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SELF-STRUCTURING OF CELLULAR AND CHANNEL TYPE IN COMPLEX SYSTEM DYNAMICS IN THE FRAMEWORK OF SCALE RELATIVITY THEORY

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In the framework of Scale Relativity Theory, by analyzing dynamics of complex system structural units based on multifractal curves, both Schrödinger and Madelung approaches are functional and complementary. The Madelung selected approach involve synchronous modes through SL(2R) transformation group based on a hidden symmetry. Moreover, coherence domains through Riemann Manifolds embedded with a Poincaré metric based on a parallel transport of direction, in a Levi Civita sense are presented. In this last context, stationary-non-stationary dynamics transition through harmonic mapping from the usual space to the hyperbolic one is manifested as cellular and channel type self-structuring.

Keywords: fractal object, scales space, self-structuring, harmonic mapping, operational procedures

1. Introduction

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Both chaos and self-structuring are accepted as one of the most fundamental properties of any complex system dynamics. Interactions between the structural units of any complex system imply mutual constraints at different scale resolutions, so that the universality of the dynamics laws for any complex system must be reflected in various theoretical models. In such a context, the regular theoretical models are based on the hypothesis that the variables characterizing the complex system dynamics are differentiable. This may not always hold true, such that the validations of the previously described type of models need to be seen as gradual and applicable on restricted domains for differentiability is respected. Since chaos and self-structuring are implying predominantly non-differentiable behaviors in the description of complex system dynamics, it is necessary to explicitly introduce the scale resolution in the dynamic equations. This implies that any variables used to describe any complex system have a dual dependence both on the space-time coordinates and on the scale resolution. For instance, instead of using variables defined by non-differentiable functions, approximations of these complex functions will be used at various scale resolutions, which become functional. Thus, all variables used to describe the complex system dynamics will work as a limit of families of functions. As such, for null scale resolution, they are non-differentiable and for non-null scale resolution, they are differentiable. The previous mathematical procedures imply adequate geometrical structures and a class of models respectively, for which the motion laws are integrated with the scale laws. Such geometrical structures are based on the concept of multifractality and the equivalent theoretical models are based on the Scale Relativity Theory. The Scale Relativity theory can be developed either in the fractal dimension $D_F = 2$ (as in Nottale models) or in an arbitrary and constant dimension (as in Multifractal Theory of Motion). Since in the Scale Relativity Theory, the complex system's structural unit's dynamics can be described by continuous but non-differentiable movement curves (multifractal motion curves), these curves exhibit self-similarity as their main property. In any of the points which are forming the curve, behaviors of holographic type emerge (every part reflects the global system). Therefore, a complex approach suggests that only holographic implementation can offer complete descriptions of the complex system dynamics [1-3].

In the present paper, by assimilating any complex fluid with a mathematic object of fractal type, in the framework of Scale Relativity Theory (SRT) [4, 5] and Multifractal Theory of Motion [6-9], various non-linear behaviors through a fractal hydrodynamic-type description as well as through a fractal Schrodinger-type description, were established. Thus, the fractal hydrodynamic-type description implies holographic implementations of dynamics through velocity fields at non-differentiable scale resolution, via fractal soliton, fractal soliton-kink and fractal minimal vortex. In this definition, various operational procedures can become functional. We can mention the fractal cubics with fractal SL(2R) group invariance through n-phaser coherence of the structural units dynamics of any complex fluid, fractal SL(2R) groups through dynamics synchronization along the complex systems structural units, fractal Riemann manifolds induced by fractal cubics and embedded with a Poincaré metric through apolar transport of cubics (parallel transport of direction, in a Levi Civita sense in a Levi Civita sense, harmonic mapping from the usual space to the hyperbolic one. These procedures become operational so that several possible scenarios towards chaos (fractal periodic doubling scenario) but without fully transitioning into chaos (non-manifest chaos) can be obtained.

In this work, from a multifractal perspective, the nonlinear dynamics of complex systems will be analyzed. In such a context, exploring a hidden symmetry under the form of synchronization groups of complex system structural units lead to the generation of Riemann manifold with hyperbolic type metric via parallel transport of direction. Then, accessing of complex systems nonstationary dynamics are performed thorough harmonic mapping from the usual space to the hyperbolic one.

