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THE INFLUENCE OF AXLE LOAD, EMU COMPOSITION AND RUNWAY PARAMETERS ON ACCELERATIONS AT START-UP

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For passenger traffic, the current trend is to have start-up accelerations between 1 and 1.2 m /s², values that will allow the choice of the optimal composition for an electric train (electric frame) intended for urban (metro), regional and interregional. In this paper, an analysis is made of the influence of the constructive parameters (axle load, total mass of the train) of the electric frame, the characteristics of the runway profile, on the starting accelerations. The correct choice of the installed power of the traction motors from the perspective of reaching the required driving speeds is also sought. The study proposed in this paper will allow the choice of optimal driving modes from the perspective of electricity consumption for the start-up mode (with three-phase asynchronous traction electric motors).

Keywords: railway vehicle, axle load, slopes, accelerations, coefficient of adhesion.

1. Introduction

Currently, research on the optimization of energy consumption is of vital importance from the perspective of the energy crisis versus the economic efficiency (costs) of operating electric traction vehicles. The approach to optimizing electricity consumption covers both the railway sector (regional, interregional or high-speed transport) and the public sector (urban and metropolitan transport). Energy efficient techniques are receiving increasing attention due to rising energy prices and environmental concerns. Railways, along with other modes of transport, are facing increasing pressure to provide smarter and more efficient energy management strategies. From this perspective, the studies are oriented both on the driving regimes for electric trains intended for the railway network as well as for electric subway and tram trains. The research approaches sought to adapt the analysis methods to the specific peculiarities of the analyzed systems (railway, urban or metropolitan).

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In order to reduce the energy consumption of the train running between stations, ensuring the travel times and passenger comfort, there are researches that guide the solution of the analysis of the travel strategy for energy-efficient operation to the multi-objective optimization method by genetic algorithms [1]. Thus, after taking into account the slope and the speed limit of the line, the multi-objective optimization model of the train's energy efficiency is established based on the train's energy consumption, running time and passenger comfort. The improved multi-objective genetic algorithm is used to optimize the target speed sequence to obtain the train operation strategy. An optimization method is obtained by considering the control structure of the automatic train operation system.

Optimizing the energy consumed for the traction mode (efficient driving strategy) for applications in railway and urban systems can also be done by developing the SmartDrive package. Analysis of the results showed that by implementing an optimal speed trajectory, energy consumption in the network can be significantly reduced [2].

From the perspective of energy optimization of subway traffic, by using optimal train trajectory and travel times, energy consumption and substation load can be significantly reduced, thus improving system performance and stability. This also has the effect of reducing investment costs in new subway stations [3]. In this way, algorithms can be developed to optimize the train trajectory simultaneously with an algorithm to optimize travel times.

The assessment of energy economy for the railway system can be achieved when the coasting technique is widely used [4], [9]. This technique consists of using, whenever possible, the kinetic energy accumulated by the train. This solution requires full acceleration starts. From this perspective, this paper proposes the analysis of the factors that influence the acceleration values in order to choose the optimal power of the traction system. It is known that the starting regime is a big consumer of energy and for this reason the analysis of the factors influencing the values of the accelerations becomes important. Combining the coasting technique with the traction-free driving regime will improve the results of the energy optimization.

Transport policies are exclusively oriented towards sustainable and environmentally friendly transport. Rail transport with electric traction is one of the most environmentally friendly branches of transport. It also offers many possibilities to improve energy efficiency and reduce energy consumption, one of which is the recovery and reuse of energy resulting from the electrodynamic braking process. The so-called "energy cooperation" of trains can be achieved [5]. An indirect optimization of the amount of recovered electrical energy with possibilities of immediate use by another vehicle is obtained by maximizing the duration of energy co-optimization. To this approach can be added the possibility of optimizing the efficiency of the recovery of braking energy in railway transport by changing the arrival time [6].

Designing a robust and energy-efficient speed profile of metropolitan lines, taking into account load variations and time delays that may be generated [7]. The strategic plans of the metropolitan and regional rail operators (system that involves frequent starting regimes) currently include actions to reduce energy consumption. The application of eco driving initiatives in lines equipped with ATO (Automatic Train Operation) systems can provide significant savings with low investment. Previous ATO studies did not consider the main uncertainties in traffic operation: train load and track delays. And from this perspective the study presented in this paper acquires an obvious importance.

