THERMAL STRESS IN SIMPLY SUPPORTED CIRCULAR PLATES

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The paper discusses the case of simply supported circular plates of various radius lengths 100 mm, 600 mm and 1000 mm, put under the action of a temperature field which varies in the direction of the radius but constant on its thickness. The expressions of the radial and annular stresses and also the radial displacement relation are determined and presented in the charts. The loading is considered below the yield limit value.

Keywords: simply supported, circular plate, thermal stress, radial displacement

1. Introduction

The various types of industrial processes necessitated conceiving and realizing complex equipments and in their structure are included, among other elements also circular plates which can be completely or partially clamped on the edge or they can be simply supported [1 - 8, 12, 13, 14 - 16]. These plates can be heated or cooled down varying according to different heat conveyance laws [9 - 11]. The thermal loading for each case [18] represents a special requirement in the determination of the generated stresses and displacements so that the material use is low. In the same context we have to take into consideration the actions of corrosion and erosion in the working environment.

This study proposes a stationary regime (heating), with different temperature values varying according to different heat conveyance laws, along the radius of the circular plate, having constant thickness, without central orifice and the peripheral outline simply supported.

We determine the radial and annular stresses and then the equivalent stresses which are compared to the allowable stress of this steel; the first one should be lower that the second (the yield limit value should not be exceeded). At the same time we assess the radial displacement of the studied plate, determining the following expression:

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$$u(r)=\frac{\left(1+\nu_p\right)\cdot\alpha_T}{r}\cdot\int_0^r \Delta T(r)\cdot r\cdot dr+C_{1T}\cdot r\,, \tag{1}$$

where α_T is the thermal deformation factor of the plate material, which is constant for the temperature zone $\Delta T(0)\cdots\Delta T\left(r_{cr}\right)$; $\Delta T\left(r\right)$ - the relative temperature, referred to T_0 the environment's temperature; ($\Delta T\left(r\right)=T_e\left(r\right)-T_0$); $T_e\left(r\right)$- the working temperature; C_{1T}- the integration constant, established function of the contour condition; r – the current radius of the plate; ν_p – the transversal contraction coefficient of the plate material (*Poisson*); r_{cr} - the exterior radius of the plate.

For the evaluation of σ_r and σ_θ, the radial and annular stresses we take into consideration the following [17, 20]:

$$\sigma_r=\frac{E_p}{1-\nu_p^2}\cdot\left(\frac{du}{dr}+\nu_p\cdot\frac{u}{r}\right)-\frac{E_p\cdot\alpha_T}{1-\nu_p}\cdot\Delta T\left(r\right); \tag{2}$$

$$\sigma_\theta=\frac{E_p}{1-\nu_p^2}\cdot\left(\frac{u}{r}+\nu_p\cdot\frac{du}{dr}\right)-\frac{E_p\cdot\alpha_T}{1-\nu_p}\cdot\Delta T\left(r\right), \tag{3}$$

where E_p is the longitudinal modulus of elasticity of the plates material.

From the expression of the radial displacement (1) it can be obtained:

$$\frac{u}{r}=\frac{\left(1+\nu_p\right)\cdot\alpha_T}{r^2}\cdot I_1+C_{1T}\,; \tag{4}$$

$$\frac{du}{dr}=-\frac{\left(1+\nu_p\right)\cdot\alpha_T}{r^2}\cdot I_1+\left(1+\nu_p\right)\cdot\alpha_T\cdot\Delta T\left(r\right)+C_{1T}\,, \tag{5}$$

where:

$$I_1=\int_0^r \Delta T\left(r\right)\cdot r\cdot dr\,. \tag{6}$$

After the required processing, it results:

$$\sigma_r=-\frac{E_p\cdot\alpha_T}{r^2}\cdot I_1+\frac{E_p}{1-\nu_p}\cdot C_{1T}\,; \tag{7}$$

$$\sigma_\theta=\frac{E_p\cdot\alpha_T}{r^2}\cdot I_1-E_p\cdot\alpha_T\cdot\Delta T\left(r\right)+\frac{E_p}{1-\nu_p}\cdot C_{1T}\,. \tag{8}$$

In our case the contour of the plate is **simply supported**, for a **null radial stress**, it is obvious that:

$$C_{1T} = \frac{(1-\nu_p)\cdot\alpha_T}{r_{cr}^2}\cdot I_2 , \tag{9}$$

where:
$$I_2 = \int_0^{r_{cr}} \Delta T(r)\cdot r\cdot dr , \tag{10}$$

with r_{cr} - the exterior radius of the plate.