2. Mathematical Model

2.1. Motion Equation

In what follows, any complex system can be assimilated with a multifractal object. Then, since in the framework of Scale relativity Theory [6-9] the dynamics of complex system structural units are described through multifractal curves, the motion equation becomes (for details see [6-9]):

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i + \frac{1}{4}(dt)^{\left[\frac{2}{f(\alpha)}\right]-1} D^{lk} \partial_l \partial_k \hat{V}^i = 0, \tag{1}$$

where

$$\begin{gathered} \hat{V}^l = V_D^l - V_F^l \\ D^{lk} = d^{lk} - i\hat{d}^{lk} \\ d^{lk} = \lambda_+^l \lambda_+^k - \lambda_-^l \lambda_-^k \\ \hat{d}^{lk} = \lambda_+^l \lambda_+^k + \lambda_-^l \lambda_-^k \end{gathered} \tag{2}$$

$$\partial_t = \frac{\partial}{\partial t}, \partial_l = \frac{\partial}{\partial x^l}, \partial_l \partial_k = \frac{\partial}{\partial x^l}\frac{\partial}{\partial x^k}, i = \sqrt{-1}, l, k = 1,2,3$$

In relations (1) and (2), the meaning of the variables and parameters are given:

- x^l is the multifractal spatial coordinate,

- t is the non-multifractal time having the role of an affine parameter of the motion curves,
- $\hat{V}^l$ is the multifractal complex velocity,
- V_D^l is the differentiable velocity independent on the scale resolution,
- V_F^l is the non-differentiable velocity dependent on the scale resolution,
- dt is the scale resolution,
- $f(\alpha)$ is the singularity spectrum of order α,
- α is the singularity index and is a function of fractal dimension D_f,
- D^{lk} is the constant tensor associated with the differentiable–non-differentiable transition,
- $\lambda_+^l(\lambda_+^k)$ is the constant vector associated with the backward differentiable–non-differentiable dynamic processes,
- $\lambda_-^l(\lambda_-^k)$ is the constant vector associated with the forward differentiable–non-differentiable dynamic processes.

It should be noted that, by using the singularity spectrum, the following patterns in the complex system dynamics can be distinguished:

i) Monofractal patterns. These imply dynamics in homogenous complex systems characterized through a single fractal dimension and having the same scaling properties in any time interval.

ii) Multifractal patterns. These include dynamics in inhomogeneous and anisotropic complex systems characterized simultaneously by a wide variety of fractal dimensions.

Consequently, *f(α)* allows the identification of the universality classes in the dynamics of any complex systems even when the strange attractors associated to these dynamics have different aspects.

The relation (1) reveals that, in the generalized case of complex systems structural units dynamics, irrespective of the fractalization type, in any point of the motion curves, the multifractal inertial, $\partial_t \hat{V}^i$, the multifractal convective, $\hat{V}^l \partial_l \hat{V}^i$, the multifractal dissipative effects, $\frac{1}{4}(dt)^{\left[\frac{2}{f(\alpha)}\right]-1} D^{lk} \partial_l \partial_k \hat{V}^i$, are achieving balance.

2.2 Schrodinger and Madelung approaches in the description of complex systems dynamics

For a large temporal scale resolution with respect to the inverse of the highest Lyapunov exponent [7-9], the class of deterministic trajectories of any complex system structural units can be substituted by the class of virtual trajectories. Then, the concept of definite trajectories is replaced by the one of density of probability. The multifractality is thus expressed by means of multi-stochasticity.

Many modes of multifractalization through stochasticization processes can be used. Among the most employed ones, the Markovian and non-Markovian stochastic processes are found [10-12]. In what follows, in the description of complex system dynamics, only multifractalizations by means of Markovian stochastic processes will be discussed. Consequently, the following constraints become operational [10-12]:

$$\lambda_+^i \lambda_+^l = \lambda_-^i \lambda_-^l = 2\lambda \delta^{il}, \tag{3}$$

where λ is a constant associated to the differentiable-nondifferentiable transitions and δ^{il} is the Kronecker pseudo-tensor. Based on constraints (3), the motion equation (1) becomes:

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i - i\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial_l \partial^l \hat{V}^i = 0. \tag{4}$$

The relations (4) show that in any point of the motion curves, the local multifractal complex acceleration, $\partial_t \hat{V}^i$, the multifractal complex convection, $\hat{V}^l \partial_l \hat{V}^i$, and the multifractal complex dissipation $i\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial_l \partial^l \hat{V}^i$ are in equilibrium.

