ON THE KINEMATICS OF THE SCORBOT ER-VII ROBOT

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Due to the really fast dynamics of the development and diversification of industrial robots and to the ever wider areas of their use, a structural optimization is desired. Using computers with high processing power of the data led to the possibility of modelling a large number of variants of structure to accomplish similar functions.

By modelling the corresponding direct and inverse problems, it is possible to determine positions in terms of the end-effector's timing and that it may impose different travel paths within a set time so that the final effect should achieve precise tasks. The practical aspect can be inferred by analyzing the kinematics of the serial robot SCORBOT ER-VII at the same time determining velocity and acceleration charts for a characteristic point on the end-effector.

Keywords: robot, kinematics model, path planning

1. Introduction

The task of a robot manipulator is to follow a path leading the end-effector towards a stable position.

In order to cause the robot to move according to a path between two or more well-defined target positions, being the required conditions to achieve these positions in a given time, at a certain speed, acceleration and so on, it determines the movement of different components of the robot [1].

The movement it makes can be achieved by several configurations of elements, in which case the robot motion must be studied within the workspace [2], [3].

Kinematics analysis is the study of the movement of robots that make up these mechanisms. Everything related to kinematic analysis, i.e. position, velocity and acceleration of all elements will be calculated in relation to a fixed given reference system. The kinematics analyses do not account for the forces and momentums acting on the robot elements.

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To browse functional trajectories, the kinematic model needs a program to generate these trajectories, which program will be able to accomplish the task either off-line or on-line.

Most robot manipulators are designed to accomplish tasks in a 3D workspace. There are two different approaches on the robot arm movement:

• specifying the location of the end-effector in 3D coordinates;

• the individual movement of each joint separately.

The component, the manipulated tool or the end-effector must follow a planned trajectory.

The kinematic model provides relations between the end-effector's position and orientation and spatial positions of the other elements, the joints.

Kinematics modeling is divided into two problems: direct kinematics and inverse kinematics.

The problem of direct kinematics is to determine the position and orientation of the end-effector giving values for the variables in the robot joints. Inverse kinematics focuses on determining values for the variables in the joints required to move the robot's end-effector in a desired position and orientation.

2. The inverse geometric model

The geometric model for geometric reverse order (inverse geometric model) consists in determining the vector of generalized coordinates (the robot coordinates) $\overline{\Theta} = \overline{\Theta}(q_1, q_2, \ldots, q_k)$ based on the vector of the operational coordinates ${}^0\overline{X} = {}^0\overline{X}(p_x, p_y, p_z, \alpha, \beta, \gamma)$ (the coordinates of the characteristic point P and the orientation angles for the end-effector in relation to the system {0}) [4],[5],[6],[7],[8].

The usual command algorithms are made up of relations which express the movements of the motors in terms of the positioning parameters of the directed body.

As direct geometric modelling is defined by the vectorial expression:

$$ {}^0\overline{X} = f(\overline{\Theta}) \tag{2.1} $$

the inverse geometric modelling (the command geometric modelling) will have the vector expression:

$$ \overline{\Theta} = f^{-1}\left({}^0\overline{X}\right) \tag{2.2} $$

It is said that a robot can be solved for a category of tasks involving a vector type ${}^0\overline{X}$ of the operational coordinates, if, knowing the direct geometric model, ${}^0\overline{X} = \overline{f}(\overline{\Theta})$ we can mathematically obtain a unique solution for the system $\overline{\Theta} = \overline{f}^{-1}\left({}^0\overline{X}\right)$.

In order to achieve the geometric command, the (m) number of the parameters ${}^0X_j \subset {}^0\overline{X}$ should not be larger than the (n) number of the degrees of freedom:

$$ {}^0\overline{X} = \left[{}^0X_j ; j = 1 \to m \right]^T = \begin{cases} \left[p_x \ p_y \ p_z \ \alpha \ \beta \ \gamma \right]^T, & if \ \ n = 6 \\ \left[p_x \ p_y \ p_z \ \alpha \ \beta \right]^T, & if \ \ n = 5 \\ \left[p_x \ p_y \ p_z \ \alpha \right]^T, & if \ \ n = 4 \\ \left[p_x \ p_y \ p_z \right]^T, & if \ \ n = 3 \\ \left[p_x \ p_y \right]^T, & if \ \ n = 2 \end{cases} \quad (2.3) $$

where the first situation shows the general movement of the end-effector and the last case relates to a translation movement within the horizontal plane.

