DYNAMICS ANALYSIS OF THE CABLE-TYING CLOSE-COUPLING MULTI-ROBOT COLLABORATIVELY TOWING SYSTEM

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The structure model of the cable-tying closing-coupling multi-robot collaboratively towing system (CTMRTS) is analyzed, which is equivalent to the multi-rigid body flexible cable system. Firstly, the configuration design of CTMRTS is introduced; The kinematic model of towing system is established, and the generalized kinematics of the system is analyzed; Subsequently, the generalized dynamic equations are given by using the Newton - Euler equation. Finally, make the towing system which composed of three robots as example, the kinematics of the towing system has been carried on numerical calculation and experiment simulation. Comparison results reveal that the proposed kinematics model is reasonable, and the system motion state is smooth. It may be lay a foundation for further research about the stability and the actual control of CTMRTS.

Keywords: towing system, multi-robots system, close-coupling, dynamics, kinematic

1. Introduction

With the development of robot technology, the study of the multi-robot collaboratively towing system has become one of hot spots in the field of the automatic control and the robot technology [1, 2]. Single robot is unable to meet the needs of current social production tasks, because its workspace, loading capacity and flexibility are limited. However, CTMRTS has many advantages, such as its good flexibility, high precise, high-loading capacity, high work efficiency and lager workspace [3] and so on. Hence, the study of the system has great practical application value [4-6].

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In recent decades, many scientific and technological workers have interest in the research of CTMRTS, the study of CTMRTS mainly focuses on the modeling of system, the trajectory planning and the control strategy and so on. At the present stage, a few literatures that use multiple robots to tow a heavy or bulky load are reported. A. Alireza etc set up the experiment platform of collaboratively towing system by using nine cables, they control the system with method of double integral sliding mode control, but it only limits the three degrees of freedom of the towed object [7], the tension of cables is discussed in the literature [8], and the workspace of the towed object is analyzed in the literature [9]. In 2009, I. Maza, K. Kondak, M. Bernard etc use three small-scale unmanned helicopters to research towing control experiment [10, 11]. A. Omran etc establish optimal forward kinematics modeling of stewart manipulator by using genetic algorithms, regression model and predicted squared error of stewart manipulator are also given in the literature [12]. Kinematics and dynamics analysis of two parallel mechanisms are analyzed by using recursive matrix relations [13]. In the above research works some basic question of system, such as the structure of hardware and software are discussed, but many substantive questions of the system aren't discussed, such as the model of kinematics, dynamics and stability of the system, so the research results can't be guidance for practical application.

B. ZI [14-16] etc set up the towing system by using six cables, the kinematics and dynamics of the system are discussed, however, it is considered three translational degrees of freedom of the towed object. And in the literature [17], one end of cables are fixed, it adjusts the towed object's position and posture by also adjusting the length of cables. J. H. ZHOU[18] etc analyzed the workspace of the towed object when considering tension constraints in the cables and system stiffness. In above research works the workspace of system is small, because all robot-ends are fixed. Y. Q. ZHENG [19, 20] etc use three mobile trolley, which can move along straight line, and tow the same object, the kinematics and dynamics of the system is analyzed. The dynamics of cable-driven parallel robot coordination towing system is analyzed in the literature [21]. But the robot-ends can only move along the straight line, thus, it restricts the workspace and motion flexibility of the system. The regulation of the tension-direction is discussed by utilizing configuration space redundancy to shape the tension null space [23].

The author has been participating and studying the system from 2012, the experiment platform has been set up by using three industrial robots, movement control CARDS, PC and the tracking device of object's position and posture. CTMRTS is a closed-loop control system, since system's complication and the problem of control redundancy, considering the safety and the experiment cost, the simulation experiment platform of this type system has a great value for verifying the kinematics model.

It is impossible to design the intelligent controller of the system if we can't establish accurate kinematic and dynamic models of this kind system. The paper mainly discussed about the kinematics and dynamics of CTMRTS. According to the characteristics of the system, the following several aspects on the kinematics and dynamics of CTMRTS are discussed. In the second section, the configuration of the system is analyzed. In the third section, the coordinate frames of the system are defined, and the generalized kinematics of the system is discussed. In the fourth section, the generalized dynamic equations of the system are given. In the fifth section, make the most difficult to control towing system which composed of three modular serial robots as example, the corresponding parameters of the system are given. Numerical calculations result with using the software MATLAB are given based on the proposed mathematical model, and then the motion simulation platform is set up by using the software UG/ADAMS, the compared results are showed in the last. The conclusion is also given finally. The conclusion will lay the foundation for further research about the stability and the actual control of CTMRTS.

2. System configuration

CTMRTS is parallel robots system which consisted of multiple modular serial robots, making it has an ability of collaboratively towing the same object, let the towed object move as expected position and posture by adjusting the position of the robot-ends or the length of cables. The rationality and the complexity of the system model are mainly depend on the rationality of the flexible cable modeling, and that the complexity of the coupling mechanics among robots, flexible cables and towed object, modeling of the flexible cable is the hardest thing. The rationality of cable model directly affects the smooth of the system movement state and the correctness of the simulation results.

The space configuration of CTMRTS is shown in figure 1. The towed object is suspended under multiple robots through flexible cables. In figure 1 the fixed coordinate frame {O} and the moving coordinate frame {P} are established, b_i($i=1,2,\cdots,n$) are points of junction between cables and robot-ends, and p_i ($i=1,2,\cdots,n$) are points of junction between cables and the towed object. The number of robots n (n≥3) can be configured according to the need of actual tasks.