2. Special cases

CASE I. *Stationary thermal field, dependent of the current radius of the plate* [8]

For this heat conveyance law we consider that:

$$\Delta T(r) = \Delta T_{ec} + \left(T_{ep} - T_{ec}\right)\cdot\frac{r}{r_{cr}} ; \tag{11}$$

$$\Delta T_{ec} = T_{ec} - T_0 , \tag{12}$$

where T_{ec} is the working temperature of the centre of the plate, and T_{ep} the existing temperature on the simply supported contour , leading to:

$$C_{1T} = (1-\nu_p)\cdot[\frac{1}{2}\cdot\Delta T_{ec} + \frac{1}{3}\cdot(T_{ep} - T_{ec})]\cdot\alpha_{T,} \tag{13}$$

and also:

$$\sigma_r = K_{11}^I\cdot(T_{ep} - T_{ec}) ; \quad \sigma_\theta = K_{12}^I\cdot(T_{ep} - T_{ec}) , \tag{14}$$

where:

$$K_{11}^I = \frac{1}{3}\cdot\left(1-\frac{r}{r_{cr}}\right)\cdot E_p\cdot\alpha_T ; K_{12}^I = \frac{1}{3}\cdot\left(1-2\cdot\frac{r}{r_{cr}}\right)\cdot E_p\cdot\alpha_T ; \tag{15}$$

In this paper the plates are considered to be made of steel with the following characteristics:

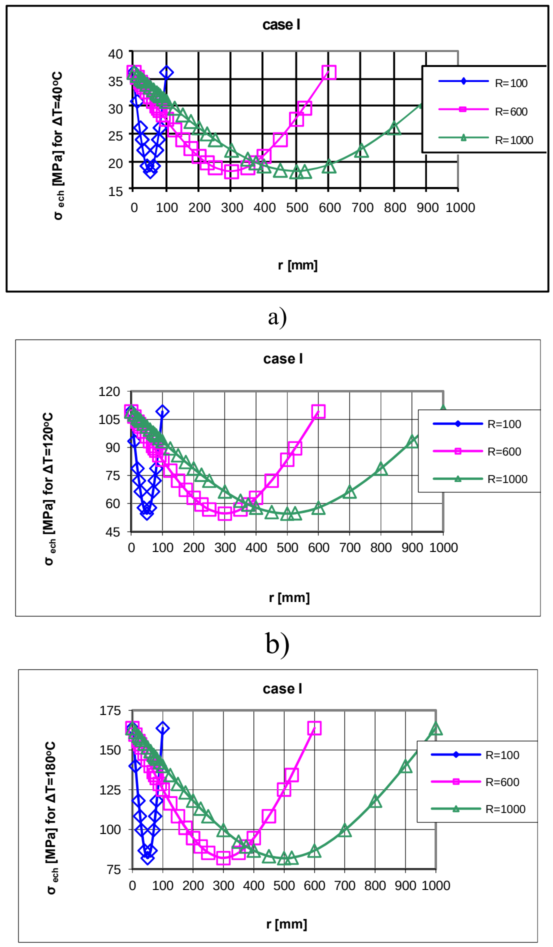
$$E_p = 2.1\cdot 10^5 [N/mm^2], \sigma_c = 309\ [N/mm^2], \sigma_r = 456\ [N/mm^2],$$

$$\alpha_T = 1.2\cdot 10^{-5}[grad^{-1}], \quad \nu_p = 0.3 , \ T_0 = T_{ep} = 20\,^oC , \ T_{ec} = 0\ldots 200\,^oC .$$

Equivalent stress is calculated with the following expression:

$$\sigma_{ech} = \sqrt{\sigma_r^2 + \sigma_\theta^2 - \sigma_r\cdot\sigma_\theta} . \tag{16}$$

With this data we draw out in Excel the charts below that show the variation of the equivalent stress values for circular plates of 100 mm (Fig. 1. a), 600 mm (Fig. 1. b) and 1000 mm (Fig. 1. c) radius length.