The connection between the column vectors ${}^0\overline{X}$ and $\overline{\Theta}$ is thus achieved by the f operator:

$$ {}^0\overline{X} = \left[{}^0X_j ; j = 1 \to m \right]^T = \left[f_j \left(q_i ; i = 1 \to n \right) ; \ j = 1 \to m ; \ m \le n \right]^T \quad (2.4) $$

Concerning the inverse geometric model, the (2.4) equation system stands for a non linear transcending system of equations for which there is no general calculation algorithm. Under certain conditions, connected to the position and the relative orientation of the neighbouring kinematic axes $\overline{k}_{i-1}, \overline{k}_i$, the (2.4) system can be solved through algebraic methods or through methods belonging to the plane geometry.

Unlike the geometric approach of the inverse geometric model which differs from a problem to another, the algebraic methods are based on the reduction of the transcending equations to algebraic ones using one unknown term and as a consequence, they can be generalized. The (2.2) equation can be written as follows:

$$ \left[q_i ; i = 1 \to n \right]^T = \left[f_i^{-1} \left({}^0X_j ; \ j = 1 \to m \right) ; \ i = 1 \to n \right]^T \quad (2.5) $$

The (2.5) equations express a certain configuration of the robot, which satisfies the known position and orientation of the final effector.

The great hindrance of the (2.1) and (2.2) equation systems is that they are non-linear. As we know, such systems are solved using numerical methods which present unavoidable errors.

The solving methods are divided into two categories:

- closed, algebrical or geometrical methods (applicable on particular cases);
- numerical methods.

Any of the previous methods lead to multiple solutions for the generalized coordinate q_i. The choice of the unique solution depends on the geometry of the mechanical structure of the robot and its interaction with the environment.

3. The kinematic model of the SCORBOT-ER VII robot

The SCORBOT-ER VII robot is a vertically articulated robot having 5 rotation joints [9].

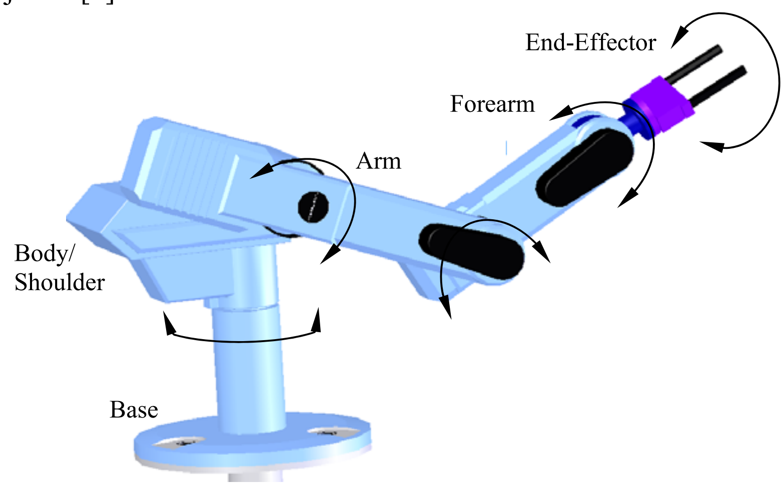


Fig. 1 Elements and joints of the SCORBOT-ER VII robot

In figure 1 both the joints and the elements/components of the robotic system can be identified.

The kinematics chain, in which the chosen reference systems are highlighted too, is presented in figure 2.

Considering the problem of determining a set of variables, respectively $\theta_1, \theta_2, \theta_3, \theta_4$, it is necessary to satisfy the demand for position of the end-effector. The demand for orientation of the end effector and determining the θ_5 variable are neglected.