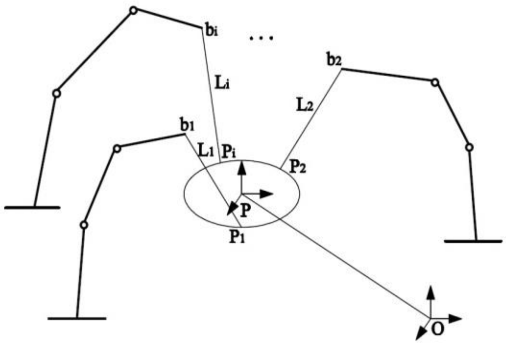

Fig. 1. The space configuration of system

3. Kinematics analysis of CTMRTS

Kinematics analysis of the system is that solve the object's position (x,y,z) and posture $(\varphi_1,\varphi_2,\varphi_3)$, in the premise that each joint angle $(\theta_{i1},\theta_{i2},\theta_{i3})$ of robots is given. Hence, the paper analyzes the system's kinematics as following two parts, firstly, it is solved that the robot-ends' position is expressed with joint angles $(\theta_{i1},\theta_{i2},\theta_{i3})$ in the fixed coordinate frame $\{O\}$. Secondly, the relationship between the towed object's position, posture and robot-ends' position is analyzed.

3.1 Kinematics analysis of robot

The three degrees of freedom joints robot is used in this paper, when three joint angles $(\theta_{i1},\theta_{i2},\theta_{i3})$ are given, the robot-end's position relative to the robot base can be easily solved through the geometry relationship:

$$\begin{cases} x_i^* = a_2 cos\theta_{i1} cos\theta_{i2} + a_3 cos\theta_{i1} cos\theta_{i3} \\ y_i^* = a_2 sin\theta_{i1} cos\theta_{i2} + a_3 sin\theta_{i1} cos\theta_{i3} \qquad i = 1,2\cdots n \\ z_i^* = a_1 + a_2 sin\theta_{i2} + a_3 sin\theta_{i3} \end{cases} \tag{1}$$

Where, a_i is the effective length of the i^{th} root link rod.

Giving the absolute joint angles of rotation θ_{i1}, θ_{i2}, θ_{i3} of serial robots, the position vectors $\left(x_i^*, y_i^*, z_i^*\right)^T$ of bi points of contact between cables and robot-ends can be expressed in the fixed frame as follows:

$$\left(x_i^*, y_i^*, z_i^*\right)^T = \{a_1 \text{ rot}(z,\theta_{i1}) + \text{rot}(z,\theta_{i1})[a_2 \text{ rot}(y,\pi/2-\theta_{i2}) + a_2 \text{ rot}(y,\pi/2-\theta_{i3})]\}[0\ 0\ 1]^T$$

With the geometry relationship (1)

Assuming that robot base coordinate frame's origin position relative to the fixed coordinate frame {O} is $({}^{o}x_i, {}^{o}y_i, {}^{o}z_i)^{\mathrm{T}}$. By transformation of coordinates, the robot-end's position in the fixed coordinate frame{O} can be obtained:

$${}^{o}b_i = (x_i, y_i, z_i)^{\mathrm{T}} = ({}^{o}x_i, {}^{o}y_i, {}^{o}z_i)^{\mathrm{T}} + (x_i^*, y_i^*, z_i^*)^{\mathrm{T}}$$

3.2 Kinematics analysis of CTMRTS

The kinematics of CTMRTS is that the function expression of the object's position and posture are expressed with the robot-ends' coordinates. Assuming that the system is composed of n(n≥3) robots, the robot-end and the towed object are regarded as mass point, and the ends of the flexible cables are regarded as pivot, thus, the system can be regard as a combination of several mass-damping system.

3.2.1 Coordinate frames of the robot-ends and the towed object

The independently controllable system coordinates can be expressed as $\boldsymbol{q} = (\boldsymbol{b_1}, \boldsymbol{b_2} \cdots \boldsymbol{b}_n; \boldsymbol{L_1}, \boldsymbol{L_2} \cdots \boldsymbol{L}_n)^{\mathrm{T}}$, the coordinates of robot-ends is $\boldsymbol{b} = (\boldsymbol{b_1}, \boldsymbol{b_2} \cdots \boldsymbol{b}_n)^{\mathrm{T}}$, the vectors of flexible cables length is $\boldsymbol{L} = (\boldsymbol{L_1}, \boldsymbol{L_2} \cdots \boldsymbol{L}_n)^{\mathrm{T}}$, the position and posture of towed object's reference point P is $\boldsymbol{y_p} = (\boldsymbol{r}, \boldsymbol{\varphi})^{\mathrm{T}}$, here, $\boldsymbol{r} = (x, y, z)^{\mathrm{T}}$ and $\boldsymbol{\varphi} = (\varphi_1, \varphi_2, \varphi_3)^{\mathrm{T}}$, the position vector $\boldsymbol{r}$ and the orientation vector $\boldsymbol{\varphi}$ of the reference point P of the towed object can describe the spatial position of the towed object in the fixed frame {O}.