The traction energy consumption of electric locomotives and electric multiple units at speed restrictions [8], [10] is also important from the perspective of acceleration analysis. The analysis of accelerations and the factors that influence their values allow the calculation of additional acceleration energy after speed restriction places.

In this introduction, we have presented different approaches to reducing energy consumption for passenger transport. The approach we propose in the article is to study how the construction and control of energy-consuming walking regimes can be optimized to reduce energy consumption.

All practical problems of train traction, such as: setting tonnages, determining speeds and travel times, solving braking problems, choosing the type of locomotive, depending on the characteristics of the towing sections, determining fuel and electricity consumption, can be solved with ease, if one knows the mathematical expression that establishes the connection between the movement of the train and the causes that produce it.

2. Train motion equation

The energy required to move a train is provided by an external energy source (electrical energy) or an internal energy source (heat engine or hydrogen electric cells). This energy is transformed into mechanical energy by which the train's running regimes are ensured (starting, movement at constant speed) [11-15]. An energy balance is presented in figure 1.

We can consider the control variable to be the applied acceleration and that the control level could be changed continuously.

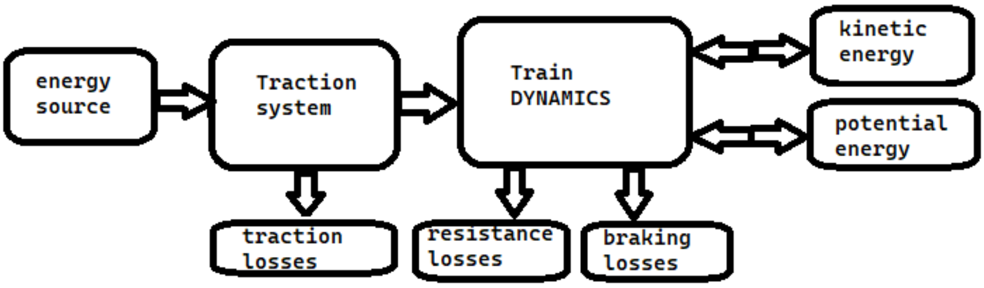

Fig 1. Energy balance of the train

If we consider the travel time *T*, *X* the distance between two stations, *u*(*t*) the acceleration applied to the train by the drive system, *V* the speed of the train and - r [*v*(*t*)] is the specific acceleration given by the forces resisting the advance, the equation of motion of the train is given by the relation:

$$v'(t) = u(t) - r[v(t)] \tag{1}$$

where

$$r(v) = a + bv + cv^2 \tag{2}$$

is given by Davis's formula, and

$$v'(t) = \frac{dv}{dt} \tag{3}$$

If we consider that the acceleration is limited to a value k = (1 m/s^2 - 1,2 m/s^2)

$$|u(t)| \le k \tag{4}$$

If we consider only the positive values of the accelerations as consuming energy (starting mode), the negative ones being able to generate energy for the cases of commanded braking, the energy consumption will be given by the relationship:

$$J(u,v) = \int_0^T u_+(t) v(t) dt \tag{5}$$

If we also add the calculation relation of the distance travelled by the train

$$\int_0^T v(t) dt = X \tag{6}$$

We have the study premises for the analysis of energy consumption and to be able to establish an optimal control strategy of the driving regimes to minimize the energy consumption of the train.

In the first stage of the study, the article aims to determine the influences of the constructive parameters of the train on the accelerations, in order to later establish the influence of the starting parameters (of the accelerations) on the energy consumption in order to subsequently develop a strategy for energy optimization of the starting regime, taking into account the conditions imposed by train traffic (speed of traffic and travel time).

3. Numerical simulations

The analysis seeks to establish the influence of the mass of the train on the accelerations at the start from 2 conditions/perspectives:
- the load on the axle under the conditions in which the number of wagons is kept constant
- the number of wagons in the train composition for constant axle load.