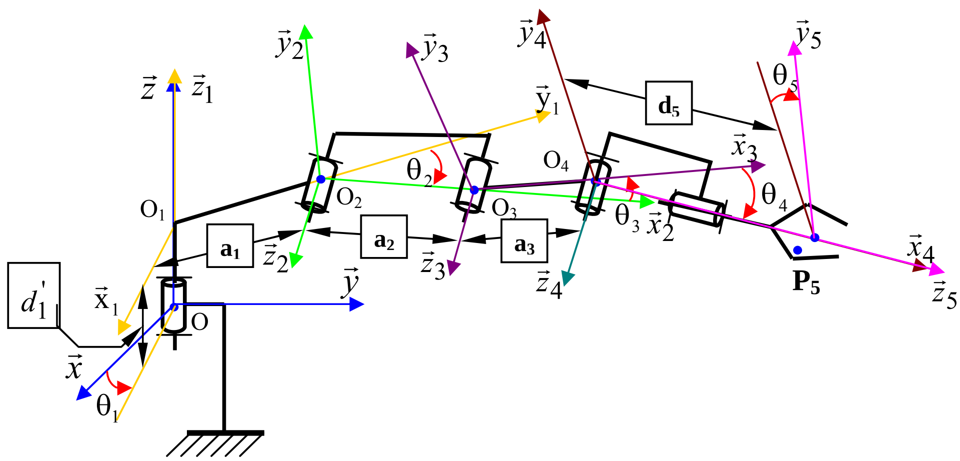


Fig. 2. Choosing the reference systems in the kinematic chain of the SCORBOT -ER VII

As far as a „Pick and Place" operation is concerned, the 4 variables can be determined geometrically, imposing some of the conditions illustrated in figure 3.

One condition could be that the end-effector is **parallel to the base plane**.

The representations in figure 2 and figure 3 are used to level the elements within the robot's configuration which are constant:

$OO_1 = d_1;\ O_1O_2 = a_1;\ O_2O_3 = a_2;\ O_3O_4 = a_3;\ O_4P_5 = d_5$.

The P_5 point has the coordinates expressed in terms of the base frame Oxyz.

Given the coordinates of P_5 $(x_P,\ y_P,\ z_P)$ on element 5 and the constant distances (a_1, a_2, a_3, d_1 and d_5), variables can be determined.

In order to determine the θ_1 variable, the fact that the projection of P_5 in the plane xOy is point C with $(x_P,\ y_P,\ 0)$ coordinates can be used. It results in:

$$\theta_1 = a\tan 2(y_P, x_P) \tag{3.1}$$

The $a\tan 2(y, x)$ function calculates the arc tangent of the two variables x and y. In terms of the standard arctan function, whose range is $(-\pi/2, \pi/2)$, it can be expressed as follows:

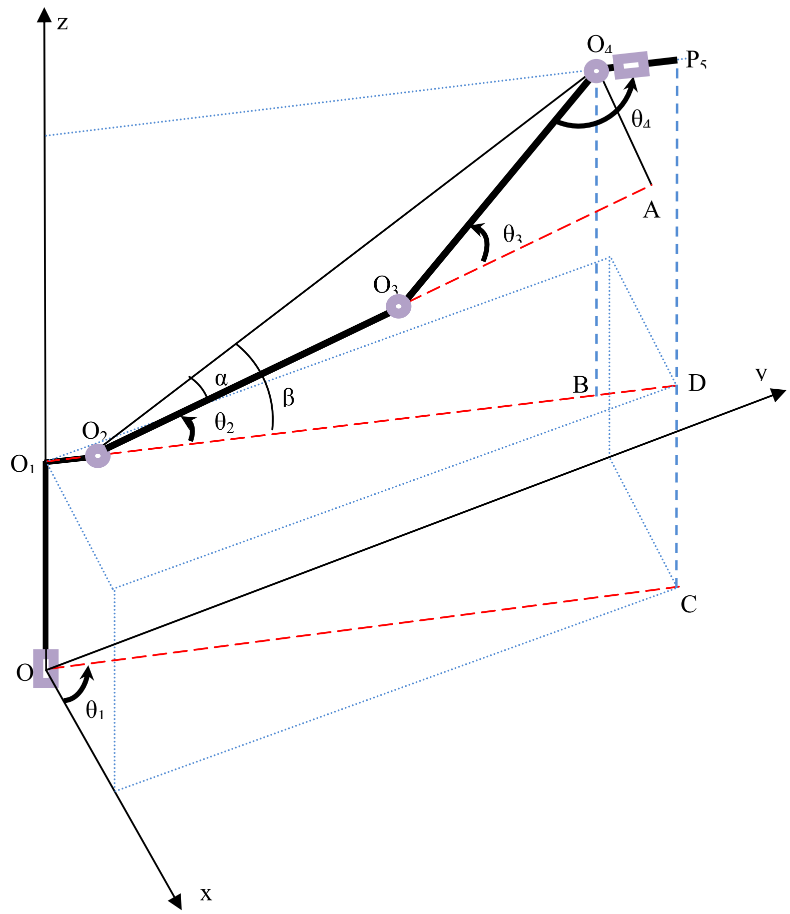


Fig. 3 Simplified representation of the robot's elements within the Oxyz reference system

$$a\tan 2(y,x)=\begin{cases}\arctan\dfrac{y}{x} & x>0\\ \arctan\dfrac{y}{x}+\pi & y\geq 0,\ x<0\\ \arctan\dfrac{y}{x}-\pi & y<0,\ x<0\\ +\dfrac{\pi}{2} & y>0,\ x=0\\ -\dfrac{\pi}{2} & y<0,\ x=0\\ \textit{undefined} & y=0,\ x=0\end{cases}$$

Horizontal distance: $O_2B = O_1D - O_1O_2 - BD = \sqrt{(x_P)^2 + (y_P)^2} - a_1 - d_5$

Vertical distance: $O_4B = DP_5 = P_5C - DC = z_P - d_1$

$$O_2O_4^2 = O_2B^2 + O_4B^2 \tag{3.2}$$

$$\begin{aligned} O_2O_4^2 &= O_2O_3^2 + O_3O_4^2 - 2 \cdot O_2O_3 \cdot O_3O_4 \cdot \cos(\pi - \theta_3) = \\ &= a_2^2 + a_3^2 + 2 \cdot a_2 \cdot a_3 \cdot \cos(\theta_3) \end{aligned} \tag{3.3}$$

From (3.2) and (3.3) results:

$$\theta_3 = a\cos\left(\frac{O_2O_4^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}\right) = a\cos\left(\frac{O_2B^2 + O_4B^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}\right) \tag{3.4}$$

Replacing the horizontal and vertical distances in (3.4), O_2B and O_4B, previously determined, this equation is obtained:

$$\theta_3 = a\cos\left(\frac{\left(\sqrt{(x_P)^2 + (y_P)^2} - a_1 - d_5\right)^2 + (z_P - d_1)^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3}\right) \tag{3.5}$$

Using the previously determined variable, the α angle in the AO_2O_4 and AO_3O_4 triangles is determined.

$$\begin{aligned} \alpha &= a\tan 2(O_4A, O_2A) = a\tan 2(O_3O_4 \cdot \sin(\theta_3),\ O_2O_3 + O_3A) \\ &= a\tan 2(a_3 \cdot \sin(\theta_3),\ a_2 + a_3 \cdot \cos(\theta_3)) \end{aligned} \tag{3.6}$$

The β angle will be determined within the BO_2O_4 triangle using the distances BO_2 and BO_4,

$$\beta = a\tan 2(BO_4, BO_2) = a\tan 2\left(z_P - d_1,\ \sqrt{(x_P)^2 + (y_P)^2} - a_1 - d_5\right) \tag{3.7}$$

The θ_2 variable is determined through the difference between the two angles, α and β:

$$\theta_2 = \beta - \alpha \tag{3.8}$$

$$\begin{aligned} \theta_2 &= a\tan 2\left(z_P - d_1,\ \sqrt{(x_P)^2 + (y_P)^2} - a_1 - d_5\right) - \\ &\quad - a\tan 2(a_3 \cdot \sin(\theta_3),\ a_2 + a_3 \cdot \cos(\theta_3)) \end{aligned} \tag{3.9}$$

The condition of being horizontal of the end-effector is expressed by the relation (3.10), $\theta_2 + \theta_3 + \theta_4 = 0$ (3.10)

From the (3.10) condition and by using (3.9) and (3.5), the value of the θ_4 angle is determined: $\theta_4 = -\theta_2 - \theta_3$ (3.11)

4. Trajectory planning

The purpose of trajectory planning is to generate input data for the motion control system so that the manipulator should execute a specific task under imposed velocity and acceleration conditions [10].