The velocity vector of the towed object in the fixed coordinate frame {O} is $\dot{\boldsymbol{s}}_p = [\boldsymbol{v}, \boldsymbol{\omega}]^{\mathrm{T}}$, with the translational velocity $\boldsymbol{v} = [\dot{x}, \dot{y}, \dot{z}]^{\mathrm{T}}$ and the angular velocity $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^{\mathrm{T}}$ (coordinates $\boldsymbol{\omega}$ in {O}). The relation between $\dot{\boldsymbol{s}}_p$ and $\dot{\boldsymbol{y}}_{\boldsymbol{p}}$ is given by $\dot{\boldsymbol{y}}_{\boldsymbol{p}} = \boldsymbol{H}(\boldsymbol{y_p})\dot{\boldsymbol{s}}_p$, here, $\boldsymbol{H}(\boldsymbol{y}_p) = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_{\omega}(\boldsymbol{\varphi}) \end{bmatrix}$. According to general rotation matrix ${}^{O}\boldsymbol{R}_P$ (3), below described, the contents of the matrix $\boldsymbol{H}_{\omega}(\boldsymbol{\varphi})$ is, in fact, the following:

$$\cos\varphi_2 \boldsymbol{H}_{\omega}(\boldsymbol{\varphi}) = \begin{bmatrix} \cos\varphi_3 & \sin\varphi_3 & 0 \\ -\cos\varphi_2 \sin\varphi_3 & \cos\varphi_2 \cos\varphi_3 & 0 \\ \sin\varphi_3 & \cos\varphi_3 \sin\varphi_2 & \sin\varphi_3 \cos\varphi_2 \end{bmatrix}$$

The vector ${}^{P}p_i$ is that the position coordinates of points p_i in the moving coordinate frame {P}. The position vector ${}^{O}T_p$ is that the position coordinates of the reference P of the towed object in the fixed coordinate system {O}. As shown in figure 2.

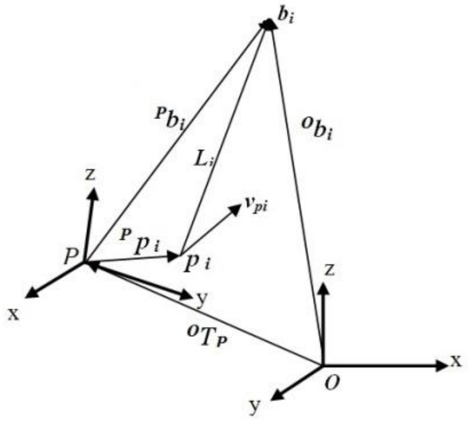

Fig. 2. The position vector that is the object relative to the robot-end

3.3. Constraint equations

CTMRTS is a parallel robot system which composed of multiple serial robots, hence, the system has a strong coupling, there exist many constraints among robots, flexible cables and towed object, such as constraints on the position level, the velocity level and the acceleration level and so forth.

3.3.1. Constraints on the Position Level

The cable vector $\boldsymbol{L}_i$ in the fixed coordinate frame {O} is expressed by the following equation:

$$\boldsymbol{L}_i = {}^O\boldsymbol{T}_P + {}^O\boldsymbol{R}_p {}^P\boldsymbol{P}_i - {}^O\boldsymbol{b}_i, \quad i = 1,2\cdots n \tag{2}$$

Where, ${}^O\boldsymbol{T}_P = [x,y,z]^{\mathrm{T}}$, ${}^P\boldsymbol{P}_i = \left[x_{pi}, y_{pi}, z_{pi}\right]^{\mathrm{T}}$.

The absolute orientation of the towed object can be described by a general rotation matrix ${}^o\boldsymbol{R}_p$. The Euler representation is adopted to describe the absolute moving towed object posture, which is composed of three successive relative rotations, namely: first rotation of angle φ_3 around the Z1-axis, followed by second rotation of angle φ_2 around the rotated Y2-axis and finally followed by third rotation of angle φ_1 around the X-axis of the final moving frame P-XYZ attached to the moving towed object. With respect to the fixed global coordinate system O-xyz, the general rotation matrix can be expressed by a multiplication of three basic rotation matrices as follows:

$${}^O\boldsymbol{R}_P = \boldsymbol{R}_1(\mathrm{Z1}, \boldsymbol{\varphi}_3)\boldsymbol{R}_2(\mathrm{Y2}, \boldsymbol{\varphi}_2)\boldsymbol{R}_3(\mathrm{X}, \boldsymbol{\varphi}_1) \tag{3}$$

According to Fig.2, the geometric constraints equations between the coordinates of the independently controllable object $\boldsymbol{q}$ and the object's position and posture vector $\boldsymbol{y_p}$ are expressed as:

$$\boldsymbol{g}_i(\boldsymbol{y_p},\boldsymbol{q}) = \boldsymbol{L_i}^{\mathrm{T}} \cdot \boldsymbol{L}_i - \|\boldsymbol{L}_i\|^2 = 0\,, \quad i = 1,2 \cdots n \tag{4}$$

So the following expression can be obtained:

$$\boldsymbol{L_i} = \begin{bmatrix} x + z_{pi}(\sin(\varphi_1)\sin(\varphi_3) + \cos(\varphi_1)\cos(\varphi_3)\sin(\varphi_2)) - y_{pi}(\cos(\varphi_1)\sin(\varphi_3) - \cos(\varphi_3)\sin(\varphi_1)\sin(\varphi_2)) \\ \quad + x_{pi}\cos(\varphi_2)\cos(\varphi_3) - x_i \\ y + y_{pi}(\cos(\varphi_1)\cos(\varphi_3) + \sin(\varphi_1)\sin(\varphi_2)\sin(\varphi_3)) - z_{pi}(\cos(\varphi_3)\sin(\varphi_1) - \cos(\varphi_1)\sin(\varphi_2)\sin(\varphi_3)) \\ \quad + x_{pi}\cos(\varphi_2)\sin(\varphi_3) - y_i \\ z + z_{pi}\cos(\varphi_1)\cos(\varphi_2) - x_{pi}\sin(\varphi_2) + y_{pi}\cos(\varphi_2)\sin(\varphi_1) - z_i \end{bmatrix} \tag{5}$$