a)

b)

c)

Fig. 1. Equivalent stress for plates of 100 mm, 600 mm and 1000 mm radius lenght and heated at: a - $\Delta T = 40\ {}^0C$; b - $\Delta T = 120\ {}^0C$ and c - $\Delta T = 180\ {}^0C$

The expression of the radial displacements is:

$$u\,(r) = k_{11}^{I} \cdot \Delta T_{ec} + k_{12}^{I} \cdot (T_{ep} - T_{ec}), \tag{17}$$

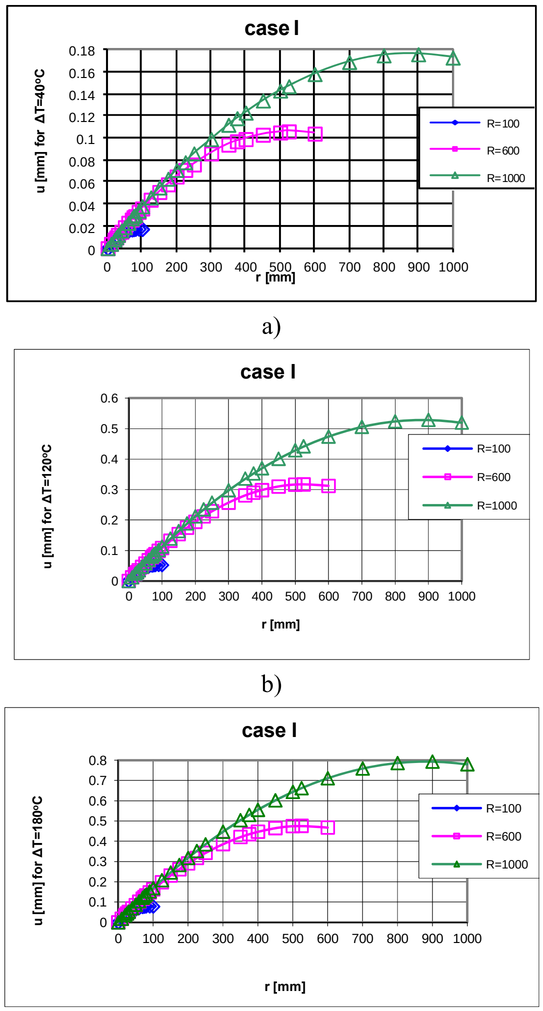
where:

$$k_{11}^{I} = \alpha_T \cdot r;\ k_{12}^{I} = \frac{1}{3} \cdot \left[\left(1 + \nu_p\right) \cdot \frac{r}{r_{cr}} + 1 - \nu_p \right] \cdot \alpha_T \cdot r, \tag{18}$$

it can be seen that the highest value is on the plate's edge.

$$u\left(r_{cr}\right) = \left[\Delta T_{ec} + \frac{2}{3} \cdot \left(T_{ep} - T_{ec} \right) \right] \cdot \alpha_T \cdot r_{cr}\,. \tag{19}$$

Using the same data and radius lengths the charts for the radial displacements are drawn (Fig. 2).



c)

Fig. 2. Radial displacements for plates of 100 mm, 600 mm and 1000 mm radius lenght and heated at: a - $\Delta T = 40\ ^0C$; b - $\Delta T = 120\ ^0C$ and c - $\Delta T = 180\ ^0C$

CASE II. *Central warmed plate. Stationary thermal field, with parabolic variation [17]*

According to the mentioned source, we choose the following heat conveyance law ($T_{e\,c}$ is the working temperature at the centre of the plate):

$$\Delta T(r) = \left(T_{ec} - T_0\right) \cdot \left(1 - \frac{r^2}{r_{cr}^2}\right) = \Delta T_{ec} \cdot \left(1 - \frac{r^2}{r_{cr}^2}\right). \tag{20}$$