In the case of the previously studied manipulated it is mandatory that the end-effector trajectory should be a segment defined by the starting point M, by the (x_M, y_M, z_M) coordinates and the final point N, by the (x_N, y_N, z_N) coordinates, the above mentioned trajectory being follow by the characteristic point P_5 within the time interval Δt (figure 4).

If we keep the imposed restriction according to which the end-effector cover the trajectory horizontally and we add the restriction according to which the covered segment would be within xOz plane, the manipulator movement will depend on the variation of the θ_2 angle.

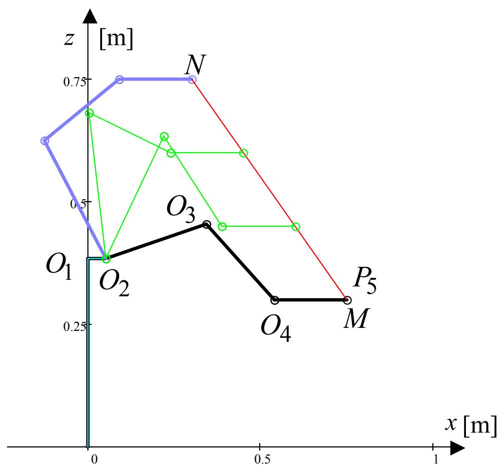


Fig. 5 MN trajectory in initial and final position of the robot manipulator

To a set of constant values representing the construction elements of the manipulator [9] $a_1 = 0.050\,m$, $a_2 = 0.300\,m$, $a_3 = 0.250\,m$, $d_1 = 0.385\,m$, $d_5 = 0.212\,m$, the M(0.750, 0, 0.300), N(0.300, 0, 0.750) trajectory and for a polynomial variation of θ_2, we can obtain velocity and acceleration variations of point P_5.

Using (3.1), (3.5), (3.9), (3.11) equations for the coordinates of points M and N, we can determine the initial and final θ_1, θ_2, θ_3 and θ_4 angles. By using these values (especial the ones for θ_2), we can determine the cubic variation on a $\Delta t = 10$ seconds time interval.

The algebrical expression is graphically represented in figure 5 and takes the form of equation (4.1).

$$\theta_2(t) = 13.4685 + 3.3946 \cdot t^2 - 0.2263 \cdot t^3 \qquad [\text{deg}] \tag{4.1}$$

According to the θ_2 angle we can determine the segment which is parallel to the trajectory as being the intersection between the segment $O_4^i O_4^f$ and the center circle O_3 and the $O_3 O_4^i$ (respectively a_3) radius (figure 6). We should observe that O_3 is mobile according to the arc trajectory having $O_2 O_3$ radius (respectively a_2), with O_2 as its center.

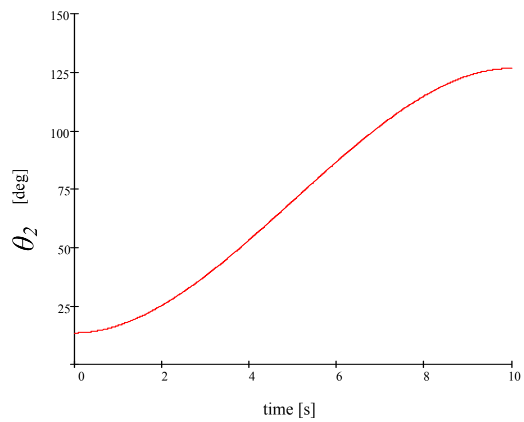


Fig. 6 Cubic variation in terms of time of the θ_2 parameter.