3.3.2. Constraints on the Velocity Level

The derivatives of the vectors of the length of cables in Eq. (2) corresponding to time t, can be shown in the following equation:

$$\dot{\boldsymbol{L}}_i = {}^{\boldsymbol{o}}\dot{\boldsymbol{T}}_{\boldsymbol{p}} + {}^{\boldsymbol{p}}\dot{\boldsymbol{P}}_{\boldsymbol{i}} - {}^{\boldsymbol{o}}\dot{\boldsymbol{b}}_{\boldsymbol{i}}\,, \; i = 1,2 \cdots n \tag{6}$$

Where, $\dot{\boldsymbol{L}}_i = {}^{\boldsymbol{o}}\dot{\boldsymbol{T}}_{\boldsymbol{p}} + {}^{\boldsymbol{p}}\dot{\boldsymbol{P}}_{\boldsymbol{i}} - [\dot{x}_i \;\; \dot{y}_i \;\; \dot{z}_i]^{\mathrm{T}}$, $\; i = 1,2 \cdots n$

The vector ${}^{\boldsymbol{p}}\dot{\boldsymbol{P}}_i$ can be expressed with angular velocity ω through the rigid body screw theory:

$${}^{\boldsymbol{p}}\dot{\boldsymbol{P}}_{\boldsymbol{i}} = \hat{\omega}^{p}\boldsymbol{p}_i \tag{7}$$

The derivative $\mathrm{d}({}^{P}\boldsymbol{P}_i)/\mathrm{d}t$, in (6) and (7), is, in fact, the following:

$$\mathrm{d}({}^{P}\boldsymbol{P}_i)/\mathrm{d}t = \Omega \cdot {}^{O}\boldsymbol{R}_P \cdot {}^{P}\boldsymbol{P}_i$$

Where the skew-symmetric matrix $\Omega = \mathrm{d}({}^{O}\boldsymbol{R}_P)/\mathrm{d}t \cdot {}^{O}\boldsymbol{R}_P^{\mathrm{T}}$ and its associated vector ω of angular velocity are both expressed in the fixed coordinate system.

Here, $\hat{\omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} = -\hat{\omega}^{\mathrm{T}}$

Hence,

$$\dot{\boldsymbol{L}}_i = {}^{\boldsymbol{o}}\dot{\boldsymbol{T}}_{\boldsymbol{p}} + \hat{\omega}^{p}\boldsymbol{p}_i - {}^{\boldsymbol{o}}\dot{\boldsymbol{b}}_{\boldsymbol{i}}\,, \; i = 1,2 \cdots n \tag{8}$$

So the following expression can be obtained, from EQ.(8):

$$\dot{\boldsymbol{L}}_{\boldsymbol{i}}=\begin{bmatrix}\dot{x}+\omega_y z_{pi}-\omega_z y_{pi}-\dot{x}_i\\ \dot{y}+\omega_z x_{pi}-\omega_x z_{pi}-\dot{y}_i\\ \dot{z}+\omega_x y_{pi}-\omega_y x_{pi}-\dot{z}_i\end{bmatrix},\ i=1,2\cdots n \tag{9}$$

The derivatives of the geometric constraints equation in Eq. (4) corresponding to time t, can be shown in the following equation:

$$\dot{\boldsymbol{g}}=\boldsymbol{G_s}(\boldsymbol{y_p},\boldsymbol{q})\dot{\boldsymbol{s}}_{\boldsymbol{p}}+\boldsymbol{G_q}(\boldsymbol{y_p},\boldsymbol{q})\dot{\boldsymbol{q}}=0,\quad i=1,2\cdots n \tag{10}$$

Constraint matrixes are shown the following equations:

$$\boldsymbol{G_s}(\boldsymbol{y_p},\boldsymbol{q})=2\begin{bmatrix}\boldsymbol{L}_1^{\mathrm{T}} & -\boldsymbol{L}_1^{\mathrm{T}}\tilde{\boldsymbol{p}}_1\\ \boldsymbol{L}_2^{\mathrm{T}} & -\boldsymbol{L}_2^{\mathrm{T}}\tilde{\boldsymbol{p}}_2\\ \vdots & \vdots\\ \boldsymbol{L}_n^{\mathrm{T}} & -\boldsymbol{L}_n^{\mathrm{T}}\tilde{\boldsymbol{p}}_n\end{bmatrix} \tag{11}$$

$$\boldsymbol{G_q}(\boldsymbol{y_p},\boldsymbol{q})=2\begin{bmatrix}\boldsymbol{L}_1^{\mathrm{T}}\boldsymbol{e}_x & 0 & 0\cdots & L_1 & 0 & 0 & \cdots & 0\\ 0 & \boldsymbol{L}_2^{\mathrm{T}}\boldsymbol{e}_x & 0\cdots & 0 & L_2 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & & \vdots\\ 0 & 0 & 0\cdots & \boldsymbol{L}_n^{\mathrm{T}}\boldsymbol{e}_x & 0 & 0 & \cdots & L_n\end{bmatrix} \tag{12}$$