In what follows, let it be admitted that the motions of the entities belonging to any complex system are irrotational. Then, the multifractal complex velocity fields from (2) take the form:

$$\hat{V}^i = -2i\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial^i \, ln\, \Psi, \tag{5}$$

where

$$\chi = -2i\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \, ln\, \Psi \tag{6}$$

is the multifractal complex scalar potential of the complex velocity fields from (5) and Ψ is the function of states. Further on, substituting (5) in (4) and using the mathematical procedures from [6-9] the motion equation (4) takes the form of the multifractal Schrödinger equation:

$$\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \partial^l \partial_l \Psi + i\lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \partial_t \Psi = 0. \tag{7}$$

Therefore, for the complex velocity fields (5), the dynamics of any complex system structural units are described through Schrödinger type “regimes” at various scale resolutions (Schrödinger’s multifractal description). (7) defines the Schrödinger scenario on the holographic implementation of complex system dynamics.

If it is chosen Ψ of the form (Madelung’s type choice):

$$\Psi = \sqrt{\rho} e^{is}, \tag{8}$$

where $\sqrt{\rho}$ is the amplitude and s is the phase, then the multifractal complex velocity fields (5) take the explicit form:

$$\hat{V}^i = 2\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\partial^i s - i\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\partial^i \ln\rho, \tag{9}$$

From (9), the real multifractal velocity fields result:

$$V_D^i = 2\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\partial^i s \tag{10}$$

$$V_F^i = \lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\partial^i \ln\rho. \tag{11}$$

In (10), V_D^i is the differential velocity field, while in (11), V_F^i is the non-differentiable velocity field.

By means of (9), (10) and (11) and using the mathematical procedures from [6-10], the motion equation (4) reduces to the multifractal Madelung equations:

$$\partial_t V_D^i + V_D^l \partial_l V_D^i = -\partial^i Q \tag{12}$$

$$\partial_t \rho + \partial_l\left(\rho V_D^l\right) = 0, \tag{13}$$

with Q the multifractal specific potential:

$$Q = -2\lambda^2(dt)^{\left[\frac{4}{f(\alpha)}\right]-2}\frac{\partial^l\partial_l\sqrt{\rho}}{\sqrt{\rho}} = -V_F^i V_F^i - \frac{1}{2}\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\partial_l V_F^l. \tag{14}$$

The equation (12) corresponds to the multifractal specific momentum conservation law, while equation (13) corresponds to the multifractal states density conservation law. The multifractal specific potential (14) implies the multifractal specific force:

$$F^i = -\partial^i Q = -2\lambda^2(dt)^{\left[\frac{4}{f(\alpha)}\right]-2}\partial^i\frac{\partial^l\partial_l\sqrt{\rho}}{\sqrt{\rho}}, \tag{15}$$

which is a measure of the multifractality of the motion curves.

Therefore, for the multifractal complex velocity fields (9), the dynamics of any complex system are described through Madelung-type "regimes" at various scale resolution (Madelung's multifractal description). (12)-(14) define the Madelung approach on the holographic implementation for complex system dynamics. In this context, several important consequences can be observed:

i) Any complex system structural units are in a permanent interaction with a multifractal medium through the multifractal specific force (15).

ii) All complex system can be identified with a multifractal fluid, the dynamics of which is described by the multifractal Madelung's equation (see (12) – (14)).

iii) The velocity field V_F^i does not represent the contemporary dynamics. Since V_F^i is missing from (13) this velocity field contributes to the transfer of the multifractal specific momentum and to the multifractal energy focus.

iv) Any analysis of Q should consider the "self" nature of the specific momentum transfer of multifractal type. Then, the conservation of the multifractal energy and the multifractal momentum that ensure the reversibility and the existence of the multifractal eigenstates.

If the multifractal tensor is considered:

$$\hat{\tau}^{il} = 2\lambda^2 (dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \rho \partial^i \partial^l \ln \rho, \tag{16}$$

the equation defining the multifractal forces that derive from the multifractal specific potential Q can be written in the form of a multifractal equilibrium equation:

$$\rho \partial^i Q = \partial_l \hat{\tau}^{il}. \tag{17}$$

Since $\hat{\tau}^{il}$ can be also written in the form:

$$\hat{\tau}^{il} = \eta\left(\partial_l V_F^i + \partial_i V_F^l\right), \tag{18}$$

with

$$\eta = \lambda (dt)^{\left[\frac{2}{f(\alpha)}\right]-1} \rho \tag{19}$$

a multifractal linear constitutive equation for a multifractal "viscous fluid" can be highlighted. In such a context, the coefficient η can be interpreted as a multifractal dynamic viscosity coefficient of the multifractal fluid.