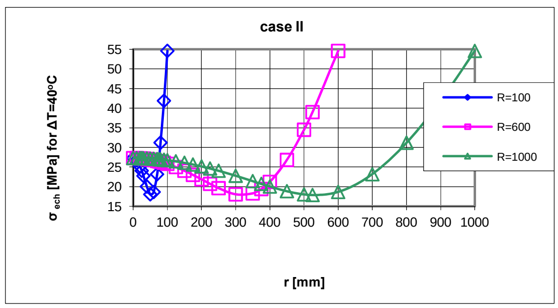
so that:
$$C_{1T} = \frac{1-\nu_p}{4} \cdot \alpha_T \cdot \Delta T_{ec}\ , \tag{21}$$

going forward, using (2) and (3) we determine the stresses:

$$\sigma_r = K_{21}^{II} \cdot \Delta T_{ec}\,;\ \sigma_\theta = K_{22}^{II} \cdot \Delta T_{ec}\,, \tag{22}$$

where:
$$K_{21}^{II} = -\frac{1}{4} \cdot \left(1 - \frac{r^2}{r_{cr}^2}\right) \cdot E_p \cdot \alpha_T; \qquad (23)_1$$

$$K_{22}^{II} = -\frac{1}{4} \cdot \left(1 - 3 \cdot \frac{r^2}{r_{cr}^2}\right) \cdot E_p \cdot \alpha_T. \qquad (23)_2$$



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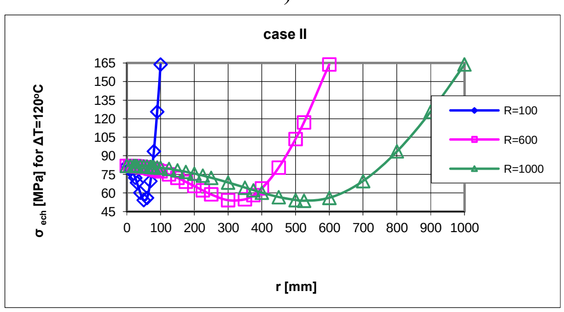

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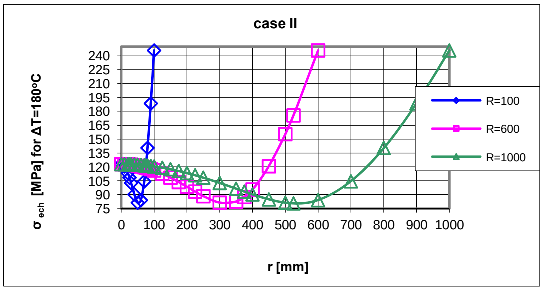

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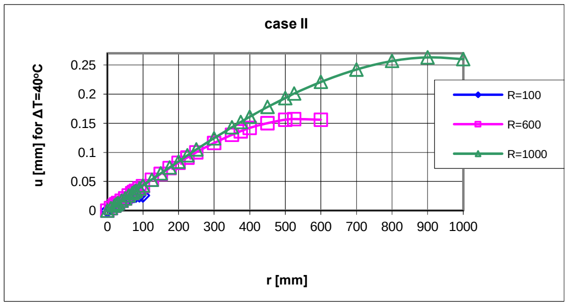
Fig. 3. Equivalent stress for plates of 100 mm, 600 mm and 1000 mm radius lenght and heated at: a - $\Delta T = 40\ ^0C$; b - $\Delta T = 120\ ^0C$ and c - $\Delta T = 180\ ^0C$



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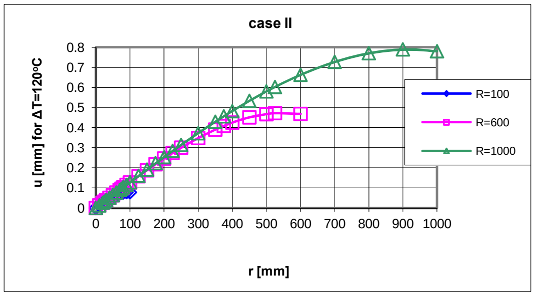

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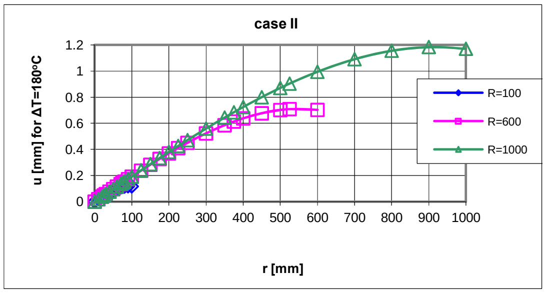

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Fig. 4. Radial displacements for plates of 100 mm, 600 mm and 1000 mm radius lenght and heated at: a - $\Delta T = 40\ {}^0C$; b - $\Delta T = 120\ {}^0C$ and c - $\Delta T = 180\ {}^0C$

The results for the equivalent stress are presented in the chart below (Fig. 3).