And where, $\boldsymbol{e}_x=[1\ 0\ 0]^{\mathrm{T}}$

3.3.3 Constraints on the acceleration level

The derivatives of Eq. (10) corresponding to time t, can be shown in the following equation:

$$\ddot{\boldsymbol{g}}=\boldsymbol{G_s}(\boldsymbol{y_p},\boldsymbol{q})\ddot{\boldsymbol{s}}_{\boldsymbol{p}}+\boldsymbol{G_q}(\boldsymbol{y_p},\boldsymbol{q})\ddot{\boldsymbol{q}}+\dot{\boldsymbol{G}}_{\boldsymbol{s}}(\boldsymbol{y_p},\boldsymbol{q})\dot{\boldsymbol{s}}_{\boldsymbol{p}}+\dot{\boldsymbol{G}}_{\boldsymbol{p}}(\boldsymbol{y_p},\boldsymbol{q})\dot{\boldsymbol{q}}=\boldsymbol{0} \tag{13}$$

4. Dynamics analysis of CTMRTS

In CTMRTS, the towed object not only is effected by itself gravity, but also by pull of cables in one direction in process of the movement. Assuming that the mass of the towed object will be all concentrated to the center of mass, and the origin point of the moving coordinate frame $\{P\}$ and the center of mass of the towed object have the same position, so the gravity will not provide torques that make the towed object rotation. It is assumed that all cables are ideal, that is, the gravity and elastic deformation of cables are ignored, T_i ($i=1,2\cdots n$) is the tension of the ith cable. m is the mass of the towed object. On the basis of Newton - Euler equation, dynamic equations of the system can be obtained:

$$\begin{bmatrix} \frac{x_1-x_{p1}}{L_1} & \frac{x_2-x_{p2}}{L_2} & \cdots & \frac{x_n-x_{pn}}{L_n} \\ \frac{y_1-y_{p1}}{L_1} & \frac{y_2-y_{p2}}{L_2} & \cdots & \frac{y_n-y_{pn}}{L_n} \\ \frac{z_1-z_{p1}}{L_1} & \frac{z_2-z_{p2}}{L_2} & \cdots & \frac{z_n-z_{pn}}{L_n} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} = \begin{bmatrix} m\ddot{x} \\ m\ddot{y} \\ mg+m\ddot{z} \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} \frac{y_1-y_{p1}}{L_1} & 0 & \cdots & 0 \\ 0 & \frac{y_2-y_{p2}}{L_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{y_n-y_{pn}}{L_n} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} \begin{bmatrix} x_{p1} \\ x_{p2} \\ \vdots \\ x_{pn} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \frac{z_1-z_{p1}}{L_1} & 0 & \cdots & 0 \\ 0 & \frac{z_2-z_{p2}}{L_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{z_n-z_{pn}}{L_n} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} \begin{bmatrix} y_{p1} \\ y_{p2} \\ \vdots \\ y_{pn} \end{bmatrix}^{\mathrm{T}} = J_x\ddot{\varphi}_1$$

(15)

$$\begin{bmatrix} \frac{x_1-x_{p1}}{L_1} & 0 & \cdots & 0 \\ 0 & \frac{x_2-x_{p2}}{L_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{x_n-x_{pn}}{L_n} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} \begin{bmatrix} z_{p1} \\ z_{p2} \\ \vdots \\ z_{pn} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \frac{z_1-z_{p1}}{L_1} & 0 & \cdots & 0 \\ 0 & \frac{z_2-z_{p2}}{L_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{z_n-z_{pn}}{L_n} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} \begin{bmatrix} x_{p1} \\ x_{p2} \\ \vdots \\ x_{pn} \end{bmatrix}^{\mathrm{T}} = J_y\ddot{\varphi}_2$$

(16)

$$\begin{bmatrix} \frac{x_1-x_{p1}}{L_1} & 0 & \cdots & 0 \\ 0 & \frac{x_2-x_{p2}}{L_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{x_n-x_{pn}}{L_n} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} \begin{bmatrix} y_{p1} \\ y_{p2} \\ \vdots \\ y_{pn} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \frac{y_1-y_{p1}}{L_1} & 0 & \cdots & 0 \\ 0 & \frac{y_2-y_{p2}}{L_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{y_n-y_{pn}}{L_n} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} \begin{bmatrix} x_{p1} \\ x_{p2} \\ \vdots \\ x_{pn} \end{bmatrix}^{\mathrm{T}} = J_z\ddot{\varphi}_3 \tag{17}$$

And ${}^{p}\boldsymbol{b}_i = {}^{p}\boldsymbol{P}_i + \boldsymbol{L}_i$.

$$^{P}\boldsymbol{b_i} = (x, y, z) - (x_{p_i}, y_{p_i}, z_{p_i}) \ .$$

$$^{O}\boldsymbol{p}_i = R_z(\varphi_3) R_y(\varphi_2) R_x(\varphi_1)\, ^{P}\boldsymbol{p_i} \ .$$

In equations from (14) to (17):

$\ddot{x}, \ddot{y}, \ddot{z}$ are the translational acceleration respectively among the directions x, y, z.

$\ddot{\varphi}_1$, $\ddot{\varphi}_2$, $\ddot{\varphi}_3$ are the angle acceleration respectively around the axis x, y, z.

J_x, J_y, J_z are the rotary inertia respectively around axis x, y, z.

x_{pi}, y_{pi}, z_{pi} are coordinates of junction points p_i in the moving coordinate frame $\{P\}$.

5. Example simulation and analysis

The number of robot *n* (*n*≥3) can be configured according to the need of actual tasks. According to the relationship between the number of robot *n* and the number of degrees of freedom of the towed object *e*, cable-tying parallel robots can be classified into the following four types[18] : (1) incompletely restrained positioning mechanisms, with *n* = *e*; (2) completely restrained positioning mechanisms, with *n* =1+ *e*; (3) redundantly restrained positioning mechanisms, with *n* >1+ *e* ; (4) under restrained positioning mechanisms, with *n*< *e*. The fourth is the most difficult to control in practical application. Hence, this paper study the kinematics and the dynamics of the under restrained system by the example which composed of three robots (*n*=3), it will lay the foundation for designing controller of this kind system.