The study (analysis) is done on an electric train (EMU), the scenarios for the analysis being:

Table 1

The parameters for the simulations of the vechicle

Structure of the train set (EMU)	4, 5, 6 wagons
Number of electric motors	12 electric motors
Gradient	0‰, 10‰, 15‰, 20‰, 25‰;
Axle loades	18t, 19t, 20t, 21t
Load status	full loaded, medium loaded, empty

The steps of analysis are:
- Determining the traction characteristics and the rolling resistant forces for an EMY consisting of 5 wagons and 12 electric motors;
- determining the acceleration for a maximum axle weight of 20 tons (for 4, 5, 6 wagons and 12 electric motors), considering the EMU full loaded;
- determining the acceleration for the train with no loads, the train with average loads and for different load axles (18, 19, 20 and 21 tonnes).

The parameters of the vehicle are given in table 2.

Table 2

The parameters of the vehicle

Vehicle type	EMU
Top speed	200 km/h
Wheel diameter in new condition	1,25 m
Wheel diameter in semi-worn condition	1,21 m
Maximum acceleration when starting	0,8 m/s²
Maximum axle load	20 tone
Total number of seats (class I + class II per wagon)	56
Passenger weight (including luggage)	90 kg

For the first case, the train is formed with 5 wagons which has 12 electric motors and it is considered to be full loaded. The train characteristics and some of the results are presented in table 3.

Table 3.

The characteristic of simulation case 1

Characteristic	Value
Number of wagons	5
Total mass of the traction unit when loaded	400 tone
Adherent mass	80 tone
Total mass of the train in an empty state	374,8 tone
Number of traction motors	12
Maximum power	4300 kW
Speed range achieved by the train	0 to 200 km/h
Overload coefficient	1
Rated speed	133 km/h

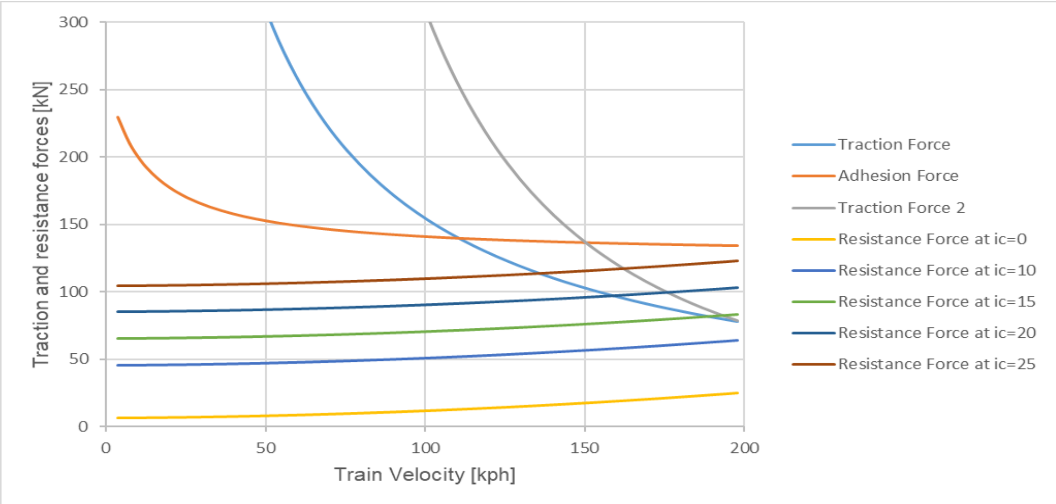

Fig.2. Traction characteristic 5 wagons and 12 electric motors

For the acceleration studies in for those different cases that are mentioned above, the basic relation to be used is:

$$v^{'}(t) = u(t) - r[v(t)]\,. \tag{7}$$

The result obtained are many for every case mentioned. In figures 2, 3 and 4 are presented the acceleration values that can be achieved by the train considering different slope gradients and different number of wagons that make up the train.