The expression of the radial displacements is:

$$u\,(r) = k_{2u}^{II} \cdot \Delta T_{ec}, \tag{24}$$

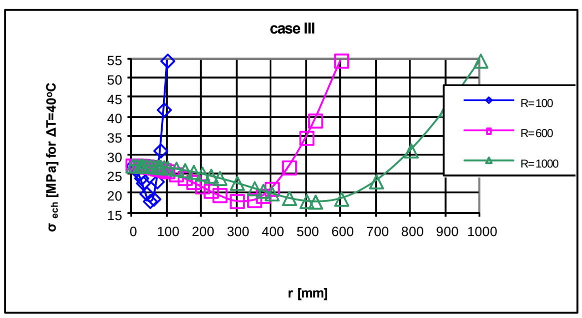
with:

$$k_{2u}^{II} = \left[\frac{1+\nu_p}{2} \cdot \left(1 - \frac{r^2}{2 \cdot r_{cr}^2} \right) + \frac{1-\nu_p}{4} \right] \cdot \alpha_T \cdot r. \tag{25}$$

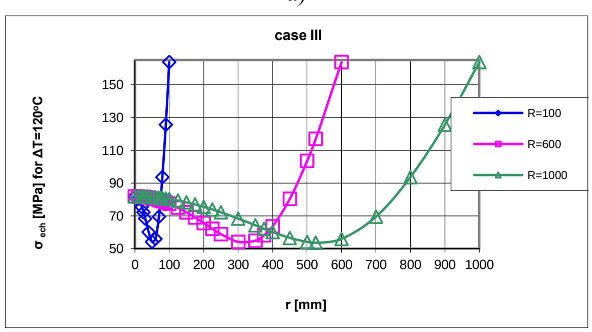
The highest value as seen in Figure 4 is at:

$$u\left(r_{cr} \right) = \frac{1}{2} \cdot \alpha_T \cdot \Delta T_{ec} \cdot r_{cr}. \tag{26}$$

CASE III. *Combined thermal fields (one independent of radius and one with descending parabolic variation from the centre of the plate to its contour)*



a)



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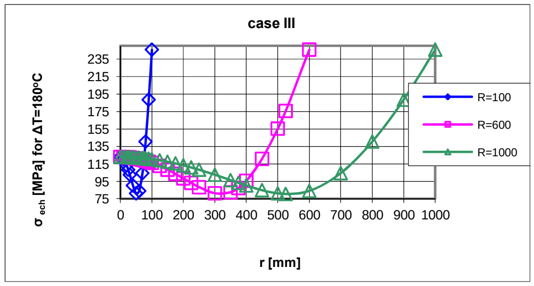


c)

Fig. 5. Equivalent stress for plates of 100 mm, 600 mm and 1000 mm radius lenght and heated at: a - $\Delta T = 40\ ^0C$; b - $\Delta T = 120\ ^0C$ and c - $\Delta T = 180\ ^0C$

This time we consider the thermal gradients having the form:

$$\Delta T(r) = \left(T_{ec} - T_0\right)\cdot\left(1 - \frac{r^2}{r_{cr}^2}\right) + \left(T_{ep} - T_0\right) = \Delta T_{ec}\cdot\left(1 - \frac{r^2}{r_{cr}^2}\right) + \Delta T_{ep}\ , \tag{27}$$

Now we have:

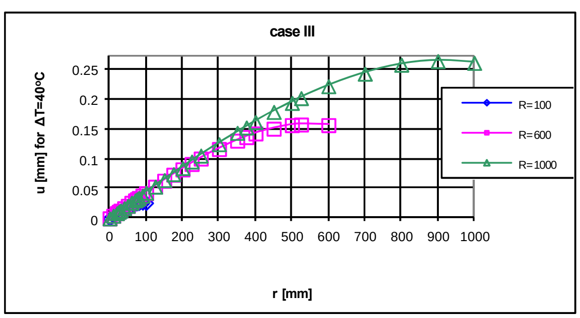
$$C_{1T} = \frac{1-\nu_p}{2}\cdot\left(\frac{1}{2}\cdot\Delta T_{ec} + \Delta T_{ep}\right)\cdot\alpha_T\ ; \tag{28}$$