5.1. Constraint Equations

The constraint equations can be obtained through the above discussion about the kinematics and the dynamics of this type system.

5.1.1. Constraints on the Position Level

The position vectors $^{P}\boldsymbol{p}_i$ （$i=1,2,3$） respectively are：

$$^{P}\boldsymbol{P}_1 = \left[\frac{\sqrt{3}}{3}l, 0, 0\right]^{\mathrm{T}}, \quad ^{P}\boldsymbol{P}_2 = \left[-\frac{\sqrt{3}}{6}l, -\frac{1}{2}l, 0\right]^{\mathrm{T}}, \quad ^{P}\boldsymbol{P}_3 = \left[-\frac{\sqrt{3}}{6}l, \frac{1}{2}l, 0\right]^{\mathrm{T}}$$

So the following expressions can be obtained:

$$\boldsymbol{L}_1=\begin{bmatrix} x+\frac{\sqrt{3}}{3}l\cos\varphi_2\cos\varphi_3-x_1 \\ y-\frac{\sqrt{3}}{3}l\cos\varphi_2\sin\varphi_3-y_1 \\ z-\frac{\sqrt{3}}{3}l\sin\varphi_2\cos\varphi_3-z_1 \end{bmatrix} \tag{18}$$

$$\boldsymbol{L}_2=\begin{bmatrix} x-\frac{\sqrt{3}}{6}l\cos\varphi_2\cos\varphi_3-\frac{1}{2}l(\sin\varphi_1\sin\varphi_2\cos\varphi_3-\cos\varphi_1\sin\varphi_3)-x_2 \\ y-\frac{\sqrt{3}}{3}l\cos\varphi_2\sin\varphi_3-\frac{1}{2}l(\sin\varphi_1\sin\varphi_2\sin\varphi_3+\cos\varphi_1\cos\varphi_3)-y_2 \\ z+\frac{\sqrt{3}}{6}l\sin\varphi_2-\frac{1}{2}l\sin\varphi_1\cos\varphi_2-z_2 \end{bmatrix} \tag{19}$$

$$\boldsymbol{L}_3=\begin{bmatrix} x-\frac{\sqrt{3}}{6}l\cos\varphi_2\cos\varphi_3+\frac{1}{2}l(\sin\varphi_1\sin\varphi_2\cos\varphi_3-\cos\varphi_1\sin\varphi_3)-x_3 \\ y-\frac{\sqrt{3}}{3}l\cos\varphi_2\sin\varphi_3+\frac{1}{2}l(\sin\varphi_1\sin\varphi_2\sin\varphi_3+\cos\varphi_1\cos\varphi_3)-y_3 \\ z+\frac{\sqrt{3}}{6}l\sin\varphi_2+\frac{1}{2}l\sin\varphi_1\cos\varphi_2-z_3 \end{bmatrix} \tag{20}$$

5.1.2. Constraints on the Velocity Level

The following constraints equations on the velocity level can be obtained:

$$\dot{\boldsymbol{L}}_1=\begin{bmatrix} \dot{x}-\dot{x}_1 \\ \dot{y}+\frac{\sqrt{3}}{3}l\omega_z-\dot{y}_1 \\ \dot{z}-\frac{\sqrt{3}}{3}l\omega_y-\dot{z}_1 \end{bmatrix} \tag{21}$$

$$\dot{\boldsymbol{L}}_2=\begin{bmatrix} \dot{x}+\frac{1}{2}l\omega_z-\dot{x}_2 \\ \dot{y}-\frac{\sqrt{3}}{6}l\omega_z-\dot{y}_2 \\ \dot{z}-\frac{1}{2}l\omega_x+\frac{\sqrt{3}}{6}l\omega_y-\dot{z}_2 \end{bmatrix} \tag{22}$$

$$\dot{\boldsymbol{L}}_3 = \begin{bmatrix} \dot{x} - \frac{1}{2} l\omega_z - \dot{x}_3 \\ \dot{y} - \frac{\sqrt{3}}{6} l\omega_z - \dot{y}_3 \\ \dot{z} + \frac{1}{2} l\omega_x + \frac{\sqrt{3}}{6} l\omega_y - \dot{z}_3 \end{bmatrix} \tag{23}$$

5.1.3 Constraints on the Acceleration Level

The following constraints equations on the acceleration level can be obtained:

$$\ddot{\boldsymbol{g}} = \boldsymbol{G}_s(\boldsymbol{y}_p, \boldsymbol{q})\ddot{\boldsymbol{s}}_p + \boldsymbol{G}_q(\boldsymbol{y}_p, \boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{G}}_s(\boldsymbol{y}_p, \boldsymbol{q})\dot{\boldsymbol{s}}_p + \dot{\boldsymbol{G}}_p(\boldsymbol{y}_p, \boldsymbol{q})\dot{\boldsymbol{q}} = \boldsymbol{0} \tag{24}$$