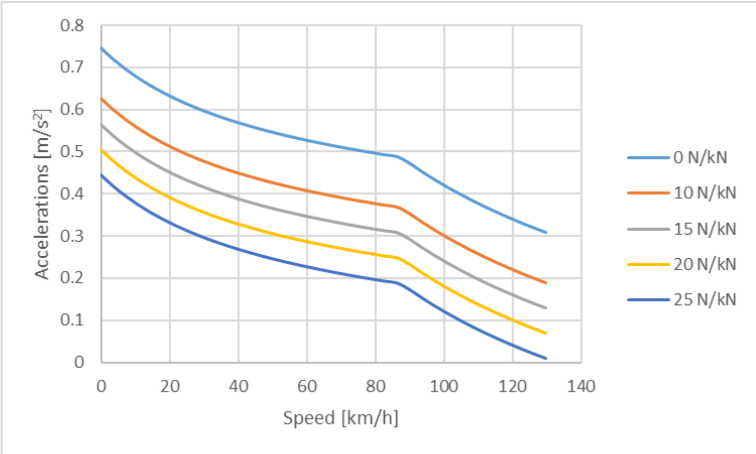

Fig.3. EMU acceleration (4 wagon train)

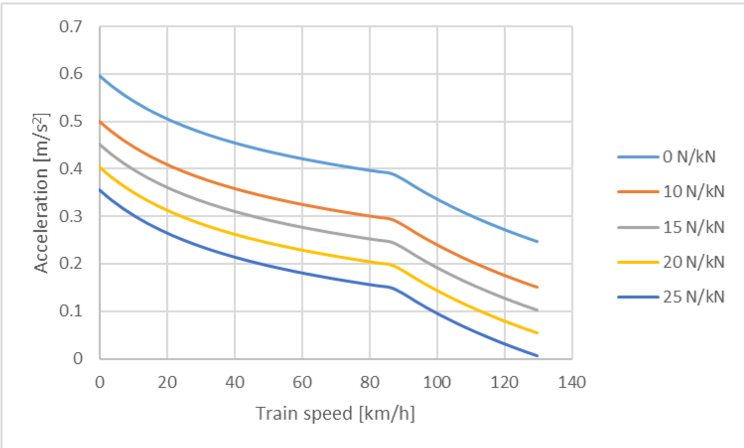

Fig.4. EMU acceleration (5 wagon train)

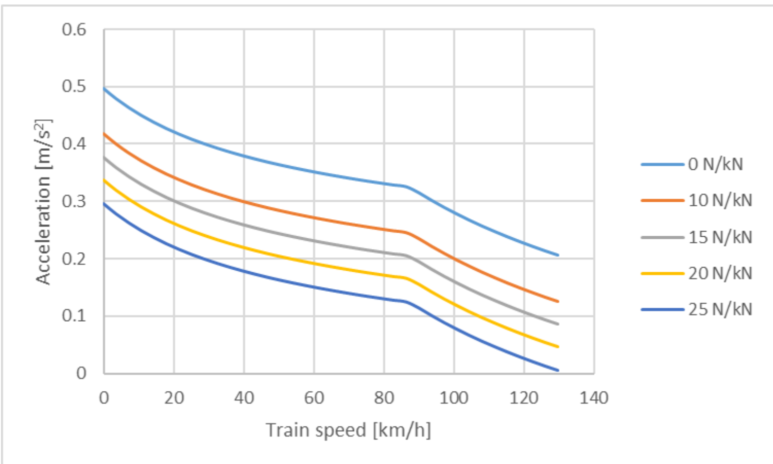

Fig.5. EMU acceleration (6 wagon train)

All the results obtained in the means of acceleration of the train are synthesized in the discussion part of this paper and analysed.

4. Discussion of the results

The number of wagons influences the starting acceleration (starting from 0.746 m/s^2 for a train consisting of 4 wagons and 0.497 m/s^2 for a train consisting of 4 wagons).

The same trend with the same proportion is observed when driving on different gradient (10‰, 15‰, 20‰, 25‰) keeping the same structure/composition of the train (4, 5, 6 wagons).

The solution for improving acceleration is to increase grip through technical solutions that improve the coefficient of adhesion (sanding on all drive wheels). With these solutions, builders manage to raise the adhesion limit. Table 4 shows the accelerations for 4, 5, 6 wagons for the loaded train condition for an axle load of 20 tons.

Table 4.

Accelerations for 4, 5, 6 wagons for the loaded train condition for an axle load of 20 tonnes

Gradient	Accelerations 4 wagons [m/s^2]	Accelerations 5 wagons [m/s^2]	Accelerations 6 wagons [m/s^2]
0 ‰	0,746	0,597	0.497
10 ‰	0,626	0,5	0,417
15 ‰	0,565	0,452	0,377
20 ‰	0,505	0,404	0,337
25 ‰	0,445	0,356	0,297

Acceleration for the empty mass of the train: the mass of the train causes a slight decrease in acceleration. Table 5 shows the accelerations for 4, 5, 6 wagons for the empty train of the train for a load on the axle of 20 tons.