We can achieve the coordinates of the mobile point O_4 by solving the quadratic equation $A \cdot X^2 + B \cdot X + C = 0$ using the following notations:

$$A = m^2 + 1 \tag{4.2}$$

$$B = 2 \cdot \left[m \cdot n - m \cdot z_{O_3}(t) - x_{O_3}(t) \right] \tag{4.3}$$

$$C = \left[x_{O_3}(t) \right]^2 + \left[z_{O_3}(t) \right]^2 - a_3^2 - 2 \cdot n \cdot z_{O_3}(t) + n^2 \tag{4.4}$$

where:

$$x_{O_3}(t) = a_1 + a_2 \cdot \cos(\theta_2(t)) \quad \text{and} \quad z_{O_3}(t) = d_1 + a_2 \cdot \sin(\theta_2(t))$$

$m = \dfrac{z_{O_4^i} - z_{O_4^f}}{x_{O_4^i} - x_{O_4^f}}$, $n = z_{O_4^f} - x_{O_4^f} \cdot \dfrac{z_{O_4^i} - z_{O_4^f}}{x_{O_4^i} - x_{O_4^f}}$, x and z the coordinates of the respective points.

The coordinates thus determined are:

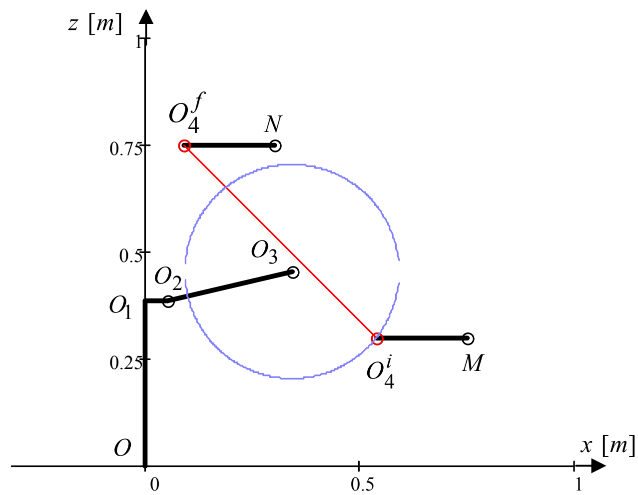


Fig. 7 Intersection between circle and the segment to determine the current point O_4

$$x_{O_4}(t) = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \tag{4.5}$$

$$z_{O_4}(t) = m \cdot x_{O_4}(t) + n \tag{4.6}$$

Using equations (4.5) and (4.6), the coordinates of the point are given by:

$$x_{P_5}(t) = x_{O_4}(t) + d_5 \qquad z_{P_5}(t) = z_{O_4}(t) \tag{4.7}$$

We can obtain velocity and acceleration of P_5, which is attached to the end-effector in figure 7.

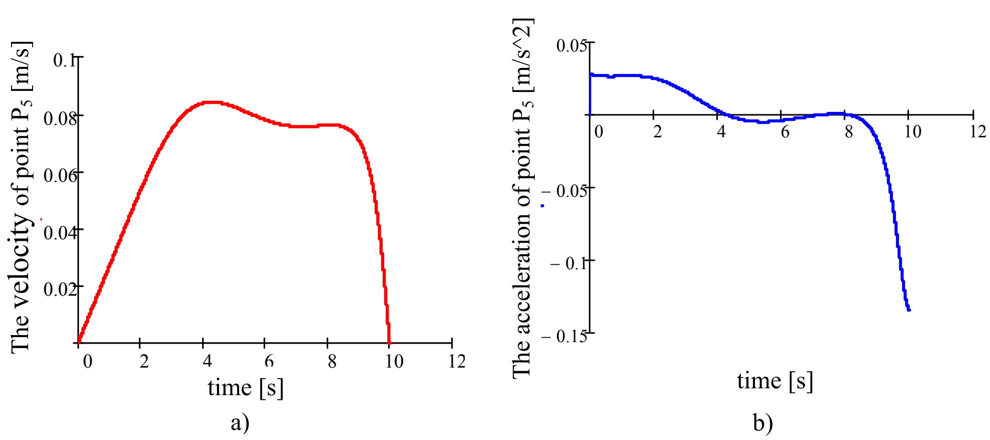


Fig. 8 Velocity and acceleration history of point P_5 for the cubic variation of θ_2 angle

If we refer to a sinusoid variation of the angle which is similar to the one in [11], θ_2 has the structure (4.8) and the shape in figure 8.

$$\theta_2(t)=\left\{\theta_i+A_0\cdot\frac{\omega_0}{2\pi}\left[t-\frac{1}{\omega_0}\sin(\omega_0\cdot t)\right]\right\}\cdot\frac{180}{\pi}\qquad[\text{deg}]\qquad(4.8)$$

For this variation parameter $\theta_i=0.235\ [rad]$, $A_0=1.975\ [rad]$ and $\omega_0=\frac{\pi}{5}\left[s^{-1}\right]$ are considered.