5.2. The dynamic equations

The following the dynamic equations can be obtained:

$$\begin{bmatrix} \frac{x_1 - x_{p1}}{L_1} & \frac{x_2 - x_{p2}}{L_2} & \frac{x_3 - x_{p3}}{L_3} \\ \frac{y_1 - y_{p1}}{L_1} & \frac{y_2 - y_{p2}}{L_2} & \frac{y_3 - y_{p3}}{L_3} \\ \frac{z_1 - z_{p1}}{L_1} & \frac{z_2 - z_{p2}}{L_2} & \frac{z_3 - z_{p3}}{L_3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} m\ddot{x} \\ m\ddot{y} \\ mg + m\ddot{z} \end{bmatrix} \tag{25}$$

$$\begin{bmatrix} \frac{y_1 - y_{p1}}{L_1} & 0 & 0 \\ 0 & \frac{y_2 - y_{p2}}{L_2} & 0 \\ 0 & 0 & \frac{y_3 - y_{p3}}{L_3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \begin{bmatrix} x_{p1} \\ x_{p2} \\ x_{p3} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \frac{z_1 - z_{p1}}{L_1} & 0 & 0 \\ 0 & \frac{z_2 - z_{p2}}{L_2} & 0 \\ 0 & 0 & \frac{z_3 - z_{p3}}{L_3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \begin{bmatrix} y_{p1} \\ y_{p2} \\ y_{p3} \end{bmatrix}^{\mathrm{T}} = J_x \ddot{\varphi}_1 \tag{26}$$

$$\begin{bmatrix} \frac{x_1 - x_{p1}}{L_1} & 0 & 0 \\ 0 & \frac{x_2 - x_{p2}}{L_2} & 0 \\ 0 & 0 & \frac{x_3 - x_{p3}}{L_3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \begin{bmatrix} z_{p1} \\ z_{p2} \\ z_{p3} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \frac{z_1 - z_{p1}}{L_1} & 0 & 0 \\ 0 & \frac{z_2 - z_{p2}}{L_2} & 0 \\ 0 & 0 & \frac{z_3 - z_{p3}}{L_3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \begin{bmatrix} x_{p1} \\ x_{p2} \\ x_{p3} \end{bmatrix}^{\mathrm{T}} = J_y \ddot{\varphi}_2 \tag{27}$$

$$\begin{bmatrix} \frac{x_1 - x_{p1}}{L_1} & 0 & 0 \\ 0 & \frac{x_2 - x_{p2}}{L_2} & 0 \\ 0 & 0 & \frac{x_3 - x_{p3}}{L_3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \begin{bmatrix} y_{p1} \\ y_{p2} \\ y_{p3} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \frac{y_1 - y_{p1}}{L_1} & 0 & 0 \\ 0 & \frac{y_2 - y_{p2}}{L_2} & 0 \\ 0 & 0 & \frac{y_3 - y_{p3}}{L_3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \begin{bmatrix} x_{p1} \\ x_{p2} \\ x_{p3} \end{bmatrix}^{\mathrm{T}} = J_z \ddot{\varphi}_3 \tag{28}$$

5.3. The Three-dimensional System Modeling

The three-dimensional virtual prototype model of towing system is established by using the software UG/ADAMS. Because all single software can't establish the flexible cable model, in this paper, the flexible cable model is established with combining software UG and ADAMS. The relating parameters of the virtual prototype model are shown in table 1.

Table 1

The relating parameters of the virtual prototype model

Parameter	Symbol	Value
The distance between p_i	l	0.5m
Mass of the towing object	m	2000kg
The rotary inertia	J_x	0.54kg.m^2
The rotary inertia	J_y	0.26kg.m^2
The rotary inertia	J_z	0.28kg.m^2
The length of cables	L_i	1m
The size of cylindrical section	$h\times\varphi d$	10mm×φ10mm
The effective length of the root link rod	a_1	310mm
The effective length of the root link rod	a_2	410mm
The effective length of the root link rod	a_3	500mm

In the work process, the cables only provide the unidirectional tension, so the cable model can be established with the thought of "micro-element". Assuming that the length of cable is L_i, it can be divided into k small cylinders, and every length of small cylinders are h, that is $L_i = k\times h$, every small cylinder is called one "micro-element", the value of h is smaller, the more realistic for motion of cable. Finally, the spherical joint is added among small cylinders. The three-dimensional towing model of system is shown in Fig. 3.

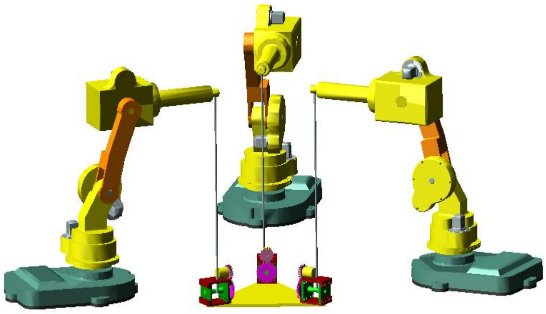

Fig. 3. The three-dimensional towing model of system

5.4. The Numerical Calculation and Simulation Analysis

The analytical expression of the object's position and posture cannot get, because of the strong coupling of the system. There is only the numerical solution of the object's position and posture that can be obtained. Based on this idea using Matlab to program, the numerical solution of the object's position and posture can obtained.

The trajectory of every robot-end is given as table 2. Here t is the motion time. Let the fixed coordinate frame $\{O\}$ and the moving coordinate frame$\{P\}$ have the same original state, that is to say the object's original position and posture in the fixed coordinate frame$\{O\}$ are $[\mathbf{0}^{1\times3}, \mathbf{0}^{1\times3}]^{\mathrm{T}}$ in the original state.