Table 5.

Accelerations for 4, 5, 6 wagons for empty train condition for an axle load of 20 tonnes

Gradient	Accelerations 4 wagons [m/s^2]	Accelerations 5 wagons [m/s^2]	Accelerations 6 wagons [m/s^2]
0 ‰	0,694	0,555	0,463
10 ‰	0,573	0,41	0,342
15 ‰	0.513	0,41	0,302
20 ‰	0,453	0,362	0,302
25 ‰	0,392	0,314	0,262

Acceleration for the average mass of the train. Table 6 shows the accelerations for 4, 5, 6 wagons for the average train condition for an axle load of 20 tons.

Table 6.

Accelerations for 4, 5, 6 wagons for average train loading condition for an axle load of 20 tonnes

Gradient	Accelerations 4 wagons [m/s^2]	Accelerations 5 wagons [m/s^2]	Accelerations 6 wagons [m/s^2]
0 ‰	0,721	0,577	0,481
10 ‰	0,6	0,48	0,4
15 ‰	0,54	0,432	0,36
20 ‰	0,48	0,384	0,32
25 ‰	0,42	0,336	0,28

Acceleration for the maximum axle mass of 21 tons of the train. Table 7 shows the accelerations for 4, 5, 6 wagons for the loaded train condition for an axle load of 21 tons.

Table 7.

Accelerations for 4, 5, 6 wagons for the loaded train condition for an axle load of 21 tonnes

Gradient	Accelerations 4 wagons [m/s^2]	Accelerations 5 wagons [m/s^2]	Accelerations 6 wagons [m/s^2]
0 ‰	0.97	0.746	0.497
10 ‰	0,501	0,626	0,418
15 ‰	0,453	0,566	0,378
20 ‰	0,405	0,506	0,338
25 ‰	0,357	0,446	0,297

Acceleration for the maximum axle mass of 18 tons of the train. Table 8 shows the accelerations for 4, 5, 6 wagons for the loaded train for an 18-ton axle load.

Table 8.

Accelerations for 4, 5, 6 wagons loaded train condition for an axle load of 18 tonnes

Gradient	Accelerations 4 wagons [m/s^2]	Accelerations 5 wagons [m/s^2]	Accelerations 6 wagons [m/s^2]
0 ‰	0.597	0,746	0,497
10 ‰	0,499	0,623	0.416
15 ‰	0,451	0,563	0,375
20 ‰	0,402	0,503	0.335
25 ‰	0,354	0,443	0,295

Acceleration for the maximum axle mass of 19 tons of the train. Table 9 shows the accelerations for 4, 5, 6 wagons for the loaded train condition for a 19-tonnes axle load.

Table 9.

Accelerations for 4, 5, 6 wagons loaded train condition for an axle load of 19 tonnes

Gradient	Accelerations 4 wagons	Accelerations 5 wagons	Accelerations 6 wagons
0 ‰	0,564	0,705	0,47
10 ‰	0,467	0,584	0,389
15 ‰	0,419	0,524	0,349
20 ‰	0,371	0,464	0,309
25 ‰	0,323	0,404	0,269

From the data above some conclusions can be made: at a fixed composition of the train 4, 5, 6 wagons, the decrease of the axle load from the maximum load of the loaded wagon to the minimum load of the empty wagon, the possibilities of acceleration of the train decrease; at a fixed axle load, a decrease in the number of wagons causes an increase in acceleration.

Those results shows that for regional trains, in order to have high accelerations, the train composition must have a minimum number of wagons (3-4 wagons), with technical solutions to improve grip, in order to have accelerations of 1 m/s². For interregional trains, the composition of the train can go up to 6 wagons, with accelerations that remain above 0.5 m/s², at the departure of the station, which is enough for an interregional train that has stops every about 100 km away. This study will allow us to evaluate later in the paper the evolution of electricity consumption for the regional train R with 4, 5, 6 wagons, respectively the interregional train IR.