3. Synchronization modes through hidden symmetries

In such a context, let it be admitted that tensors (19) or (21) become fundamental in dynamic processes of any complex system structural units. Then, their characteristic equation is given by the cubic:

$$a_0 X^3 + 3a_1 X^2 + 3a_2 X + a_3 = 0, \qquad a_0, a_1 a_2 a_3 \in \mathbb{R} \tag{20}$$

If (20) has real roots [14,15]:

$$X_1 = \frac{h + \bar{h}k}{1 + k},$$
$$X_2 = \frac{h + \varepsilon \bar{h}k}{1 + \varepsilon k}, \tag{21}$$
$$X_3 = \frac{h + \varepsilon^2 \bar{h}k}{1 + \varepsilon^2 k}$$

with h, $\bar{h}$ the roots of Hessian and $\varepsilon \equiv (-1 + i\sqrt{3})/2$ the cubic root of unity $(i = \sqrt{-1})$, the values of variables h, $\bar{h}$ and k can be "scanned" by a simple transitive group with real parameters. This group can be revealed through Riemann-type spaces associated with the previous cubic. The basis of this approach is the fact that the simply transitive group with real parameters [13-15]:

$$X_l \leftrightarrow \frac{aX_l + b}{cX_l + d}, l = 1,2,3 \;\; a, b, c, d \in R \tag{22}$$

where X_l are the roots of the cubic (20), induces the simply transitive group in the quantities h, $\bar{h}$ and k, whose actions are:

$$h \leftrightarrow \frac{ah + b}{ch + d},$$
$$\bar{h} \leftrightarrow \frac{a\bar{h} + b}{c\bar{h} + d}, \tag{23}$$
$$k \leftrightarrow \frac{c\bar{h} + d}{ch + d}k$$

The structure of this group is of SL(2R) type

$$[B^1, B^2] = B^1,$$
$$[B^2, B^3] = B^3, \tag{24}$$
$$[B^3, B^1] = -2B^2$$

where B^l are the infinitezimal generators of the group:

$$B^1 = \frac{\partial}{\partial h} + \frac{\partial}{\partial \bar{h}}$$
$$B^2 = h\frac{\partial}{\partial h} + \bar{h}\frac{\partial}{\partial \bar{h}} \tag{25}$$
$$B^3 = h^2\frac{\partial}{\partial h} + \bar{h}^2\frac{\partial}{\partial \bar{h}} + (h - \bar{h})k\frac{\partial}{\partial k}$$

and admit the absolute invariant differentials

$$\omega^1 = \frac{dh}{(h-\bar{h})k}$$
$$\omega^2 = -i\left(\frac{dk}{k} - \frac{dh + d\bar{h}}{h-\bar{h}}\right) \tag{26}$$
$$\omega^3 = -\frac{kd\bar{h}}{h-\bar{h}}$$

and the 2-form (the metric):

$$ds^2 = \left(\frac{dk}{k} - \frac{dh + d\bar{h}}{h-\bar{h}}\right)^2 - 4\frac{dhd\bar{h}}{(h-\bar{h})^2} \tag{27}$$

In real terms

$$h = u + iv, \bar{h} = u + iv, k = e^{i\theta} \tag{28}$$

and for

$$\Omega^1 = \omega^2 = d\theta + \frac{du}{v}$$
$$\Omega^2 = \cos\theta\frac{du}{v} + \sin\theta\frac{dv}{v} \tag{29}$$
$$\Omega^3 = -\sin\theta\frac{du}{v} + \cos\theta\frac{dv}{v},$$

the connection with Poincaré representation of the Lobachevsky plane can be obtained. Indeed, the metric is a three-dimensional Lorentz structure:

$$ds^2 = -(\Omega^1)^2 + (\Omega^2)^2 + (\Omega^3)^2 = -\left(d\theta + \frac{du}{v}\right)^2 + \frac{du^2 + dv^2}{v^2} \tag{30}$$

This metric reduces to that of Poincaré in case where $\Omega^1 \equiv 0$ which defines the variable θ as the „angle of parallelism" of the hyperbolic planes (the connection). In fact, recalling that

$$\frac{dk}{k} - \frac{dh + d\bar{h}}{h-\bar{h}} = 0 \leftrightarrow d\theta = -\frac{du}{v} \tag{31}$$

represent the connection form of the hyperbolic plane, the relation (29) then represents the general Bäcklund transformations in that plane [16-19]. In such a conjecture it is noted that, if the cubic is assumed to have distinct roots, the condition (31) is satisfied, if, and only if, the differential forms Ω^1 is null [20].