Table2

The trajectory of every robot-end

l	$x_i/(m)$	$y_i/(\mathrm{m})$	$z_i/(\mathrm{m})$
1	$0.012t$	$0.014t$	$0.0155t$
2	$0.013t$	$0.012t$	$0.014t$
3	$-0.017t$	$0.013t$	$0.015t$

The motion of the towing system is simulated in the software ADAMS, The test data of the object's position and posture is imported to Matlab for drawing movement curve, and compared with the numerical calculation results, the comparative results are shown in Fig. 4 to Fig.9.

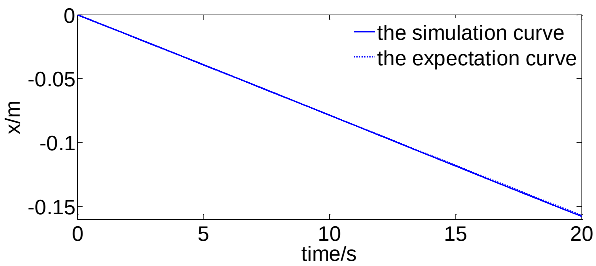

Fig.4. The curves of x

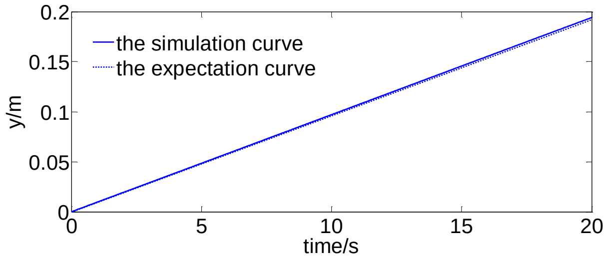

Fig.5. The curves of y

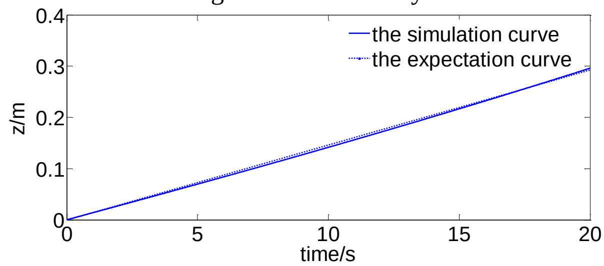

Fig.6. The curves of z

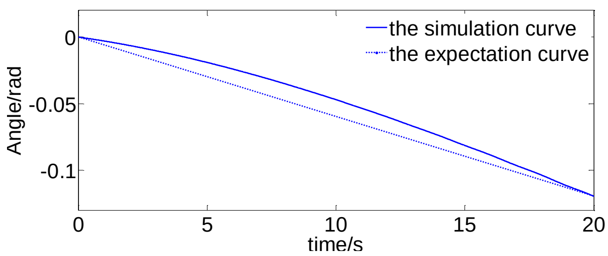

Fig.7. The curves of φ_1

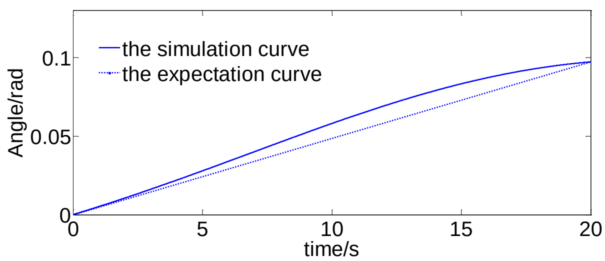

Fig.8. The curves of φ_2

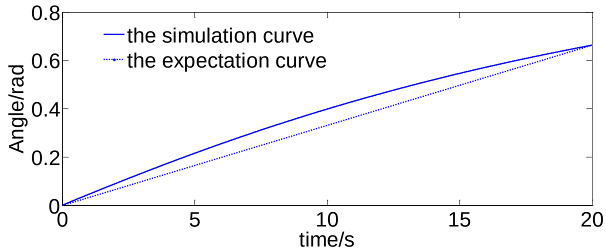

Fig.9. The curves of φ_3

In Fig. 4 to Fig.9, "the expectation curve" is the numerical calculation results.

From Fig. 4 to Fig.9, the simulation results by software ADAMS have a high fit with the numerical calculation results, the fitting is 92%, it shows the proposed kinematics and the dynamics of CTMRTS is right. But they still have errors, the main reasons that cause errors may have the following two points:

(1) The difference between the cable model of simulation and the ideal cable model of the theoretical derivation.

(2) When the velocity of every robot-end is given suddenly, due to the presence of inertia of the towed object, its state of system motion also has inertia.

The physical experiment platform of the towing system for has been set up by using three robots, in order to verify the kinematics and the dynamics of this type system. The further work is shown as following: the data of the object's position and posture will be gotten by position detection tracking device, by comparing the numerical calculation results with the experiment data, it can be determined that the kinematic model and the dynamic model of the towing system is right or not. Then the stability of CTMRTS will be researched in different case, and the comprehensive evaluation for the stability of the system will be obtained through comparing the stability of the system in different case. This is also the key research problem of the project in the further.

6. Conclusion

This paper presents the kinematics and the dynamics of CTMRTS with 6 Degrees of Freedom. In this investigation, the space configuration of CTMRTS is designed. The generalized kinematics of the system is given. Subsequently, the generalized dynamic equation is given by using the Newton - Euler equations. Finally, make the towing system which composed of three robots as example, the position kinematics of the towing system has simulated with the software ADAMS. The simulation results have a high fit with the numerical calculation results. It reveals that the proposed kinematics model is reasonable. In addition, the methods presented in this work could be used as a guide for further research about the stability and the actual control of CTMRTS.

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