The above analysis presented in this article represents the first stage of a study that aims to establish an algorithm for optimizing electricity consumption for traffic regimes on electric trains (electric traction vehicles).

4. Energy consumption simulations

The calculation of the energy required for the starting regime starts from the relationship:

$$E = \int_0^T m_t \cdot v(t) \cdot v(t) dt \tag{8}$$

where m_t is train mass.

The analysis was made for the speed range 0-200 km/h in alignment and center bearing three cases [11-15]:

a) starting mode with constant traction force
b) starting mode with constant acceleration
c) starting mode with traction force varying according to the law of variation of adhesion force

Case a) *Starting mode with constant traction force* is presented in figure 5 and the data from the simulation is synthesized in table 10.

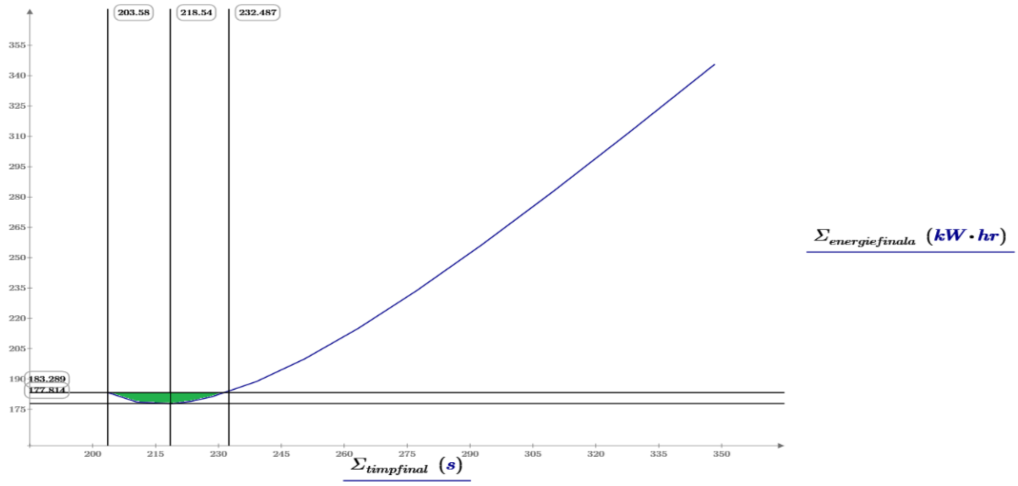

Fig. 5. Energy consumption for the case of constant traction force

Table 10.

Synthesized data for constant traction force starting mode

Case 1: Constant traction force	Energy [kW*hr]	Time [s]	Speed range [kph]	The transition speed on the constant power characteristic [kph]
Starting rate	183.289	203.58	0-91-200	91
Minimum rate	177.814	218.944	0-110-200	110
Equivalent growth rate	183.289	231.556	0-123-200	123

Case b) *Starting mode with constant acceleration* is presented in figure 6 and the data from the simulation is synthesized in table 11.

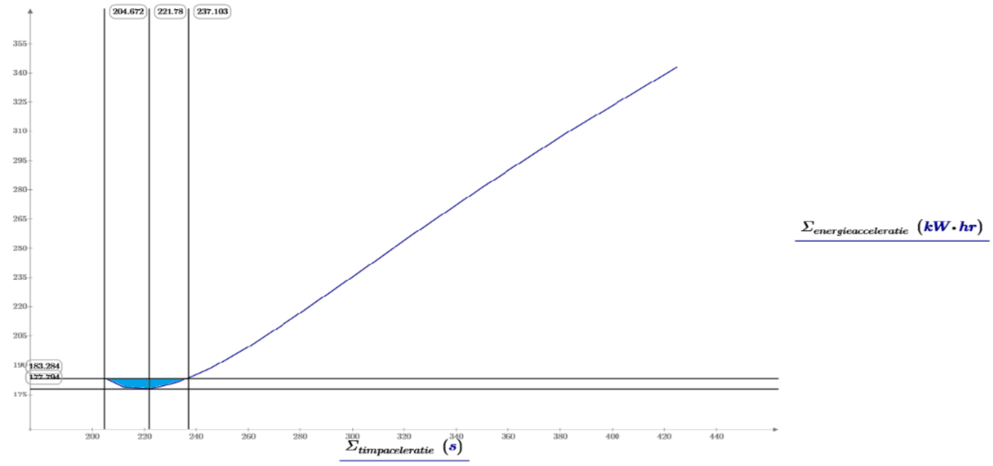

Fig.6. Energy consumption for the case of constant acceleration

Table 11.