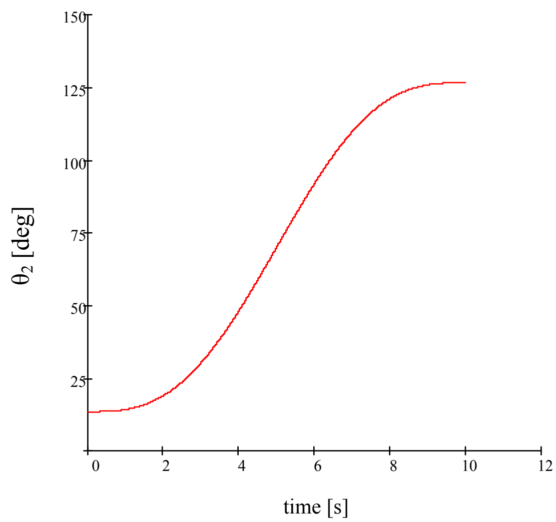
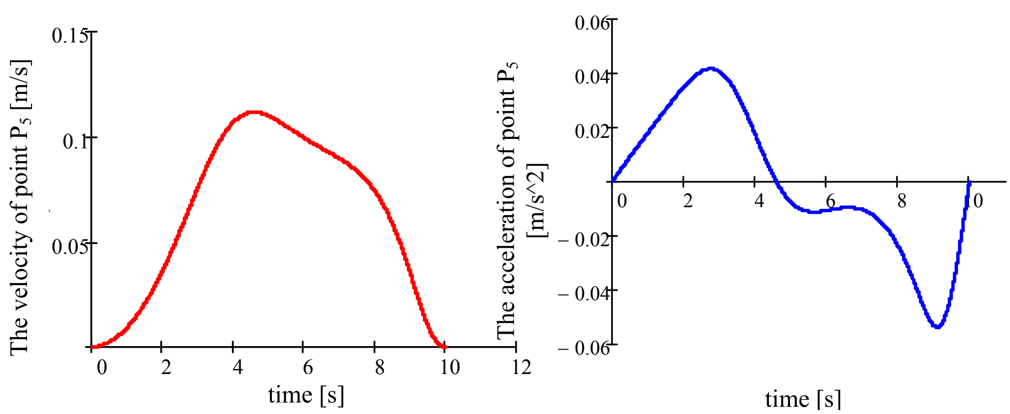


Fig. 9 Sinusoid variation in terms of time of the θ_2 parameter.



a) b)

Fig. 10 Velocity and acceleration history of point P_5 for the sinusoid variation of the θ_2 angle.

Velocity and acceleration history of point P_5 are represented in figure 9 a) and b) and are expressed in $[m/s]$ and $\left[m/s^2\right]$.

5. Conclusions

This paper wishes to illustrate the stages of planning and the motion control of a robot with rotary couplers. In order to achieve complex trajectories, we can plan and control the movement, interpolating trajectories belonging to the straight line on a plane.

The method presented here allows the calculation of the geometrical parameters of the decoupled Scorbot-ERVII robot, analysing separately the position equations and the orientation ones. Even if this procedure simplifies and facilitates the calculation effort, the analytical approach of the kinematic control solutions still remains a complex matter.

The base concept of the proposed approach is constituted by the fact that determining variables involves geometrical modelling of the robotic structure, which leads to multiple solutions, meaning that for a certain positioning, several configurations are obtained in which case it is necessary to intervene in the choice of the variable sets to generate the task.

The comparative study of the graphic representation generates the possibility of an optimal approach to the real work version in terms of the task imposed to the end-effector and its load.

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