So in Figure 5 we can see the radial and annular stresses calculated from the expressions below:

$$\sigma_r = K_{31}^{III}\cdot\Delta T_{ec}\ ; \qquad \sigma_\theta = K_{32}^{III}\cdot\Delta T_{ec}\ ; \tag{29}$$

where:

$$K_{31}^{III} = -\frac{1}{4}\cdot\left(1 - \frac{r^2}{r_{cr}^2}\right)\cdot E_p\cdot\alpha_T\ ;\quad K_{32}^{III} = -\frac{1}{4}\cdot\left(1 - 3\cdot\frac{r^2}{r_{cr}^2}\right)\cdot E_p\cdot\alpha_T\ . \tag{30}$$



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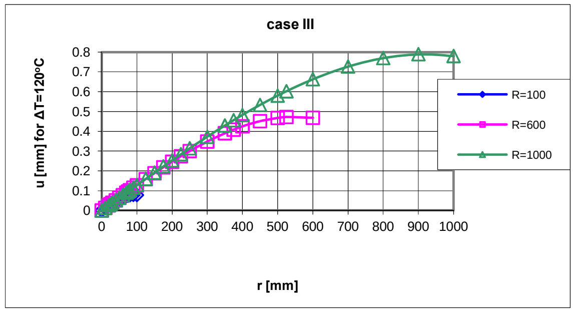

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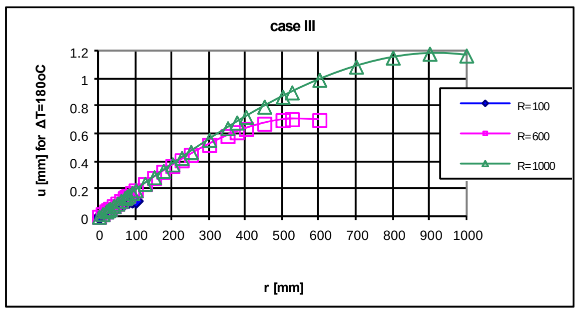

c)

Fig. 6. Radial displacements for plates of 100 mm, 600 mm and 1000 mm radius lenght and heated at: a - $\Delta T = 40\ ^0C$; b - $\Delta T = 120\ ^0C$ and c - $\Delta T = 180\ ^0C$

The expression of the radial displacements is:

$$u\,(r) = k_{31}^{III} \cdot \Delta T_{ec} + k_{32}^{III} \cdot \Delta T_{ep}\,, \tag{31}$$

and the results are in Figure 6:

$$k_{31}^{III} = \frac{1}{4} \cdot \left[\left(3 + \nu_p\right) - \left(1 + \nu_p\right) \cdot \left(\frac{r}{r_{cr}} \right)^2 \right] \cdot \alpha_T \cdot r; \quad k_{32}^{III} = \alpha_T \cdot r\,. \tag{32}$$

It can be seen, in Figure 6, that radial displacement is null at the centre of the plate but on the edge it can be determined with:

$$u\left(r_{cr}\right) = \left(\frac{1}{2} \cdot \Delta T_{ec} + \Delta T_{ep} \right) \cdot \alpha_T \cdot r_{cr}\,. \tag{33}$$

3. Conclusions

In this paper we take into consideration three heat conveyance laws applied along the radius of the circular plate to determine the equivalent stress values and the radial displacements in the given work conditions. At the same time, for this analysis we chose circular plates of different radius lengths, all of them being simply supported on their contour.

Knowing all this the equivalent stress is characterized by:

- Case I - the minimum point is positioned at the middle of the plate's radius and we observe equal values for the stress at the centre and edge of the plate (Fig. 1);
- Case II - the minimum point is also at the middle of the plate's radius, but the maximum value of the stress is situated on the leaned edge of the plate (Fig. 3);
- Case III – is similar with Case II (Fig. 5).

In all three cases we can observe a corresponding variation of the radial displacement values, growing from the centre to the edge of the plate (Fig. 2, 4,6). At the same time we have to point out that there are differences in the values recorded at the same radius for the three plates.