Synthesized data for constant acceleration starting mode

Case 1: Constant traction force	Energy [kW*hr]	Time [s]	Speed range [kph]	The transition speed on the constant power characteristic [kph]
Starting rate	183.284	204.672	0-91-200	91
Minimum rate	177.794	221.78	0-110-200	110
Equivalent growth rate	183.284	237.103	0-123-200	123

Case c) *Starting mode with traction force varying according to the law of variation of adhesion force* is presented in figure 7 and the data from the simulation is synthesized in table 13.

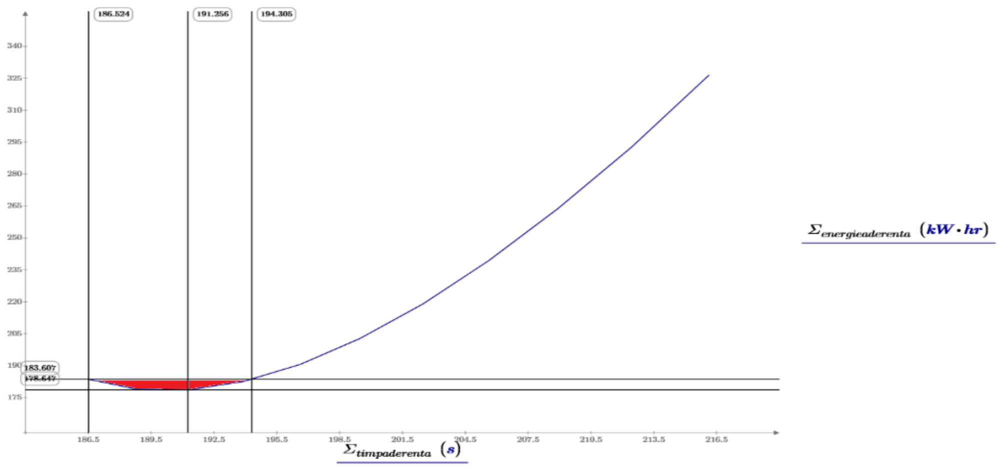

Fig.7 Energy consumption for the case - traction force that varies according to the law of variation of the adhesion force

Table 12.

Synthesized data for traction force that varies according to the law of variation of the adhesion force starting mode

Case 1: Constant traction force	Energy [kW*hr]	Time [s]	Speed range [kph]	The transition speed on the constant power characteristic [kph]
Starting rate	183.607	186.524	0-91-200	91
Minimum rate	178.647	191.256	0-110-200	110
Equivalent growth rate	183.607	194.305	0-120-200	120

In table 13, the starting rates are detailed for all 3 cases: constant traction force, constant acceleration and traction force varies according to the law of variation of adhesion:

Table 13.

Starting rates

Starting rates	Energy sum [kW*hr]	Time sum [s]
Constant traction force	183.289	203.58
Constant acceleration	183.284	204.672
Traction force varies according to the law of variation of adhesion	183.607	186.524

In table 14, the minimum rates are detailed for all 3 cases: constant traction force, constant acceleration and traction force varies according to the law of variation of adhesion:

Table 14.

Minimum rates

Minimum rates	Energy sum [kW*hr]	Time sum [s]
Constant traction force	177.814	218.944
Constant acceleration	177.794	221.78
Traction force varies according to the law of variation of adhesion	178.647	191.256

In table 15, the equivalent growth rates are detailed for all 3 cases: constant traction force, constant acceleration and traction force varies according to the law of variation of adhesion:

Table 15.