Therefore, for the metric (27) with restriction (31), the relation becomes:

$$ds^2 = \frac{dh d\bar{h}}{\left(h - \bar{h}\right)^2} = \frac{du^2 + dv^2}{v^2} \tag{32}$$

The parallel transport of the hyperbolic plane actually represents the apolar transport of the cubics (20).

Therefore, the group (23) can be assimilated with a "synchronization" group between the different structural units of the complex system, process in which participates, obviously, the amplitudes of each of them, in the sense that they are correlated, not only their phases. The usual synchronization, manifested through the phase shift of the complex system structural units, is, in this case, only a very particular case.

As a justification of the present theory, we can cite reference papers in the field of plasma plume characterization and laser ablation studies [21-32]. The same results were obtained in the medical field, when investigating by fractal analysis the images obtained with X-rays, on the human brain and on the lungs [33-35]. The advantage of interpreting the pictures is to establish a pixel topology and, depending on the calculation of the fractal dimension and the lacunarity, to determine the diseases that affect these vital organs, as well as their temporal evolution.

4. Self-structuring through harmonic mappings

In the following non-stationary dynamics in complex systems through harmonic map generation will be discussed. Indeed, let it be assumed that the complex system dynamics are described by the variables $\left(Y^j\right)$, for which the following multifractal metric was discovered:

$$h_{ij} dY^i dY^j \tag{33}$$

in an ambient space of multifractal metric:

$$\gamma_{\alpha\beta} dX^\alpha dX^\beta \tag{34}$$

In this situation, the field equations of the complex system dynamics are derived from a variational principle, connected to the multifractal Lagrangian:

$$L = \gamma^{\alpha\beta} h_{ij} \frac{dY^i dY^j}{\partial X^\alpha \partial X^\beta} \tag{35}$$

In the current case, (33) is given by (32) with the constraint (31), the field multifractal variables being h and $\bar{h}$ or, equivalently, the real and imaginary part of h. Therefore, if the variational principle:

$$\delta \int L\sqrt{\gamma} d^3x \tag{36}$$

is accepted as a starting point where $\gamma = \left|\gamma_{\alpha\beta}\right|$, the main purpose of the complex system dynamics research would be to produce multifractal metrics of the multifractal Lobachevski plane (or relate to them). In such a context, the multifractal Euler equations corresponding to the variational principle (36) are:

$$\begin{aligned} (h-\bar{h})\nabla(\nabla \mathrm{h}) &= 2(\nabla \mathrm{h})^2 \\ (z-\bar{h})\nabla(\nabla\bar{h}) &= 2(\nabla\bar{h})^2 \end{aligned} \tag{37}$$

which admits the solution:

$$h = \frac{\cosh\left(\Phi/2\right) - \sinh\left(\Phi/2\right)e^{-i\alpha}}{\cosh\left(\Phi/2\right) + \sinh\left(\Phi/2\right)e^{-i\alpha}}, \alpha \in \mathbb{R} \tag{38}$$

with α real and arbitrary, as long as $\left(\Phi/2\right)$ is the solution of a Laplace-type equation for the free space, such that $\nabla^2\left(\Phi/2\right) = 0$. For a choice of the form $\alpha = 2\Omega t$, in which case a temporal dependency was introduced in the complex system dynamics, (38) becomes:

$$h = \frac{i[e^{2\Phi}\sin(2\Omega t) - \sin(2\Omega t) - 2i\, e^{\Phi}]}{e^{2\Phi}[\cos(2\Omega t) + 1] - \cos(2\Omega t) + 1} \tag{39}$$

In Figs. (1-3) multiple nonlinear behaviors of complex dynamics at scale resolutions in dimensionless coordinates are presented:

i) nonlinear behaviors at a global scale resolution (Figs. 1a, b),

ii) nonlinear behaviors at a differentiable scale resolution (Figs. 2a, b),

iii) nonlinear behaviors at a non-differentiable scale resolution (Figs. 3a, b).

Let it be noted that, whatever the scale resolution, complex system dynamics prove themselves to be reducible to self-structuring patterns. The structures are present in pairs of two large patterns that are intercommunicated in an intermittent way. In the 0-20 range for *Ω* and *t* the resulting structures are communicating with each other via a channel created along the symmetry axis for t ~ 10. This channel is also seen for different (*Ω;t*) coordinates, which is interpreted as an intermittency in the structure bonding.