We can now draw a final observation which says that for every heat conveyance law a careful analysis of the stress resulted is needed so that the maximum equivalent stress is lower than the admissible resistance value, the yield limit value of the material. In some practical situations it is very important, as well, to know the value of the maximum radial displacement developed in given work conditions.

R E F E R E N C E S

[1] *R. I., Iatan*, Metodă generală de calcul al unei îmbinări de tip placă circulară–înveliş cilindric (I), Simpozionul Național de Mecanica Ruperii, 19- 20 octombrie, 2000, Midia, pp. 61

[2] *R. I., Iatan*, Metodă generală de calcul al unei îmbinări de tip placă plană-înveliş cilindric (II), Buletinul Universității Petrol-Gaze din Ploieşti, LII, Seria Tehnică, nr. 2, 2000, pp. 171

[3] *V. V., Jinescu,* Calculul şi construcția utilajului chimic, petrochimic şi de rafinării, 1, Editura Didactică şi Pedagogică, Bucureşti, 1983

[4] *V. V., Jinescu*, Utilaj tehnologic pentru industrii de proces, 1 – 4, Editura Tehnică, Bucureşti, 1983

[5] *Al., Pavel*, Elemente de inginerie mecanică, Editura Didactică şi Pedagogică, Bucureşti, 1981

[6] *S. D., Ponomariov, ş. a.,* Calculul de rezistență în construcția de maşini (traducere din limba rusă), Editura Tehnică, Bucureşti, 1963

[7] *T. G., Lim, M. C., Wang,* Bending of annular sectorial Mindlin Plates using Kirchhoff results, European Journal Mechanical. A/Solids, 19, 2000, pp. 1041

[8] *R. I, Iatan, Carmen., Popa*, Thermal Stress in Monolayer Plane and Circular Plates. Variable Temperature in the Direction of Radius, Rev. Chim. (Bucuresti), 60, nr. 5, 2009, pp. 513

[9] *V. V., Jinescu, Mihaela, Păunescu*, Calculul utilajului chimic la funcționarea în regim tranzitoriu, Rev. Chim. (Bucureşti), 26, nr. 7, 1975, pp. 582

[10] *V. V., Jinescu*, Durata regimului tranzitoriu al utilajului chimic, Rev. Chim. (Bucureşti), 26, nr. 9, 1975, pp. 751

[11] *V. V., Jinescu, Mihaela, Păunescu,* Efectul solicitării la oboseală asupra duratei regimului tranzitoriu, Rev. Chim. (Bucureşti), 28, nr. 3, 1977, pp. 244

[12] *I., Marțian,* Plăci plane circulare cu deformații mari supuse acțiunii unei variații de temperatură, Studii şi Cercetări de Mecanică Aplicată, 46, nr. 4, 1987, pp. 331

[13] *N. J., Reddy, ş. a.,* Axisymmetric Bending of Functionally Graded Circular and Annular Plates, European Journal Mechanical. A/ Solids, 18, 1999, pp. 185

[14] *L. S., Lee,* Integration Scheme for Elastic Deformation and Stresses, Journal of Applied Mechanics, 66, nr. 12, 1999, pp. 978
[15] *G., Monegato,* On the Existence of a Solution for a Solid Circular Plate Bilaterally Supported Along Two Antipodal Boundary Arcs and Loaded by a Central Transverse Concentrated Force, Journal of Applied Mechanics, 68, nr. 9, 2001, pp. 809
[16] *M., Sheplak, J., Dugundji*, Large Deffections of Clamped Circular Plates under Initial Tension and Transitions to Membrane Behavior, Journal of Applied Mechanics, 65, nr. 3, 1998, pp. 107
[17] *Luminița, Bibire*, Particularități ale comportamentului materialelor din compunerea unor utilaje solicitate la oboseală termică, Simpozionul Național de Mecanica Ruperii, 19-20 Octombrie 2000, Midia, pp. 161- 165
[18] *T., Pană, D. T., Pastramă,* Integritatea structurilor mecanice, Editura Fair Partners, Bucureşti, 2000
[19] *N. I., Constantinescu, T., Tacu,* Calcule de rezistență pentru utilaje tehnologice, Editura Tehnică, Bucureşti,1979
[20] *R. I, Iatan, Carmen, POPA,* Solitări termo-mecanice în plăci circulare netede, Editura MATRIX ROM, Bucureşti, 2010