Equivalent growth rates

Equivalent growth rates	Energy sum [kW*hr]	Time sum [s]
Constant traction force	183.289	231.556
Constant acceleration	183.284	237.103
Traction force varies according to the law of variation of adhesion	183.607	194.305

After this study, one can proceed to the establishment of an optimization strategy, which we will address in a later article, but which starts from the following considerations:

- the rail transport (all kind of rail transport, passengers and freight trains) represents an efficient form of transport comparing with road transport and air transport, but it must meet the needs of its potential users, like the low transport time, cost efficient, safety, comfort, accessibility, links to various places and routes. The rail transport can provide energy-efficient transport on a dedicated corridor free of external interference, both for passengers and for freight trains. The energy-efficient way of transport entitles the construction and operating of high-performance rail vehicles. All the transport operators go after reducing operating cost and maintenance costs of their fleet. Nowadays, the costs for energy consumption of the trains represents a major part of these costs. How to reduce these costs became a prime concern for all railway companies. The energy consumption, traffic speed and drive time are connected. A controversial competition is represented by the optimization energy consumption in relation to the required drive time.

- the power used to drive the vehicle comes in two forms: electric power or mechanical power (supplied by diesel engine). Energy balance shows that part of energy is consumed by resistances to advance (determined by the circulation on the railways profile) and the brake system. The remaining energy is found in the kinetic and potential energy. It is known that the energy consumption depends on drive patterns selected and of their sequence. It is confirmed that the optimal driving strategy for a train takes the form of a power-speed hold-coast-brake strategy unless the track contains steep grades.

The data used in the study of train control are: T is the time allowed for the journey, x is the distance between two stations, $u(t)$ is the accelerations applied to the train, $v(t)$ is the speed of the train, and $-r(v(t))$ is the resistive acceleration due to the friction. The movement of the train is governed by the Newton law

$$\ddot{x}(t) = u(t) - r(v(t)) \tag{9}$$

where $r(v)$, $v \in [0,\infty)$ is strictly increasing and convex function and the acceleration $u(t)$ (control variable) is limited by the relation $|u(t)| \leq 1$. The theory (see energy consumption) involves also the positive part of $u(t)$, defined by

$$u_{+}(t) = \frac{1}{2}(u(t) + |u(t)|) \tag{10}$$

The increasing and convex function $r(v)$ is exemplified by the formula

$$r(v) = a + bv + cv^2, \mathrm{v} \in [0,\infty) \tag{11}$$

where a, b, c are known real numbers subject to $a > 0$, $b > 0$, $c > 0$.

The issue of discover the prime method of driving the train to the next station, can be developed as an optimal control problem (local energy minimization principle). We are aiming to discover sequence of control settings that will get the train to the next station on a reduced time, and with the least possible energy consumption. Newton's law determines the movement equation of the train. In the following problem, x and v are state variables and u is the control variable.

From the date from the simulation, it is seen that using the traction force that varies according to the law of variation of adhesion, the time needed to obtain the maximum speed is lower, so the energy consumption is lower then the other two cases.

6. Conclusions

Energy is one of the main issues that determines world politics. Energy efficiency has become a mandatory component of the national and European Union energy strategy. The energy consumed by transport vehicles also accounts for a large part of the world's energy consumption. Therefore, today we focus on energy saving/optimization/energy consumption and energy recovery in railway, urban and metropolitan systems (regarding the energy consumption of electric traction, rail-guided vehicles), which are frequently used in systems of transport.

The presented analysis allows a correct choice of the performance parameters of the traction vehicles from the perspective of the energy consumption optimization strategy for the start-up mode.

The study proposed in this paper will allow the choice of optimal driving modes from the perspective of electricity consumption for the start-up mode.

The solution for improving acceleration is to increase grip through technical solutions that improve the coefficient of adhesion (sanding on all drive wheels).

Changing the load on the axle does not influence the maximum acceleration that the vehicle can develop, a substantial improvement can only be achieved by increasing the adhesion limit, by performing a rigorous control of the traction force up to the limit of the appearance of the stick-slip phenomenon.

However, the maximum acceleration that a train can develop will decrease with the increase in the number of wagons in the train, that is, with the increase in the total mass of the train, not from the load on the axle, but from the number of wagons.

There are no big differences regarding the choice of acceleration, it is observed that there is a starting point, a minimum point and then the growth is made by a linear value. If one of the options has to be chosen, it had been seen that if starting with a traction force that varies according to the law of variation of adhesion has a lower consumption demands due to a smaller time period needed to obtain the maximum speed.

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