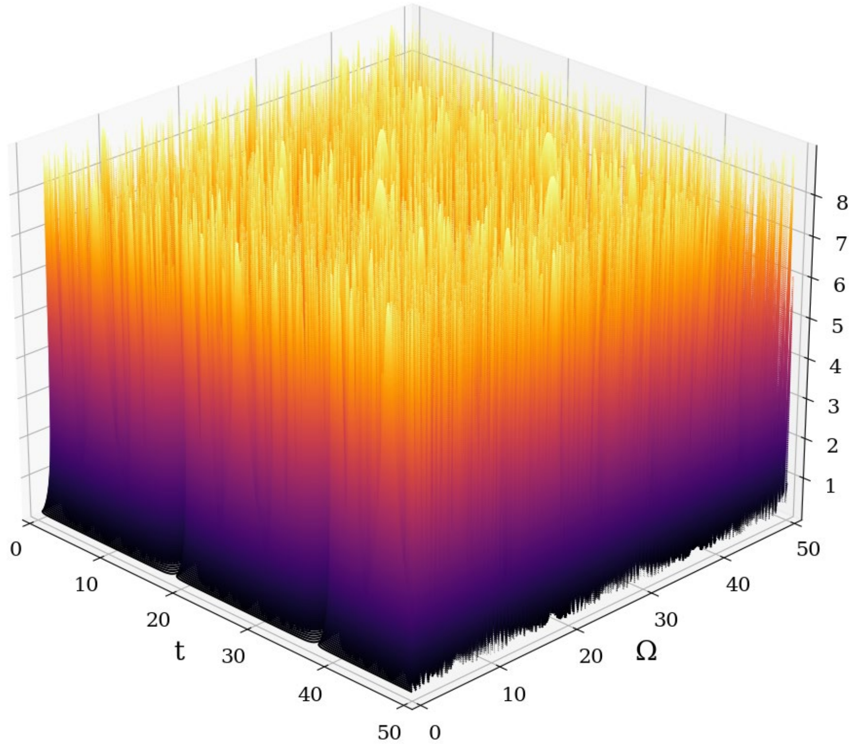

Fig. 1.a. 3D dynamics at global scale resolution of $h(\Omega, t)$ with $\Phi = 3$.

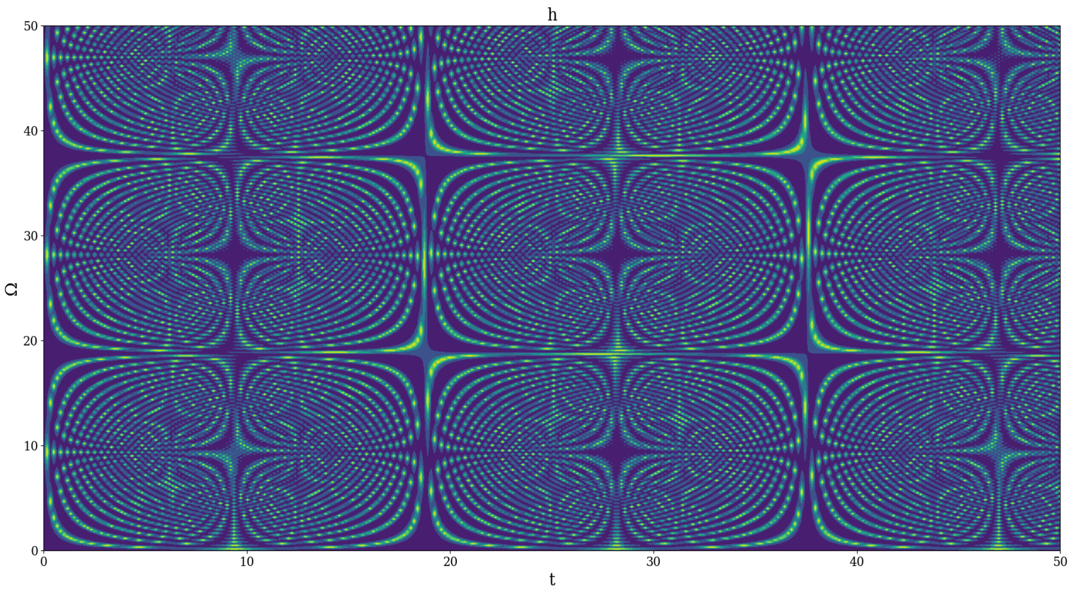

Fig. 1.b. 2D dynamics at global scale resolution of $h(\Omega, t)$ with $\Phi = 3$.

