the initial state positrons, the QED radiative corrections to the elastic relic neutrino-positron scattering in MFs existing in close vicinity to white dwarfs and neutron stars is negligibly small and can be neglected. Thus, in the tree-level approximation of the perturbation theory and the four-fermion (low-energetic) approximation of the electroweak interactions theory, the results obtained for the differential probability of the relic neutrino-positron scattering and average energy of the scattered relic neutrino in this work, within the accepted physical conditions, are justified.

6. Conclusions

We have substantiated that when relic neutrinos scatter by the ultra-relativistic cosmic positrons possessing the energy in the range $282\ GeV \lesssim \varepsilon \lesssim 1.58\ PeV$ and passing through the close vicinity to white dwarfs or neutron stars where the strengths of the MFs vary in the range $1.45 \times 10^5 G \lesssim B \lesssim 4.41 \times 10^{12}\ G$, energy is transferred from the positrons to the relic neutrinos at the expense of the anti-Stokes transitions. As a result, the positrons lose their energies very intensely and the average energy of the scattered relic neutrinos is amplified at least about $\sim 10^{14}$ times and even $\sim 10^{18}$ times. This leads to the strong amplification of the energy of the relic neutrinos in the final state. We have also determined that the average energy of the scattered neutrino is sensitive to the neutrino flavour. It enables the experimentators to distinguish the flavour of the neutrinos incoming from the close vicinity to white dwarfs and neutron stars.

Thus, the obtained results in this work enable us to come to the conclusion that (i) the elastic scattering of the lightest relic neutrinos by positrons passing through the close vicinity of white dwarfs and neutron stars is the alternative mechanism responsible for the production of high energy ($\sim 10^2\ GeV - \sim PeV$) astrophysical neutrinos, (ii) the close vicinity of white dwarfs and neutron stars is a unique natural intermediate laboratory for amplification of the energy of the lightest relic neutrinos and (iii) the detection of the scattered lightest relic neutrinos coming from the close vicinity of white dwarfs and neutron stars is quite possible due to the $\nu_l n \to p l^-$ reaction.

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