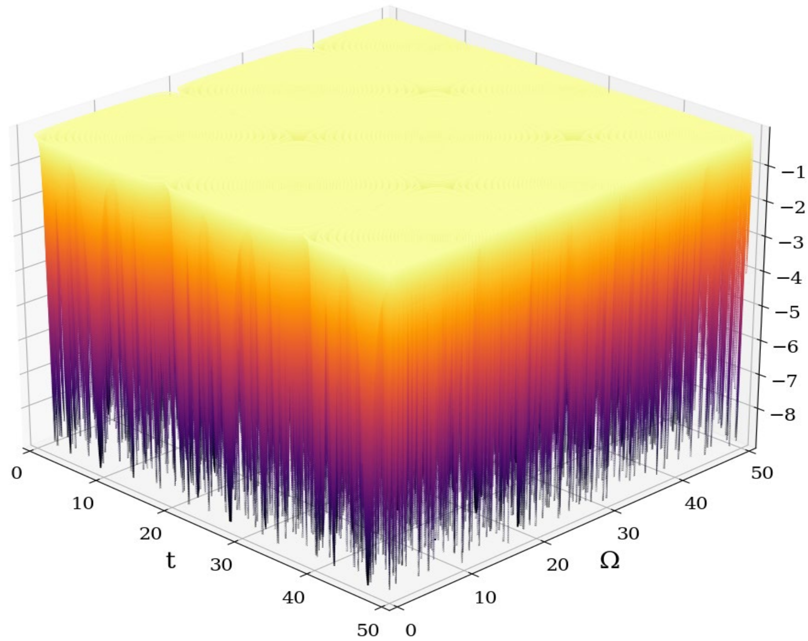

Fig. 2.a. 3D dynamics at differentiable scale resolution of $Re[h(\Omega, t)]$ with $\Phi = 3$.

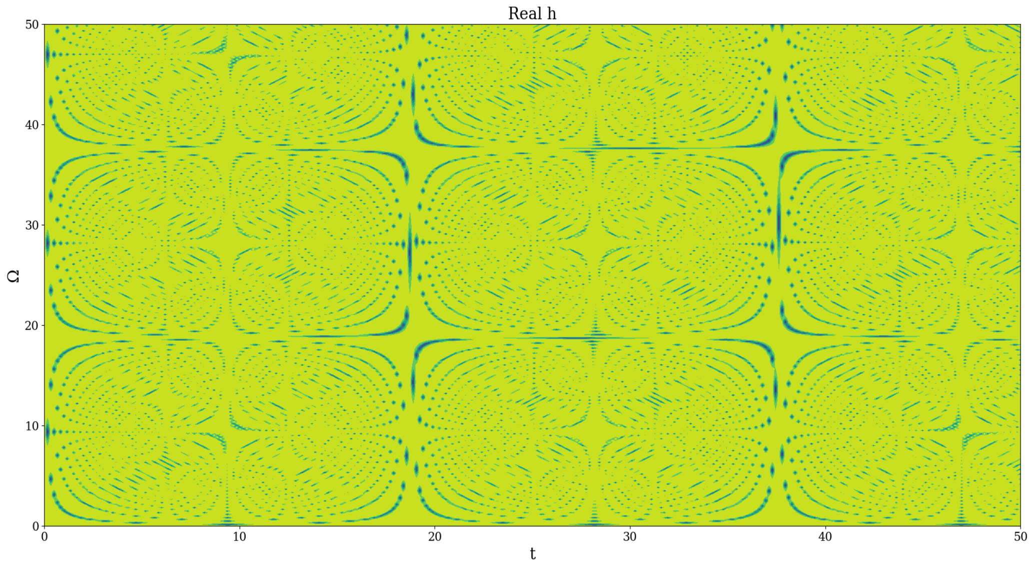

Fig. 2.b. 2D dynamics at differentiable scale resolution of $Re[h(\Omega, t)]$ with $\Phi = 3$

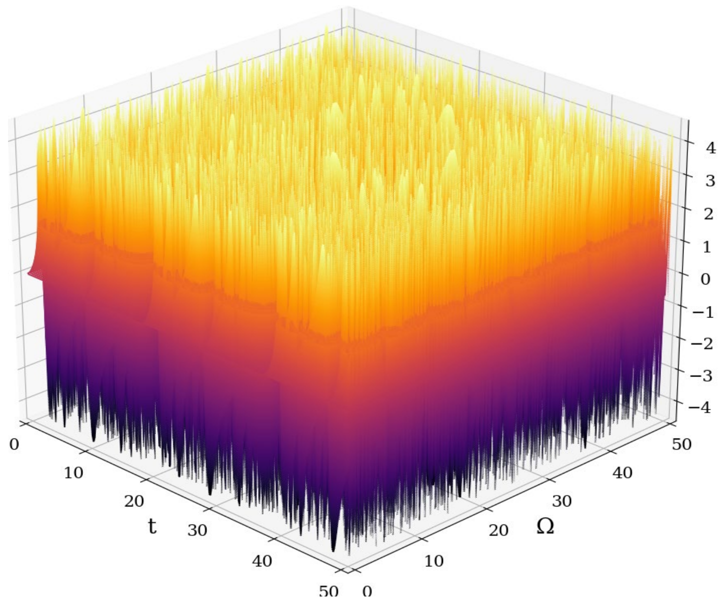

Fig. 3.a. 3D dynamics at non-differentiable scale resolution of $Im[h(\Omega, t)]$ with $\Phi = 3$.

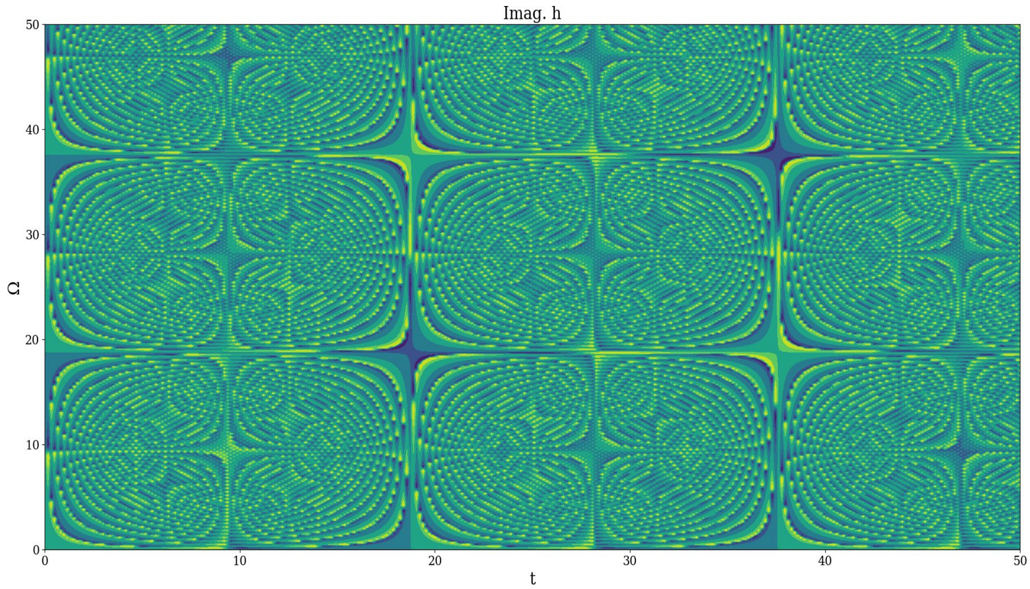

Fig. 3.b. 2D dynamics at non-differentiable scale resolution of $Im[h(\Omega, t)]$ with $\Phi = 3$.

5. Conclusions

The main conclusions of the present paper are the following:

i) By considering that any complex system dynamics can be assimilated with a mathematical object of multifractal type, various non-linear behaviors in the framework of the Multifractal Theory of Motion are developed.

ii) Schrödinger's and Madelung's multifractal description of any complex systems dynamics become operational through the multifractal motion curves.

iii) Exploring at various scale resolutions some hidden symmetries of stationary dynamics on the Madelung description, synchronization modes are seen forming through the SL (2R) group between the complex system structural units.

iv) The space associated to cubics was structured at various scale resolution as Riemann's manifolds (multifractal Riemann's manifold).

v) When a parallel transport of direction in Levi-Civita sense became functional, the metric was reduced to that of Poincare with the angle of parallelism of the hyperbolic plane defining the connections.

vi) In the presented framework, access to non-stationary dynamics at various scale resolutions became possible via harmonic mapping from the usual space to the hyperbolic one. Then self-structuring of cellular and channel types are produced.

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